PHY 905 Section 6
Accelerator Physics
Spring 2020
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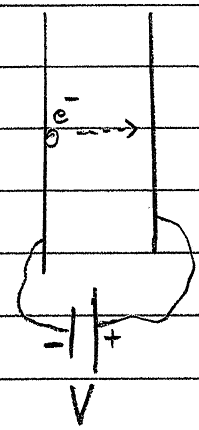
Problem Set #1
Tuesday, Jan 7, 2020 due Tuesday, Jan 14

Problem #1 20 pts

Particle Velocities.

5 pts a) An electron is accelerated between the plates of a parallel plate capacitor biased to potential V starting from rest at the minus electrode.



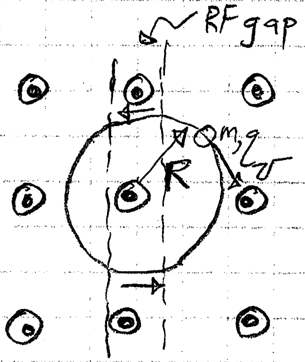
Give formulas for the final speed v of the electron in the nonrelativistic approximation and relativistically.

5 pts b) Plug numbers in a) for $V = 10\,V$ and $V = 10\,kV$ for both the nonrelativistic and relativistic formulas. Express answers in m/sec and $\beta = v/c$ units.

5 pts c) Protons in medical accelerators have kinetic energy ~100 MeV. Estimate γ, β, and the speed in m/sec of the protons.

5 pts d) In heavy-ion accelerators ion kinetic energies are typically measured in MeV/neucleon. Show how this approximately fixes the particle axial β. Explain why this is desireable in resonant RF accelerators.

Problem #2 — 20 pts — Cyclotron - single particle.



$B = \text{const}$ uniform and $\perp$ to particle orbit

m = mass particle

q = charge particle

R = radius particle orbit

v = speed particle

$\mathcal{E}$ = particle kinetic energy

A/ Neglecting the change in orbit radius R over one transit, follow steps in class to show that the nonrelativistic period τ of the orbit is

$$\tau = \frac{2\pi}{\omega_c} \qquad \omega_c = \frac{qB}{m} = \text{cyclotron freq} = \text{const}$$

Discuss implications for a particle entering an oscillating RF field gap each transit. Will the frequency of the RF field need to change?

B/ If the particle speed $v \simeq \text{const}$ over one lap, show relativistically that the

$$\frac{d\vec{p}}{dt} = -m\gamma \frac{v^2}{R}\hat{r} \qquad \vec{p} = \gamma m \vec{v}$$

$$\gamma = (1 - \vec{v}^2/c^2)^{-1/2} \simeq \text{const.}$$

C/ Derive a relativistically correct formula for τ to characterize how it varies with particle kinetic energy

$$\mathcal{E} = (\gamma - 1)mc^2$$

Discuss implications on this result for operation of a cylotron as the particle becomes relativistic.

25 pts

Problem #3 Ion Diode: Child-Langmuir Current Density

Consider a hot-plate type ion diode of voltage V_0 and gap length d. Let the current density J be composed of two species

species 1: mass m_1 charge q
species 2: mass m_2 charge q

Set

$$J = J_1 + J_2 \quad \text{with} \quad J_1 = \alpha J$$
$$J_2 = (1-\alpha) J$$

What is the "effective mass" m_{eff} in terms of m_1 and m_2 that should be used in the resulting Child-Langmuir Law:

$$J = \frac{4}{9} \varepsilon_0 \left(\frac{2q}{m_{eff}} \right)^{1/2} \frac{V_0^{3/2}}{d^2}$$