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PROBLEM SET ~~#~~ 7

30 POINTS

PROBLEM 1

CONSIDER THE PARAMETERS FOR A PROTON STORAGE RING IN WHICH THERE IS A SINGLE CIRCULATING BUNCH:

$$N_0 = 2\times10^{14} \text{ protons / bunch}$$

$$l_{b\,eff} \simeq 124\ \text{m} = \text{effective bunch length}$$

$$r_{pipe} = 0.1\ \text{m} = \text{pipe radius}$$

$$\beta = 0.875 = \frac{v_z}{c} \text{ (ion velocity)}$$

a) WHAT IS THE SINGLE BUNCH BEAM INDUCED MULTIPACTING PARAMETER δ_s?

b) WHAT IS THE CHARACTERISTIC ENERGY GAIN PER ELECTRON?

c) ASSUME AN IONIZATION CROSS SECTION OF $10^{-22}\ \text{m}^2$, AND A DESORPTION COEFFICIENT 4, AND A LINEAR PUMP RATE S OF $0.1\ \text{m}^3\ \text{s}^{-1}\ \text{m}^{-1}$. WILL THE PRESSURE BE STABLE IN THIS MACHINE AGAINST RUNAWAY DESORPTION? (Assume the beam fills the ring for making your estimate.)

d) Assume that $r_{b0} = 0.01$ m. ESTIMATE THE ELECTRON oscillation frequency in the field of the ion beam.

(2) Consider the effects of Coulomb collisions for a beam with an initial temperature $T_{\perp 0} \neq T_{\parallel 0}$. Assume τ = const and that

(20 POINTS)

$$\frac{dT_\perp}{dt} = -\frac{1}{2}\frac{dT_\parallel}{dt} = \frac{-(T_\perp - T_\parallel)}{\tau}$$

SHOW THAT FOR INITIAL VALUES $T_\perp = T_{\perp 0}$ and $T_\parallel = T_{\parallel 0}$, that the general solution (when τ is assumed constant) is:

$$T_\parallel = A - \frac{2}{3} B \exp[-\alpha t]$$

$$\& \quad T_\perp = A + \frac{1}{3} B \exp[-\alpha t]$$

CALCULATE $A, B, \& \ \alpha$ in terms of $T_{\perp 0}, T_{\parallel 0}, \& \ \tau$.

WHAT ARE THE VALUES OF $T_\perp$ & $T_\parallel$ AS $t \to \infty$?

Problem 3, 25 Points

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TCE Problem 9

S.M. Lund

In class we derived rms envelope equations for a coasting beam:

$$r_x'' + \kappa_x(s) r_x - \frac{2Q}{r_x + r_y} - \frac{\varepsilon_x^2}{r_x^3} = 0$$

$$r_y'' + \kappa_y(s) r_y - \frac{2Q}{r_x + r_y} - \frac{\varepsilon_y^2}{r_y^3} = 0$$

Take:

$$\left.\begin{aligned} \kappa_x &= k_{\beta 0}^2 + \delta\kappa_x \\ \kappa_y &= k_{\beta 0}^2 + \delta\kappa_y \\ Q &= Q + \delta Q \\ \varepsilon_x &= \varepsilon + \delta\varepsilon_x \\ \varepsilon_y &= \varepsilon + \delta\varepsilon_y \end{aligned}\right\} \quad \text{Continuous focus equilibrium + Perturbations}$$

and

$$\left.\begin{aligned} r_x &= r_m + \delta r_x \\ r_y &= r_m + \delta r_y \end{aligned}\right\} \quad \text{Matched Envelope + Perturbations}$$

$r_m = \text{const.}$

A/ Expand to linear order as in class notes and derive equilibrium and perturbed envelope eqns

B/ Show that the linear order envelope eqns in A/ decouple when taking

$$\delta r_+ = \frac{\delta r_x + \delta r_y}{2}$$

$$\delta r_- = \frac{\delta r_x - \delta r_y}{2}$$

and give

$$\delta r_+'' + k_{\beta 0}^2 \delta r_+ + \frac{Q}{r_m^2}\delta r_+ + \frac{3\varepsilon^2}{\delta r_m^4}\delta r_+ = \delta P_+$$

$$\delta r_-'' + k_{\beta 0}^2 \delta r_- + \frac{3\varepsilon^2}{\delta r_m^4}\delta r_- = \delta P_-$$

with

$$\delta P_+ = -r_m\left(\frac{\delta\kappa_x + \delta\kappa_y}{2}\right) + \frac{1}{r_m}\delta Q + \frac{\varepsilon}{r_m^3}\left(\delta\varepsilon_x + \delta\varepsilon_y\right)$$

$$\delta P_- = -r_m\left(\frac{\delta\kappa_x + \delta\kappa_y}{2}\right) + \frac{\varepsilon}{r_m^3}\left(\delta\varepsilon_x - \delta\varepsilon_y\right)$$

C/ Take:

$$k_{\beta 0} = \frac{\sigma_0}{L_p}$$

$$\sigma = \sqrt{\sigma_0^2 - \frac{Q}{(r_m/L_p)^2}} = \frac{\varepsilon L_p}{r_m^2}$$

$$\sigma_+ = \sqrt{2\sigma_0^2 + 2\sigma^2}$$

and show that the eqn for δr_+ in B/ becomes:

$$L_p^2 \frac{d^2}{ds^2}\left(\frac{\delta r_+}{r_m}\right) + \sigma_+^2\left(\frac{\delta r_+}{r_m}\right) = \frac{-\sigma_0^2}{2}\left(\frac{\delta\kappa_x}{k_{\beta 0}^2} + \frac{\delta\kappa_y}{k_{\beta 0}^2}\right) + (\sigma_0^2 - \sigma^2)\frac{\delta Q}{Q} + \sigma^2\left(\frac{\delta\varepsilon_x}{\varepsilon} + \frac{\delta\varepsilon_y}{\varepsilon}\right)$$

Solution of the eqn in this form was extensively developed in the lectures.

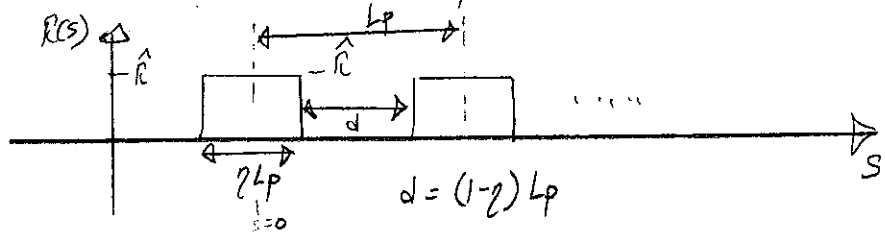
S. M. Lund

TCF Problem 6

Approximate Matched Envelope Solution for a Solenoid

A/ For a solenoidal transport channel:

Please see class notes and parallel discussion for quadrupole channel.



We can Fourier-series expand $\kappa_x(s) = \kappa_y(s) \equiv \kappa(s)$ as

$$\kappa(s) = \sum_{n=0}^{\infty} \kappa_n \cos\left(\frac{2n\pi s}{L_p}\right)$$

and identify

$$\kappa_0 = \int_0^{L_p} \frac{ds}{L_p}\, \kappa(s)$$

$$\kappa_n = \frac{2}{L_p}\int_0^{L_p} ds\, \cos\left(\frac{2n\pi s}{L_p}\right) \kappa(s) \qquad n = 1, 2, \ldots$$

calculate κ_0 and κ_n

B/ The matched beam envelope equation is: (Larmor frame)

$$r_{xm}'' + \kappa(s)\, r_{xm} - \frac{Q}{r_{xm}} - \frac{\varepsilon_x^2}{r_{xm}^3} = 0$$

$$r_{xm}(s+L_p) = r_{xm}(s) \quad ; \quad r_{xm} = r_{ym} \quad , \quad \begin{matrix}\varepsilon_x = \text{const.} \\ Q = \text{const.}\end{matrix}$$

Expand the matched envelope solution similarly:

$$r_{xm}(s) = r_b\left[1 + \Delta \cos\left(\frac{2\pi s}{L_p}\right)\right] + r_b \sum_{n=2} \Delta_n \cos\left(\frac{2n\pi s}{L_p}\right)$$

expansion coefficients.

where

$$r_b = \text{const}$$

is the average beam radius and

$$\Delta = \text{const.} \qquad |\Delta| \ll 1$$

is an expansion term and Δ_n are dimensionless expansion coefficients we take to be small relative to Δ.

Insert the expansions

- Neglect all terms $\mathcal{O}(\Delta^2)$ and higher
- Neglect fast oscillation terms $\sim \cos\left(\frac{2n\pi s}{L_p}\right)$ with $n \geq 2$

Derive avg and leading-order force balance terms.

C/ Explain (no need to calculate explicitly) how the constraints in B/ can be used to calculate.

Max $[r_{xm}]$: Max beam excursion in lattice period, in terms of lattice parameters, avg. beam radius, and emittance

Q : Beam perveance as a function of lattice parameters, avg. beam radius, and emittance.

D/ The undepressed particle phase advance can be calculated as

$$\cos\sigma_0 = \cos(2\Theta) - \frac{(1-\eta)}{\eta}\,\Theta\sin(2\Theta)$$

$$\Theta = \sqrt{\hat{\kappa}}\,\frac{\eta L_p}{2}$$

expand this expression to leading order in Θ to obtain a simpler design expression relating σ_0 and lattice parameters.

Results from part C/ and D/ can be employed to design solenoidal transport channels.