Physics 905

Accelerator Physics

Problem Set # 11

Tues. Mar 31, 2020 due Tues. April 7, 2020

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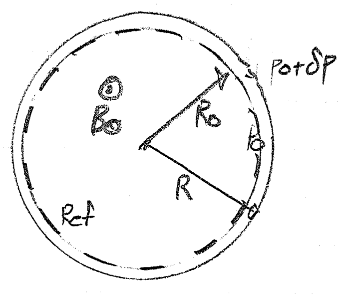
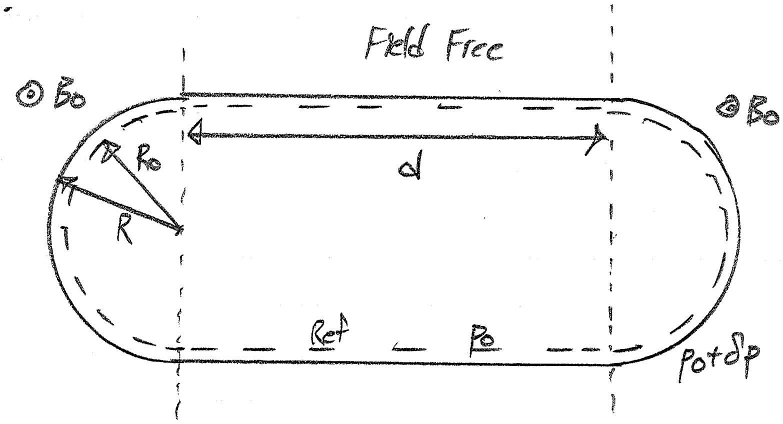
Problem #1 : Slip Factor 20 pts

5 pts
a) Consider a circular accelerator / storage ring composed of a uniform magnetic field $B_y = B_0\hat{y}$ = const. The ideal ref path for particle of momentum p_0 has radius R_0 in the plane $\perp$ to B_y. A particle with momentum $p = p_0 + \delta p$ will have a different closed path and radius R.



Calculate the slip factor η in terms of γ for this situation.

5 pts b) Next, repeat a) for a racetrack accelerator with two uniform dipole bends seperated by a field free drift of distance d. Calculate the slip factor in terms of γ and d/R_0.



10 pts c) For $d = 2R_0$ in c), plot η as a function of γ and note where it changes sign. Is this the "transition gamma"? What speed in $\beta = v/c$ does this correspond to?

30 pts.

Problem #2 Pillbox Cavity

Consider a pillbox cavity or radius $r_c = \frac{75}{2}$ cm and axial length $l = 50$ cm.

2pts a) What is the resonant freq. of the fundamental TM_{010} mode?

2pts b) What is the resonant freq. of the next highest TM_{011} mode?

6pts c) What value of β will have a transit-time-factor of $T = 1/2$ for this cavity, operating at the fundamental frequency? Use the single-gap transit-time-factor derived in class. Feel free to use a numerical root finder. β should be greater than this value for $T > 1/2$.

5pts d) Explain how the cavity might be modified to increase the acceleration efficiency (larger T for given β greater than value found in c). Qualitative only.

15pts e) For the cavity operating at the fundamental
Assume an RF voltage $V_0 = E_0 l = 500$ kV and CU with a conductivity of $\frac{1}{\sigma} = 1.7 \times 10^{-8}\ \Omega \cdot m$, calculate:

U = stored EM energy

R_{surf} = RF surface resistance

$\langle P_{loss} \rangle_t$ = Avg. power loss.

Q = Quality factor.

R_s = shunt impedance

Use formulas derived in the class notes.

Problem #3 Coaxial Half-Wave Resonator   70 pts

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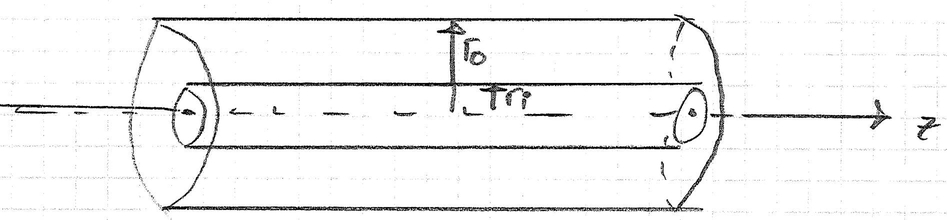
Consider a coaxial transmission line with inner conductor radius r_i and outer radius r_o:



5 pts
a) Between the conductors an EM wave satisfies:

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E} = 0$$
$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{B} = 0$$

Examine the equation for $\vec{E}$ and show that an EM wave with a radial electric field in traveling wave form

$$E_r(r,\theta,z,t) = E_r(r)\, e^{\pm ikz + i\omega t}$$

$i = \sqrt{-1}$
$\omega = \text{const}$ Angular Freq.
$k = \text{const}$ Axial Wavenumber

is supported. Show that $k = \pm\frac{\omega}{c}$ and $E_r = \text{const}/r$ satisfies the wave equation, i.e.

$$E_r(r,\theta,z,t) = \frac{C_1}{r}\, e^{\pm i\frac{\omega}{c}z + i\omega t} \qquad C_1 = \text{const.}$$

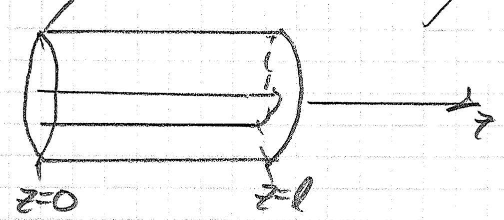
Does this satisfy needed boundary conditions at $r = r_i, r_o$ if $\vec{E} = E_r\hat{r}$ if the inner and outer conductors are perfectly conducting?

2 pts
b) A forward and backward traveling wave E_r can be superimposed in a cavity to obtain:

$$E_r(r,\theta,z,t) = \frac{-V}{r}\sin\left(\frac{\omega z}{c}\right)\cos(\omega t + \phi)$$

$V = \text{const}$ RF Amplitude parameter.
$\phi = \text{const}$ RF phase

For a cavity with axial length l and coordinates:

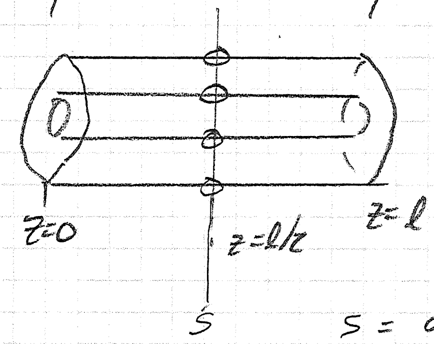


Show that $\omega = \frac{p\pi c}{l}$, $p = 1, 2, 3, \ldots$ to meet $\vec{E}$ boundary conditions.

10 pts. c) Find a corresponding form of $\vec{B} = B_\theta \hat{\theta} = B_\theta(r,z,t)\hat{\theta}$ consistent with the EM standing wave $E_r(r,\theta,z,t)$ in part b) and show it meets needed cavity boundary conditions and satisfies all Maxwell equations between conductors.

3 pts. d) For a cavity $l = 1\,m$ long what is the lowest allowed resonant frequency $f_{rf} = \omega/(2\pi)$. How many RF wavelengths long is the structure? Is there any dependance of the resonant frequency on r_i and r_o?

5 pts. e) Imagine the cavity has an apeture hole cut through at $z = l/2$, and is operated in the $p=1$ mode.



Assume apeture small and does not perturb cavity field.

s = axial accelerator coordinate.

Sketch what the axial electric field E_s should look like at time $\omega t + \phi = 0$. Let $s=0$ correspond to $r=0$.

How should the RF phase advance be timed so that the particle will gain energy in each half ($s=-r_o \to s=-r_i$ and $s=r_i \to s=r_o$) of the RF structure? Justify your answer.

5 pts. f) For $P=1$, will a particle traversing the gaps be significantly deflected by the magnetic field B_θ for any RF phase? Justify your answer.

10 pts. g) How would you define a transit-time-factor for the two-gap cavity. Give a workable expression for

$$\Delta W = q E_0 L T \cos\phi \qquad \text{Panofsky Eqn.}$$

Define E_0, L, and T to make this work.

No need to simplify answer for T and take:

$$E_0 \equiv \int_{\text{gaps},\ \omega t+\phi=0} |E_s|\, ds = 2V \ln(r_o/r_i)$$

to account for the sign change between gaps.

5pts

h) Would the two gap transit time factor defined in problem set #6, problem #3 calculated for $\beta \approx$ const be a good approximation for this cavity if the gaps are tuned to be the same length? Why? Qualitative answer only.

10pts

i) Calculate the stored RF energy U of this cavity for arbitrary p. Show that:

$$U = \frac{\pi \varepsilon_0}{2} V^2 l \ln(r_o/r_i)$$

Hint: parallel steps applied to the pillbox cavity in the class notes.

10pts

j) Calculate the average RF power loss $\langle P_{loss} \rangle_{rf}$ of this cavity for arbitrary p. Show that

$$\langle P_{loss} \rangle_{rf} = \frac{\pi}{2} \frac{V^2 \varepsilon_0 R_{surf}}{\mu_0} \left[l \left(\frac{1}{r_p} + \frac{1}{r_o} \right) + 4 \ln(r_o/r_i) \right]$$

R_{surf} = surface Resistance

Hints: parallel steps applied to the pillbox cavity in the class notes. Do not forget end contributions at $z = 0, l$.

If we take $\vec{H}$ in complex, harmonic form,

$$\langle P_{loss} \rangle_{rf} = \frac{R_{surf}}{2} \int_{\text{Cavity Surface}} |H_{tangent}|^2 d^2x$$

$$H = H(r,z) e^{i\omega t} \quad \text{here.}$$

5pts

k) Calculate the cavity Q, using the results of i) and j) and the definition of Q given in class. Show that

$$Q = \frac{p\pi}{R_{surf}} \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{\ln(r_o/r_i)}{l\left(\frac{1}{r_i} + \frac{1}{r_o}\right) + 4 \ln(r_o/r_i)}$$