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US Particle Accelerator School
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"Beam Physics with Intense Space Charge"

Lecturers:
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Class material including a course overview, lecture schedule, a more
detailed course description, lecture notes (pdf copies), problems sets,
and the final exam can be found on the course web site:

https://people.nscl.msu.edu/~lund/uspas/bpisc_2015

Topical Course Outline:

"Beam Physics with Intense Space-Charge"

Note:  This outline and the distribution files are arranged in logical
presentation order.  In the actual class, there are some deviations from
this order due to practical constraints. The actual order of material
presented is given in the lecture schedule on the course web site.

Lecturer abbreviations:
   JJB - J.J. Barnard
   SML - S.M. Lund

1.  Introduction to the Physics of Beams and Basic Parameters  (JJB)
  1.1 Particle equations of motion
  1.2 Dimensionless parameters: Perveance, phase advance, space charge
      tune depression
  1.3 Plasma physics of beams: collisions, Debye Length
  1.4 Klimontovich equation, Vlasov equation, Liouville's theorem
  1.4 Emittance and brightness

2.  Envelope Equations-I  (JJB)
  2.1 Paraxial Ray Equation
  2.2 Envelope equations for axially symmetric beams
  2.3 Cartesian equations of motion
      2.3.1 Quadrupole focusing
      2.3.2 Space charge force for elliptical beams
  2.4 Envelope equations for elliptically symmetric beams

3.  Current Limits in Accelerators and Centroid equations-I (JJB)
  3.1 Axisymmetric beams
     3.1.1 Solenoids
     3.1.2 Einzel Lenses
  3.2 Elliptically symmetric beams
     3.2.1 Derivation of space charge term in envelope equation with
           elliptical symmetry
     3.2.2 Current limit for quadrupoles using Fourier transforms
  3.3 Current limit for continuous focusing
     3.3.1 Calculation of sigma_0 (using matrix multiplication)
     3.3.2 Comparison of quadrupole current limit (from Fourier transform,
     and matrix methods)
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  3.4 Centroid equations (first order moments)
     3.4.1 Space charge and focusing forces
  3.5 Image forces (effect on centroid and envelope)

4.  Transverse Particle Dynamics (SML)
  4.1  Particle Equations of Motion
    4.1.A Introduction: The Lorentz Force Equation
    4.1.B Applied Fields
    4.1.C Machine Lattice
    4.1.D Self Fields
    4.1.E Equations of Motion in s and the Paraxial Approximation
    4.1.F Axial Particle Kinetic Energy
    4.1.G Summary: Transverse Particle Equations of Motion
    4.1.H Overview of Analysis to Come
    4.1.I Bent Coordinate System and Particle Equations of Motion with
          Dipole Bends and Axial Momentum Spread
    Appendix A: Gamma and Beta Factor Conversions
  4.2  Transverse Particle Equations of Motion in Linear Focusing Channels
    4.1.A Introduction
    4.2.B Continuous Focusing
    4.2.C Alternating Gradient Quadrupole Focusing - Electric Quadrupoles
    4.2.D Alternating Gradient Quadrupole Focusing - Magnetic Quadrupoles
    4.2.E Solenoidal Focusing
    4.2.F Summary of Transverse Particle Equations of Motion
    Appendix A: Quadrupole Skew Coupling
    Appendix B: The Larmor Transform to Express Solenoidal Focused
                Particle Equations of Motion in Uncoupled Form
    Appendix C: Transfer Matrices for Solenoidal Focusing
  4.3  Description of Applied Focusing Fields
    4.3.A Overview
    4.3.B Magnetic Field Expansions for Focusing and Bending
    4.3.C Hard Edge Equivalent Models
    4.3.D 2D Transverse Multipole Magnetic Moments
    4.3.E Good Field Radius
    4.3.F Example Permanent Magnet Assemblies
  4.4 Transverse Particle Equations of Motion with Nonlinear Applied Fields
    4.4.A Overview
    4.4.B Approach 1: Explicit 3D Form
    4.4.C Approach 2: Perturbed Form
  4.5  Linear Equations of Motion Without Space-Charge, Acceleration, and
       Momentum Spread
    4.5.A Hill's equation
    4.5.B Transfer Matrix Form of the Solution to Hill's Equation
    4.5.C Wronskian Symmetry of Hill's Equation
    4.5.D Stability of Solutions to Hill's Equation in a Periodic Lattice
  4.6  Hill's Equation: Floquet's Theorem and the Phase-Amplitude Form
       of the Particle Orbit
    4.6.A Introduction
    4.6.B Floquet's Theorem
    4.6.C Phase-Amplitude Form of the Particle Orbit
    4.6.D Summary: Phase-Amplitude Form of the Solution to Hill's Equation
    4.6.E Points on the Phase-Amplitude Formulation
    4.6.F Relation Between the Principal Orbit Functions and the
          Phase-Amplitude Form Orbit Functions
    4.6.G Undepressed Particle Phase Advance
    Appendix C: Calculation of w(s) from Principal Orbit Functions
  4.7  Hill's Equation: The Courant-Snyder Invariant and the
       Single-Particle Emittance
    4.7.A Introduction
    4.7.B Derivation of the Courant-Snyder Invariant
    4.7.C Lattice Maps
  4.8 Hill's Equation: The Betatron Formulation of the Particle Orbit
      and Maximum Orbit Excursions
    4.8.A Formulation
    4.8.B Maximum Orbit Excursions
  4.9  Momentum Spread Effects and Bending
    4.9.A Overview
    4.9.B Chromatic Effects
    4.9.B Dispersive Effects
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  4.10 Acceleration and Normalized Emittance
    4.10.A Introduction
    4.10.B Transformation to Normal Form
    4.10.C Phase−Space Relations Between Transformed and Untransformed Systems
  4.11 Calculation of Acceleration Induced Changes in gamma and beta
    4.10.A Introduction
    4.10.B Solution of the Longitudinal Equations of Motion
    4.10.C Longitudinal Solution via Energy Gain
    4.10.D Quasistatic Potential Expansion
  Contact Information
  References
  Acknowledgments

5.  Transverse Equilibrium Distribution Functions (SML)
  5.1  Vlasov Model
    Vlasov−Poisson System
    Review: Lattices: Continuous, Solenoidal, and Quadrupole
    Review: Undepressed Particle Phase Advance
  5.2  Vlasov Equilibria
    Equilibrium Conditions
    Single Particle Constants of the Motion
    Discussion: Plasma Physics Approach to Beam Physics
  5.3 The KV Equilibrium Distribution
    Hill's Equation with Linear Space−Charge Forces
    Review: Courant−Snyder Invariants
    Courant−Snyder Invariants for a Uniform Density Elliptical Beam
    KV Envelope Equations
    Canonical Form of the KV Distribution Function
    Matched Envelope Structure
    Depressed Particle Orbits
    rms Equivalent Beams
    Discussions/Comments on the KV Model
  Appendix A: Self−Fields of a Uniform Density Elliptical Beam in Free Space
    (handwritten notes)
    Derivation #1: Direct
    Derivation #2: Simplified
  Appendix B: Canonical Transforms of the KV Distribution
    (handwritten notes)
    Canonical Transforms
    Simplified Moment Calculations
  5.4  The Continuous Focusing Limit of the KV Distribution
    Reduction of Elliptical Model
    Wavenumbers of Particle Oscillations
    Distribution Form
    Discussion
  5.5  Continuous Focusing Equilibrium Distributions
    Equilibrium Form
    Poisson's Equation
    Moments and rms Equivalent Beam Envelope Equation
    Example Distributions
  5.6  Continuous Focusing: The Waterbag Equilibrium Distribution
    Distribution Form
    Poisson's Equation
    Solution in Terms of Accelerator Parameters
    Equilibrium Properties
  5.7  Continuous Focusing: The Thermal Equilibrium Distribution
    Overview
    Distribution Form
    Poisson's Equation
    Solution in Terms of Accelerator Parameters
    Equilibrium Properties
  5.8  Continuous Focusing: Debye Screening in a Thermal Equilibrium Beam
    Poisson's Equation for the Perturbed Potential Due to a Test Particle
    Solution for Characteristic Debye Screening
  5.9  Continuous Focusing: The Density Inversion Theorem
    Relation of Density Profile to the Full Distribution Function
    Example Application to the KV Distribution
  5.10 Comments on the Plausibility of Smooth, non−KV Vlasov Equilibria
       in Periodic Focusing Lattice
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    Discussion
  Contact Information
  References
  Acknowledgments

6.  Transverse Particle Resonances with Application to Circular Accelerators
    (SML)
  6.1 Overview
    Hill's Equation Review: Betatron Form of Phase−Amplitude Solution
    Transform Approach
    Random and Systematic Perturbations Acting on Orbits
  6.2 Floquet Coordinates
    Transformation of Hill's Equation to a Simple Harmonic Oscillator
    Phase−Space Structure of Solution
    Expression of the Courant−Snyder Invariant
    Phase−Space Area Transform
  6.3 Perturbed Hill's Equation in Floquet Coordinates
    Transformation Result for x−Equation
  6.4 Sources and Forms of Perturbation Terms
    Power Series Expansion of Perturbations
    Connection to Multipole Errors
  6.5 Perturbed Solution: Resonances
    Fourier Expansion of Perturbations and Resonance Terms
    Resonance Conditions
  6.6 Machine Operating Points: Restrictions Resulting from Resonances
    Tune Restrictions from Low Order Resonances
  6.7 Space−Charge Effects
    Coherent and Incoherent Tune Shifts
    Laslett Limit
  Contact Information
  References
  Acknowledgments

7.  Injectors and Longitudinal Physics Part I (JJB)
  7.1  Diodes and Injectors
    7.1.1 Space−charge limited flow and child−Langmuir law
    7.1.2 Pierce electrodes
    7.1.3 Transients in injectors and Lampel−Tiefenback solution
  7.2 Injector Choices

8.  Longitudinal Physics Part II (JJB)
  8.1  Acceleration −− introduction
  8.2  Space charge of short bunches (in rf−accelerators)
  8.3  Space charge of long bunches (g−factor model)
  8.4  Longitudinal 1D Vlasov equation
  8.5  Longitudinal fluid equation
  8.4  Longitudinal space charge waves
  8.5  Longitudinal rarefaction waves and bunch end control

9.  Longitudinal Physics Part III (JJB)
  9.1  Longitudinal cooling from acceleration
  9.2  Longitudinal resistive instability
  9.3  Bunch compression
  9.4  Longitudinal envelope equation
  9.4  Neuffer distribution function

10. Transverse Centroid and Envelope Descriptions of Beam Evolution (SML)
  10.1  Overview
  10.2  Derivation of Centroid and Envelope Equations of Motion
    Statistical Averages
    Particle Equations of Motion
    Distribution Assumptions
    Self−Field Calculation: Direct and Image
    Coupled Centroid and Envelope Equations of Motion
  10.3  Centroid Equations of Motion
    Single Particle Limit: Oscillation and Stability Properties
    Effect of Driving Errors
    Effect of Image Charges
  10.4  Envelope Equations of Motion
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    KV Envelope Equations
    Applicability of Model
    Properties of Terms
  10.5  Matched Envelope Solutions
    Construction of Matched Solution
    Symmetries of Matched Envelope: Interpretation via KV Envelope Equations
    Examples
  10.6  Envelope Perturbations
    Perturbed Equations
    Matrix Form: Stability and EigenMode Symmetries
    Decoupled Modes
    General Mode Limits
   10.7 Envelope Modes in Continuous Focusing
    Normal Modes: Breathing and Quadrupole Modes
    Driven Modes
   10.8  Envelope Modes in Periodic Focusing Channels
    Solenoidal Focusing
    Quadrupole Focusing
    Mode Launching
   10.9  Transport Limit Scaling Based on Envelope Models
         (see handwritten notes)
    Overview
    Example Calculation for a Periodic FODO Quadrupole Transport Channel
    Discussion on Application of Formulas in Design
    Results of More Detailed Models
  10.10 Centroid and Envelope Descriptions via
        1st Order Coupled Moment Equations
    Formulation
    Example Illustration − Familiar KV Envelope Model
 Contact Information
 References
 Acknowledgments

11. Continuous Focusing Envelope Modes and Beam Halo  (JJB)
  11.1 Envelope modes of unbunched beams in continuous focusing
  11.2 Envelope modes of bunched beams in continuous focusing
  11.3 Halos from mismatched beams
    11.3.1 What is halo? Why do we care
    11.3.2 Qualitative picture of halo formation: mismatches
           resonantly drive particles to large amplitude
    11.3.3 Core/particle models
    11.3.4 Amplitude phase analysis

12. Transverse Kinetic Stability (SML)
  12.1 Overview: Machine Operating Points
    Notions of Beam Stability
    Tiefenback Experimental Results for Quadrupole Transport
  12.2 Overview: Collective Modes and Transverse Kinetic Stability
    Possibility of Collective Internal Modes
    Vlasov Model Review
    Plasma Physics Approach to Understanding Higher Order Instability
  12.3 Linearized Vlasov Equation
    Equilibrium and Perturbations
    Linear Vlasov Equation
    Method of Characteristics
    Discussion
  12.4 Collective Modes on a KV Equilibrium Beam
    KV Equilibrium
    Linearized Equations of Motion
    Solution of Equations
    Mode Properties
    Physical Mode Components Based on Fluid Model
    Periodic Focusing Results
  12.5 Global Conservation Constraints
    Conserved Quantities
    Implications
  12.6 Kinetic Stability Theorem
    Effective Free Energy
    Free Energy Expansion in Perturbations
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    Perturbation Bound and a Sufficient Condition for Stability
    Interpretation and Example Applications
  12.7 rms Emittance Growth and Nonlinear Forces
    Equations of Motion
    Coupling of Nonlinear Forces to rms Emittance Evolution
  12.8 rms emittance Growth and Nonlinear Space−Charge Forces
    Equations of Motion
    rms Equivalent Beam Forms
    Wangler’s Theorem
  12.9 Uniform Density Beams and Extreme Energy States
    Variational Formulation
    Self−Field Energy Minimization
  12.10 Collective Relaxation and rms Emittance Growth
    Conservation Constraints
    Relaxation Processes
    Emittance Growth Bounds from Space−Charge Nonuniformities
  12.11 Halo Induced Mechanism of Higher−Order Instability
    Halo Model for an Elliptical Beam
    Pumping Mechanism
    Stability Properties
  12.12 Phase Mixing and Landau Damping in Beams
    (to be added in future versions)
  Contact Information
  References
  Acknowledgments

13. Pressure, Scattering, and Electron Effects (JJB)
  13.1 Beam/beam Coulomb collisions
  13.2 Beam/residual−gas scattering
  13.3 Charge−changing processes
  13.4 Wall effects
    13.4.1 gas pressure instability
  13.5 Electron cloud processes
    13.5.1 Multiple−bunch beam−induced multipacting
    13.5.2 Single−bunch beam−induced multipacting
  13.6 Electron−ion instability

14. Heavy Ion Fusion and Final Focus (JJB)
  14.1 An application of intense beams: Heavy Ion Fusion
    14.1.1 Requirements
    14.1.2 Targets for inertial confinement fusion
    14.1.3 Accelerator
    14.1.4 Drift compression
    14.1.5 Final focus
  14.2 Final focus
    14.2.1 Predicting spot size using envelope equation
           and estimate of effects from chromaticity
  14.3 Experiments for Heavy Ion Fusion

15. Numerical Simulations (SML)
  15.1 Why Numerical Simulation?
  15.2 Classes of Intense Beam Simulations
    15.2.A Overview
    15.2.B Particle Methods
    15.2.C Distribution Methods
    15.2.D Moment Methods
    15.2.E Hybrid Methods
  15.3 Overview of Basic Numerical Methods
    15.3.A Discretizations
    15.3.B Discrete Numerical Operations
      − Derivatives
      − Quadrature
      − Irregular Grids and Axisymmetric Systems
    15.3.C Time Advance
      − Overview
      − Euler and Runge−Kutta Advances
      − Solution of Moment Methods
  15.4 Numerical Methods for Particle and Distribution Methods
    14.4.A Overview
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    14.4.B Integration of Equations of Motion
      − Leapfrog Advance for Electric Forces
      − Leapfrog Advance for Electric and Magnetic Forces
      − Numerical Errors and Stability of the Leapfrog Method
      − Illustrative Examples
    15.4.C Field Solution
      − Electrostatic Overview
      − Green’s Function Approach
      − Gridded Field Solution: Equation and Boundary Conditions
      − Methods of Gridded Field Solution
      − Spectral Methods and the FFT
    15.4.D Weighting: Depositing Particles on the Field Mesh and
           Interpolating Fields to the Particles
      − Overview of Approaches
      − Approaches: Nearest Grid Point, Cloud in Cell, Area, Splines
    15.4.E Computational Cycle for Particle in Cell Simulations
   15.5 Diagnostics
   15.6 Initial Distribution and Particle Loading
   15.7 Numerical Convergence
   15.8 Practical Considerations
      15.8.A Overview
      15.8.B Fast Memory
      15.8.C Run Time
      15.8.D Machine Architectures
   15.9 Overview of the WARP Code
   15.10 Example Simulations
  Contact Information
  Acknowledgments
  References

16. Summary of Lectures by John J. Barnard (JJB)
  16.1 Emittance and phase space review
  16.2 Particle equations of motion (radial and Cartesian)
  16.3 Summary of 6 statistical envelope equations and two equations based
       on particular distribution functions
  16.4 Current limits
  16.5 Using envelope equations to estimate spot size
  16.6 Longitudinal dynamics summary
  16.7 Instability summary
  16.8 Halo summary
  16.9 Electron, gas, pressure, and scattering effects summary
  16.10 Summary of HIF
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