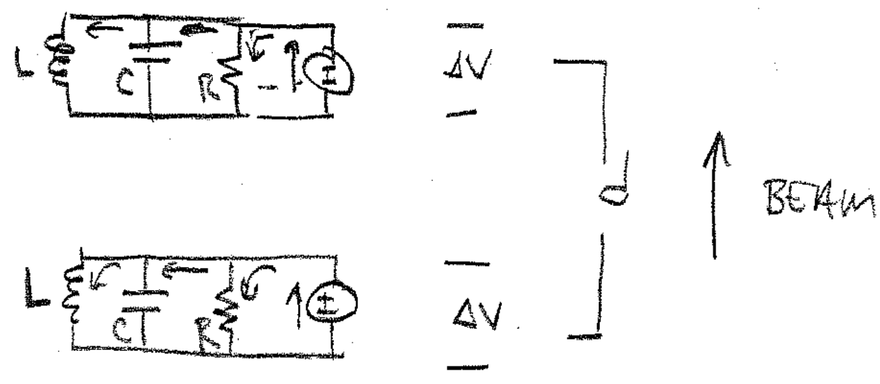
PROBLEM (1)

PROBLEM SET 5
LUND/BARNARD
2015



Assume that, as was done in class each acceleration gap or module is represented by a lumped circuit element with successive elements separated by a distance d.

Assume that the beam is a current source I_0 with perturbation I_1, AND THAT AT EACH GAP THE BEAM ACQUIRES A VOLTAGE INCREMENT ΔV

$$I = I_0 + I_1 e^{-i\omega t} \qquad \Delta V = \Delta V_0 + \Delta V_1 e^{-i\omega t}$$

SHOW THAT THE IMPEDANCE FOR SUCH A SYSTEM IS:

$$Z^*(\omega) = \frac{R^*}{1 - i\omega C^+ R^* + iR^*/\omega L^*}$$

where $R^* = \frac{R}{d}$, $C^+ = Cd$, $L^* = L/d$ and

$$E_1 = \frac{-\Delta V}{d} = -I_1 Z^*(\omega)$$

NOTE THAT THE VOLTAGE DROP ACROSS AN ELEMENT IS GIVEN BY:

$$\Delta V = \begin{cases} L\, dI/dt & \text{for inductor with inductance } L \\ Q/C & \text{for capacitor with capacitance } C \\ IR & \text{for resistor with resistance } R \end{cases}$$

HERE I = CURRENT = dQ/dt $\quad Q$ = charge

PROBLEM 2: WHEN LONGITUDINAL EMITTANCE IS INCLUDED IN THE NON-RELATIVISTIC LONGITUDINAL ENVELOPE EQUATION, DESCRIBING THE LENGTH L OF A PULSE WITH PARABOLIC LINE CHARGE DENSITY UNDERGOING BUNCH COMPRESSION, :

$$\frac{d^2 L}{ds^2} = \frac{16\,\varepsilon_z^2}{L^3} + \frac{12\, g\, q\, Q_c}{4\pi\epsilon_0\, m v^2 L^2}$$

where Q_c is the total charge in the bunch,

$\varepsilon_z^2 \equiv$ longitudinal emittance $= 25\left[\langle z^2\rangle\langle z'^2\rangle - \langle z z'\rangle^2\right]$.

SHOW THAT THE INITIAL VELOCITY TILT ΔV REQUIRED TO COMPRESS THE BEAM TO "STAGNATION" [i.e. to the point where $\dot{L} = 0$) IS GIVEN BY:

$$\Delta V^2 = \frac{16\,\varepsilon_z^2}{L_0^2}\left[C^2 - 1\right] + \frac{24\, g\, q\, Q_c}{4\pi\epsilon_0 L_0 m v_0^2}\left[C - 1\right]$$

WHERE $L_0 = L$ at $t = 0$,

$L_f = L$ at the stagnation point

& $C \equiv L_0/L_f$ = compression ratio > 1

& $\Delta V = -L_0'$

Problem 2;

TPR Problem 1

S.M. Lund

P.V.

1/ Consider the driven harmonic oscillator equation for $U(\varphi)$:

$$\frac{d^2 U(\varphi)}{d\varphi^2} + \nu_0^2 U(\varphi) = \underbrace{A\cos(\nu\varphi) + B\sin(\nu\varphi)}_{\text{driving term.}}$$

ν = constant driving frequency.
A, B constant amplitudes.

The general solution for $U(\varphi)$ can be expanded as

$$U(\varphi) = U_h(\varphi) + U_p(\varphi)$$

where U_h is the general solution to the homogeneous equation:

$$\frac{d^2 U_h}{d\varphi^2} + \nu_0^2 U_h = 0$$

$$\Rightarrow U_h = C_1\cos(\nu_0\varphi) + C_2\sin(\nu_0\varphi)$$

C_1, C_2 constants

and U_p is any particular solution to

$$\frac{d^2 U_p}{d\varphi^2} + \nu_0^2 U_p = A\cos(\nu\varphi) + B\sin(\nu\varphi)$$

a) For $\nu \neq \nu_0$ show that a solution U_p exists proportional to the driving term and find the constant of proportionality.

TPR Problem 1 S.M. Lund Pg/

b) Use the results of part a) to construct the solution ($\nu \neq \nu_0$) for $U(\varphi)$ satisfying the initial conditions at $\varphi = 0$:

$$U(\varphi = 0) = U_0$$

$$\left.\frac{dU}{d\varphi}\right|_{\varphi=0} = \mathring{U}_0 \qquad ; \qquad \frac{dU}{d\varphi} \equiv \mathring{U}$$

c) Set $\nu = \nu_0 + \delta\nu$ and find the leading order form of the solution valid for $|\delta\nu/\nu_0| \ll 1$ and $|\delta\nu \varphi| \ll 1$.
What does this limit imply on the amplitude of the particle oscillation as $\nu \to \nu_0$

d) What do these results imply for a general periodic forcing function:

$$\frac{d^2 U(\varphi)}{d\varphi^2} + \nu_0^2 U(\varphi) = f(\varphi) \quad \text{forcing function}$$

$$f(\varphi + 2\pi) = f(\varphi)$$

How does this fit in with the analysis of machine tunes carried out in the class notes?