USPAS Accelerator Physics
Problem Set 1 - 85 pts.

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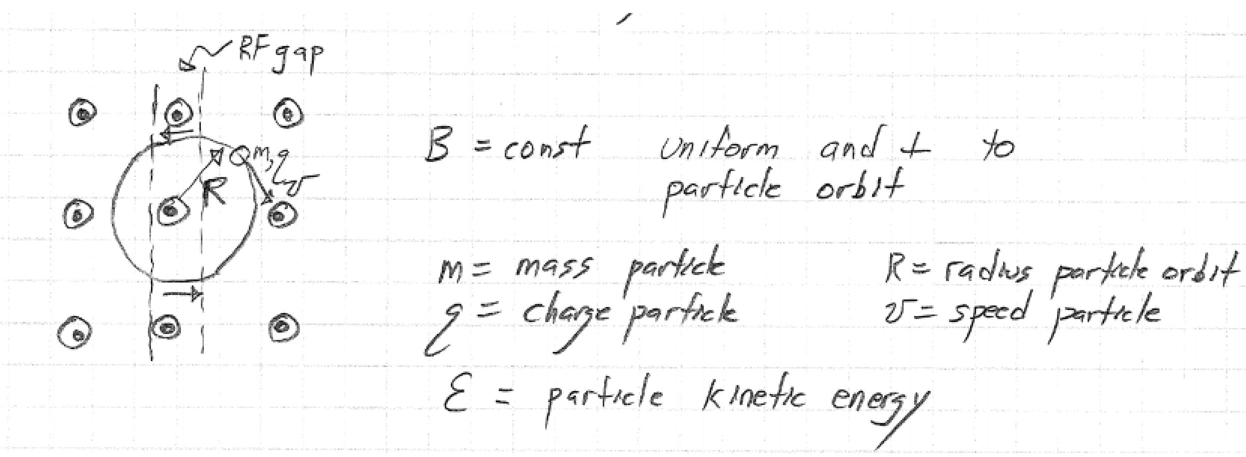
Problem 1

P003 Cyclotron - Single Particle 20 pts.



a) 5 pts: Neglecting the change in orbit radius, R, over one transit, follow the steps in class to show that the non-relativistic period τ of the orbit is:

$$\tau = \frac{2\pi}{\omega_c}$$

where $\omega_c = \frac{qB}{m}$ is the cyclotron frequency and is constant.

Discuss the implications for a particle entering the oscillating RF field gap each transit. Will the frequency of the RF field need to change?

b) 5 pts: If a particle with mass, m, is traveling relativistically with approximately constant velocity, v, over one turn in a cyclotron, show that:

$$\frac{d\vec{p}}{dt} = -\frac{m\gamma v^2}{R}\hat{r}$$

where $\vec{p} = \gamma m\vec{v}$ and $\gamma = 1/\left(1 - \frac{\vec{v}^2}{c^2}\right)^{\frac{1}{2}} \approx$ const.

c) 10 pts: Derive a relativistically correct formula for the period, τ, to characterize how it varies with particle kinetic energy, $\mathcal{E} = (\gamma - 1)\, mc^2$.

Discuss the implications of this result for the operation of a cyclotron as the particle becomes relativistic.

Problem 2

P002 Ion Diode: Child-Langmuir Current Density 25 pts.

Consider a hot-plate type ion diode of voltage V_0 and gap length d. Let the current density J be composed of two species with masses m_1 and m_2 and the same charge q. Set $J = J_1 + J2$ with $J_1 = \alpha J$ and $J_2 = (1 - \alpha)\, J$. What is the 'effective mass' m_{eff} in terms of m_1 and m_2 that should be used in the resulting Child-Langmuir Law:

$$J = \frac{4}{9}\epsilon_0 \left(\frac{2q}{m_{\text{eff}}}\right)^{\frac{1}{2}} \frac{V_0^{\frac{3}{2}}}{d^2}$$

Problem 3

P027 Cylindrical Current Sheet 25 pts.

An axially long cylindrical current sheet carrying current density J is placed in vacuum along $\hat{\mathbf{z}}$-axis. The current density on the sheet in cylindrical coordinates is:

$$J(r, \theta) = \frac{I_0}{2a} \cos(\theta) \delta(r - a)$$

where I_0 is the current of the sheet, r is the radial coordinate, θ is the azimuthal angle, and a is the radius of the current sheet. Show that this current sheet produces a pure dipole field $\mathbf{B} = B_y \hat{\mathbf{y}}$ inside ($r < a$) the sheet with:

$$B_y = -\frac{\mu_0 I_0}{4a}$$

Problem 4

P029 Magnetic Field Symmetry 15 pts.

In many cases, the design of magnets used in accelerators exploit some idealized symmetry. Due to the symmetry conditions, some multipole field components vanish. Indicate the allowed a_n and b_n terms for the interior magnetic field under the following symmetry conditions:

1) Up-down symmetry: $J(r, \phi) = J(r, 2\pi - \phi)$

2) Up-down anti-symmetry: $J(r, \phi) = -J(r, 2\pi - \phi)$

3) Left-right symmetry: $J(r, \phi) = J(r, \pi - \phi)$

Hint: For a magnetic field at $z = x + iy \equiv r'e^{i\theta}$ from a wire at $z_0 = x_0 + iy_0 \equiv re^{i\phi}$:

$$B = \frac{\mu_0 J(r, \phi)}{2\pi(r'e^{i\theta} - re^{i\phi})}$$

Prove the following relation:

$$b_n + ia_n \propto \int_0^{2\pi} e^{-i(n+1)\phi} J(r, \phi) \mathrm{d}\phi$$