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Intrabeam collisions, gas and electron effects in intense beams

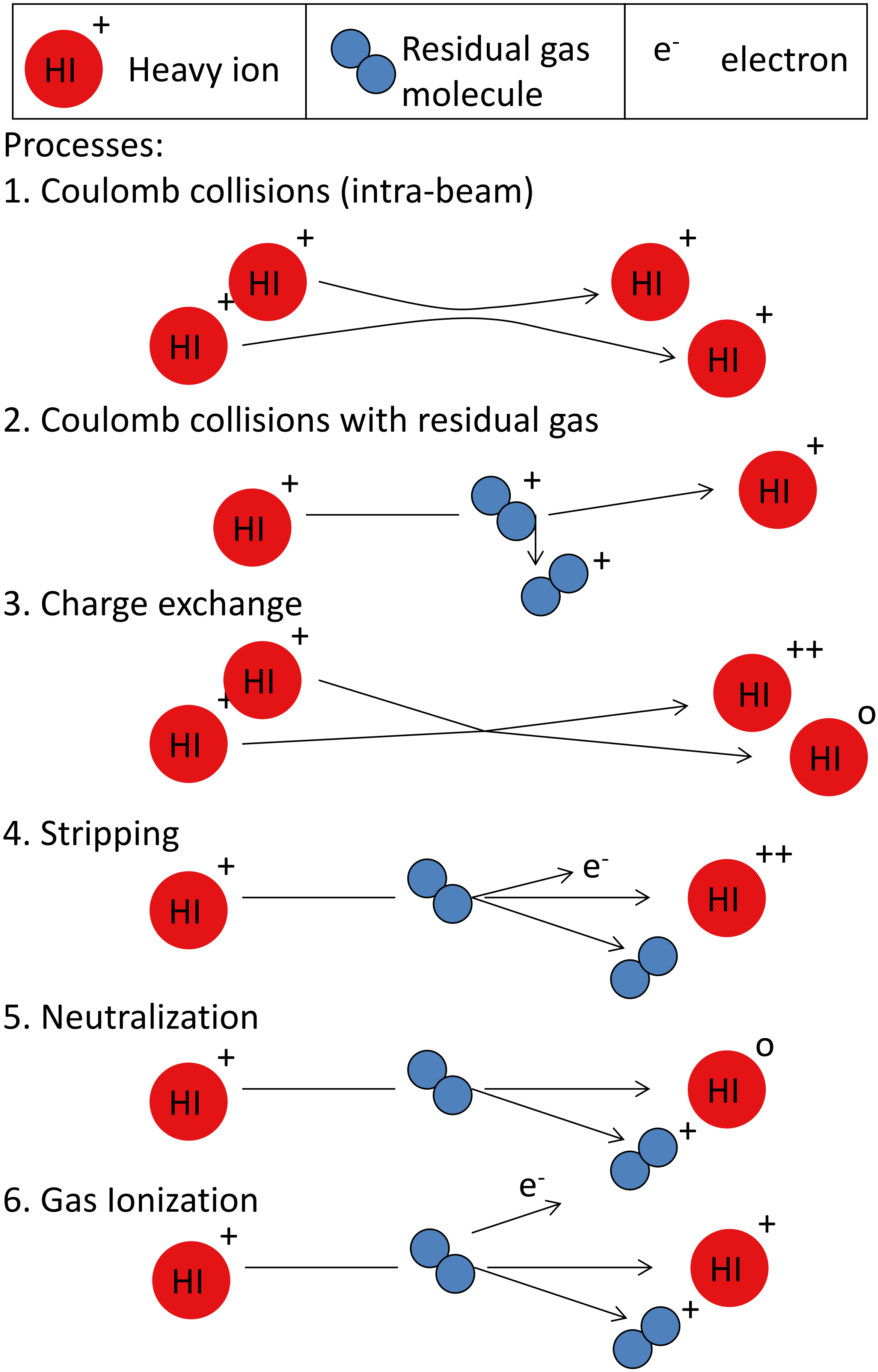
1. Beam/beam coulomb collisions
2. Beam/gas scattering
3. Charge changing processes
4. Gas pressure instability
5. Electron cloud processes
6. Electron-ion instability

Gas and electron effects

-Effects are quite different depending on q, m of species being accelerated

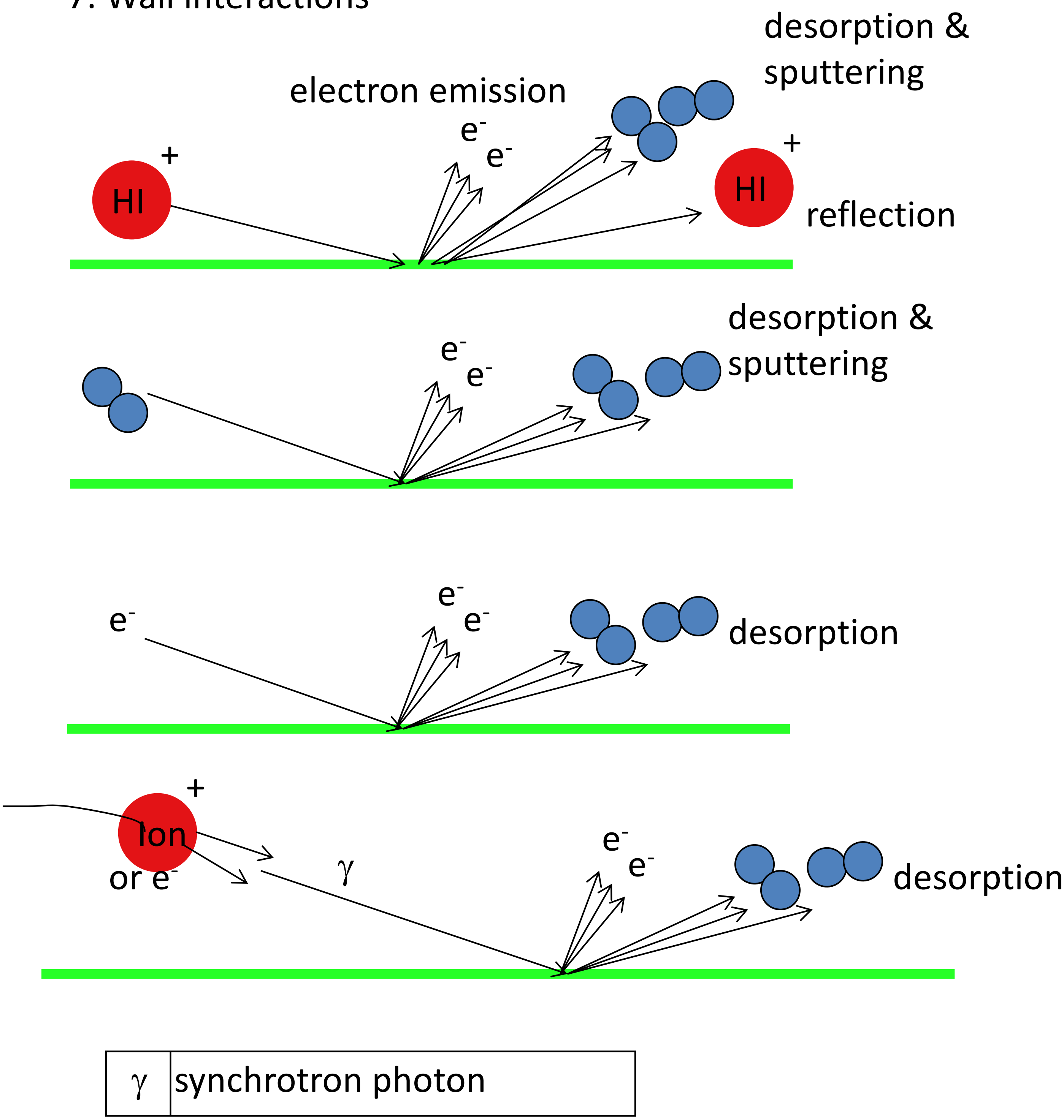
-Circular accelerators vs. Linacs
($t_{residence}$ ~ ms to days vs. 10's of μs)

-Long pulse vs. short pulse
(t_{pulse} ~ 10's of μs vs. 10's of ns)



HI +
Heavy ion
Residual gas molecule
e- electron
Processes:
1. Coulomb collisions (intra-beam)
2. Coulomb collisions with residual gas
3. Charge exchange
4. Stripping
5. Neutralization
6. Gas Ionization

7. Wall interactions



1. COLLISIONS WITHIN BEAM REISER 6.4

CONSIDER EFFECTS OF COULOMB COLLISIONS IN A CONTINUOUS BEAM PROPAGATING THROUGH a continuous FOCUSING CHANNEL WITH $T_{\perp 0} \neq T_{\parallel 0}$

(IF $T_{\perp 0} = T_{\parallel 0} \Rightarrow$ BEAM ALREADY RELAXED).

FROM ICHIMARU & ROSENBLUTH, PHYS FLUIDS 13, 2778, (1970):

$$\frac{dT_\perp}{dt} = -\frac{1}{2}\frac{dT_\parallel}{dt} = \frac{-(T_\perp - T_\parallel)}{\tau}$$

(SINCE $T_x = T_y = T_\perp$, $T_\parallel$ CHANGES AT TWICE THE RATE OF $T_\perp$)
(SINCE $2k_B T_\perp + k_B T_\parallel = \text{const}$)

τ = RELAXATION TIME

$$= \frac{15 (k_B T_{eff}/mc^2)^{3/2} (4\pi\epsilon_0)^2 m^2 c^3}{8\pi^{1/2} q^4 \ln\Lambda \; n} = \left(\frac{15\pi^{1/2}}{8\ln\Lambda}\right)\nu_c^{-1}$$

COULOMB COLLISION APPROXIMATE RATE FOR LARGE ANGLES (PAGE 9 OF INTRODUCTION NOTES)

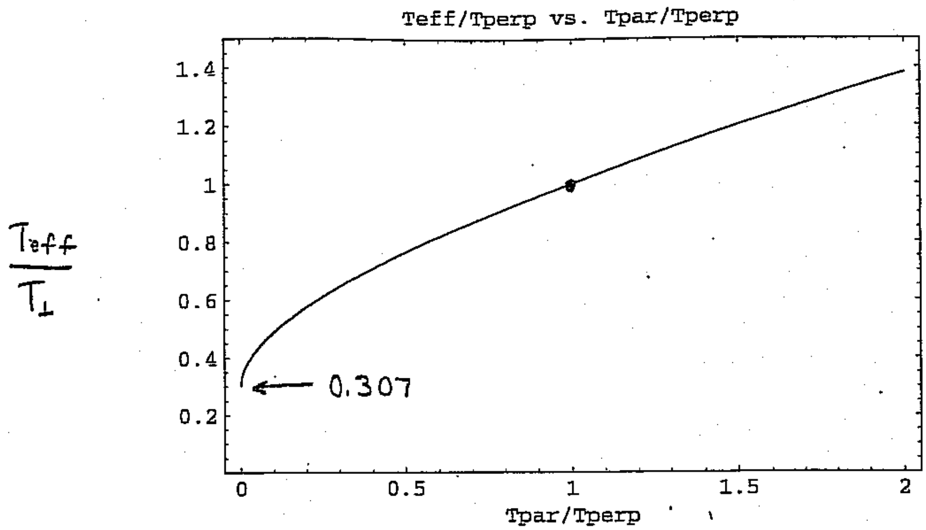
$$\nu_c \sim \pi\left(\frac{q^2}{4\pi\varepsilon_0 k_B T}\right)^2 n_0 \left(\frac{k_B T}{m}\right)^{1/2}$$

$$\ln\Lambda = \begin{cases} \ln \dfrac{(\epsilon_0 k T_{eff})^{3/2} 12\pi}{q^3 n^{1/2}} & \text{for } \lambda_D < r_b \\[2ex] \ln \dfrac{12\pi\epsilon_0 k T_{eff} r_b}{q^2} & \text{for } \lambda_D > r_b \end{cases}$$

$$T_{eff} = T_\perp\left[\frac{15}{4}\int_{-1}^{1}\frac{\mu^2(1-\mu^2)\,d\mu}{[(1-\mu^2) + \mu^2(T_\parallel/T_\perp)]^{3/2}}\right]^{-2/3}$$

T_{eff} is an appropriate average of $T_\perp$ & $T_\parallel$

and depends only on $T_\perp/T_\parallel$



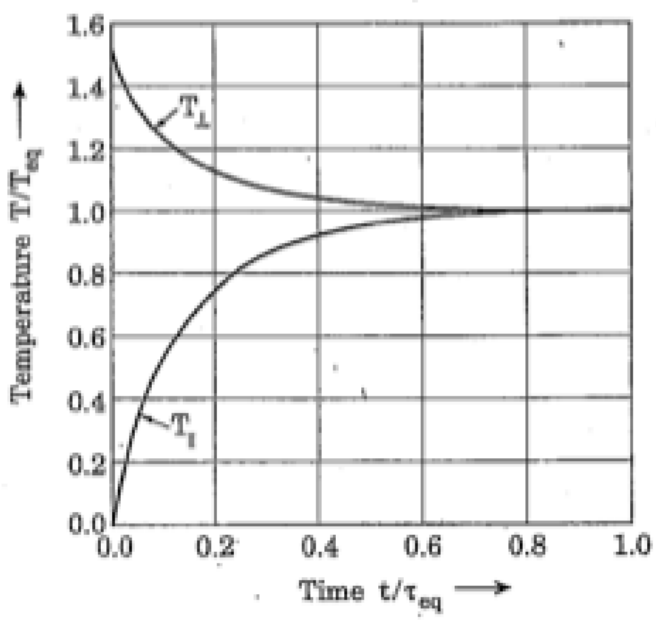
For $T_{\parallel 0} = 0$

$$T_\perp = \frac{2}{3} T_{\perp 0}\left(1 + \frac{1}{2} e^{-3t/\tau_{\text{eff}}}\right), \qquad (6.156\text{a})$$

(APPROXIMATE SOLUTIONS)

$$T_\parallel = \frac{2}{3} T_{\perp 0}\left(1 - e^{-3t/\tau_{\text{eff}}}\right), \qquad (6.156\text{b})$$

$T_{eff} = 0.42\, T_{eq}$



FROM REISER p.527

$$\tau = \left(\frac{15\pi^{1/2}}{8\ln\lambda}\right)\frac{m^{1/2}}{\pi n_0}\left(\frac{4\pi\varepsilon_0}{q^2}\right)^2\left(k_B T_{eff}\right)^{3/2}$$

$$\tau_{eq} = \tau(T_{eff} = T_{eq})$$

$$T_{eff} = T_\perp\left[\frac{15}{4}\int_{-1}^{1}\frac{\mu^2(1-\mu^2)\,d\mu}{[(1-\mu^2)+\mu^2(T_\parallel/T_\perp)]^{3/2}}\right]^{-2/3}$$

BOERSCH EFFECT

AREN'T COLLISIONS NEGLIGIBLE? (NOT ALWAYS)

PUTTING IN NUMBERS:

FOR IONS:

$$\tau_{eff} = 4.3 \cdot 10^{-4}\,\mathrm{s}\ \frac{(A^{1/2})}{Z^4}\left(\frac{kT_{eff}}{1\,\mathrm{eV}}\right)^{3/2}\left(\frac{15}{\ln\Lambda}\right)\left(\frac{10^{10}\,\mathrm{cm}^{-3}}{n}\right)$$

$$\ln\Lambda = \ln\left[\frac{1.5\cdot 10^{5}\,(kT/1\,\mathrm{eV})^{3/2}}{Z^3\,(n/10^{10}\,\mathrm{cm}^{-3})}\right]$$

EXAMPLE: 2 MeV INJECTOR

$\tau_{eff} \simeq 8.8\cdot 10^{-4}\,\mathrm{s}$ for $A = 39$, $kT_{eff} = 0.3\,\mathrm{eV}$, $Z = 1$, $\ln\Lambda = 8.5$, $n = 10^{10}\,\mathrm{cm}^{-3}$

$$t_{transit} \simeq \frac{2d}{v} \approx \frac{2\,(2m)}{(.01)\,3\cdot 10^{8}} = 1.3\,\mu\mathrm{s}$$

So $\tau_{eff} \gg t_{transit} \Rightarrow$ collisions are rare BUT

$$T_{\parallel\,\substack{accel\\cool}} = \frac{1}{2}\left(\frac{kT_0}{qV}\right) kT_0 = 2.5\cdot 10^{-9}\,\mathrm{eV} \quad \text{for } \begin{matrix} kT_0 = 0.1\,\mathrm{eV} \\ qV = 2\,\mathrm{MeV}\end{matrix}$$

$$T_{\parallel\,collisions} \simeq \frac{2}{3} T_{\perp 0}\left(1 - \exp\left(-3t/\tau_{eff}\right)\right) \simeq 2T_{\perp 0}\left(\frac{t_{transit}}{\tau_{eff}}\right) = .006\,\mathrm{eV} \text{ for } T_{\perp 0} = 1\,\mathrm{eV}$$

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SO $T_{\parallel}$ FROM "BOERSCH EFFECT"

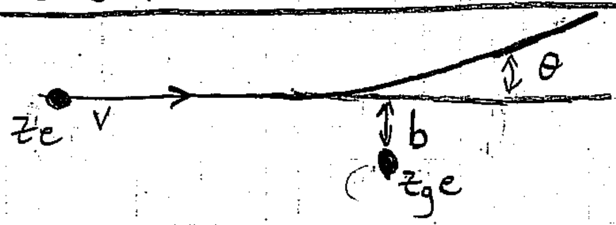
$\gg$ $T_{\parallel}$ FROM LONGITUDINAL COOLING

100 SHEETS
U.S.A.

COULOMB COLLISIONS IN RESIDUAL GAS (REISER 6.4.3)

JACKSON CHAPTER 13

(RUTHERFORD SCATTERING)



$$\frac{dp_x}{dt} = \frac{Z Z_g e^2}{4\pi\epsilon_0 r^2}\,\frac{b}{r} \quad\Rightarrow\quad \Delta p = \int_{-\infty}^{\infty} \frac{dp_x}{dt}\frac{dt}{dz}\,dz$$

$$= \frac{Z Z_g e^2 b}{4\pi\epsilon_0 v} \underbrace{\int_{-\infty}^{\infty} \frac{dz}{(z^2+b^2)^{3/2}}}_{2/b^2}$$

$$= \frac{2 Z Z_g e^2}{4\pi\epsilon_0 v b}$$

$$\theta \cong \frac{\Delta p}{p} = \frac{2 Z Z_g e^2}{4\pi\epsilon_0 p v b} \quad\Rightarrow\quad \frac{db}{d\theta} \sim \frac{1}{\theta^2}$$

DIFFERENTIAL CROSS SECTION FOR SCATTERING WITH IMPACT PARAMETER b INTO SOLID ANGLE $d\Omega$ AT ANGLE θ SATISFIES

$$\underbrace{2\pi b\, db}_{\text{AREA}} = \frac{d\sigma}{d\Omega}\underbrace{2\pi \sin\theta\, d\theta}_{\text{SOLID ANGLE}}$$

$$\Rightarrow \frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right| = \left(\frac{2 Z Z_g e^2}{4\pi\epsilon_0 p v}\right)^2 \frac{1}{\theta^4}$$

ELECTRON SCREENING PUTS CUTOFF AT SMALL θ (LARGE b) SO BETTER TO USE

$$\frac{d\sigma}{d\Omega} = \left(\frac{2 Z Z_g e^2}{4\pi\epsilon_0 p v}\right)^2 \frac{1}{(\theta^2+\theta_{min}^2)^2}$$

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$$\frac{d\sigma}{d\Omega} = \left(\frac{2 z Z_g e^2}{4\pi\epsilon_0 p V}\right)^2 \frac{1}{(\theta^2 + \theta_{min}^2)^2}$$

AVERAGE ANGLE SQUARED FOR A SINGLE SCATTERING IS:

$$\overline{\theta}^2 = \frac{\int \theta^2 \frac{d\sigma}{d\Omega} 2\pi \sin\theta \, d\theta}{\int \frac{d\sigma}{d\Omega} 2\pi \sin\theta \, d\theta} \cong \frac{\int_0^{\theta_{max}} \frac{\theta^3}{(\theta^2+\theta_{min}^2)^2} d\theta}{\int_0^{\theta_{max}} \frac{\theta}{(\theta^2+\theta_{min}^2)^2} d\theta}$$

$$\cong 2\theta_{min}^2 \ln\left(\frac{\theta_{max}}{\theta_{min}}\right)$$

ASSUMES $\theta_{max}^2 >> \theta_{min}^2$

& $\ln\left(\frac{\theta_{max}}{\theta_{min}}\right) >> 1$

MULTIPLE COLLISIONS

AFTER TRAVERSING DISTANCE s, AND UNDERGOING N_s COLLISIONS, THE MEAN SQUARE ANGLE $\overline{\Theta^2}$

$$\left[\sigma_s = \pi\left(\frac{2 z Z_g e^2}{4\pi\epsilon_0 p V}\right)^2 \frac{1}{\theta_{min}^2}\right]$$

$$\overline{\Theta^2} = N_s \overline{\theta}^2 = n_g \sigma_s s \, \overline{\theta}^2$$

$$= 8\pi n_g \left(\frac{z Z_g e^2}{4\pi\epsilon_0 m c^2 \gamma \beta^2}\right)^2 \ln\left(\frac{\theta_{max}}{\theta_{min}}\right) s$$

JACKSON ARGUES θ_{max} ARISES FROM DISTRIBUTED NATURE OF NUCLEUS (NOT POINT CHARGE) AND θ_{min} ARISES FROM SCREENING OF ELECTRONS OR UNCERTAINTY PRINCIPLE

$$\ln\frac{\theta_{max}}{\theta_{min}} \simeq \ln[(204 Z_g^{-1/3})^2] = 2\ln[204 Z_g^{-1/3}]$$

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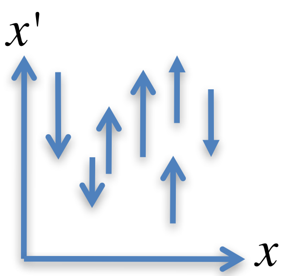
So $\overline{\Theta}^2 = N_s\overline{\theta}^2 = n_g\sigma_s s\overline{\theta}^2$

$$= 8\pi n_g\left(\frac{ZZ_g e^2}{4\pi\varepsilon_0 mc^2\gamma\beta^2}\right)^2 \ln\left(\frac{\theta_{\max}}{\theta_{\min}}\right)s = 16\pi n_g\left(\frac{ZZ_g e^2}{4\pi\varepsilon_0 mc^2\gamma\beta^2}\right)^2 \ln\left(204 Z_g^{-1/3}\right)s$$

Now $\Theta^2 = \langle x'^2\rangle + \langle y'^2\rangle = 2\langle x'^2\rangle$

$$\Rightarrow \frac{d}{ds}\langle x'^2\rangle = 4\pi n_g\left(\frac{ZZ_g e^2}{4\pi\varepsilon_0 mc^2\gamma\beta^2}\right)^2 \ln\left(\frac{\theta_{\max}}{\theta_{\min}}\right)$$
$$\equiv C_{sc}$$

How does scattering change the envelope equations?
We assume the scattering locally changes the transverse momentum, without directly changing the position (thin lens).



So after an incremental distance δs

$$x \to x_0 \qquad x' \to x_0' + \delta x'$$

$$\delta\langle x'^2\rangle = \langle (x_0'+\delta x')^2 - x_0'^2\rangle = 2\underbrace{\langle x_0'\delta x'\rangle}_{0} + \langle \delta x'^2\rangle = \langle \delta x'^2\rangle = C_{sc}\delta s$$

$$\delta\langle xx'\rangle = \langle (x_0'+\delta x')x_0 - x_0'x_0'\rangle = \underbrace{\langle x_0'\delta x'\rangle}_{0} = 0$$

$$\delta\langle x^2\rangle = \langle x_0^2 - x_0^2\rangle = 0$$

And the moment equations become:

$$\frac{d}{ds}\langle x^2\rangle = 2\langle xx'\rangle$$

$$\frac{d}{ds}\langle xx'\rangle = \langle xx''\rangle + \langle x'^2\rangle$$

$$\frac{d}{ds}\langle x'^2\rangle = 2\langle x'x''\rangle + C_{sc}$$

For $x'' = -K(s)x + \dfrac{2Q}{r_x + r_y}\dfrac{x}{r_x}$ and if energy loss is negligible then the envelope equations becomes:

$$r_x'' + \frac{2Q}{r_x + r_y} + K(s)r_x + \frac{\varepsilon_x^2}{r_x^3} = 0$$

$$\frac{d\varepsilon_x^2}{ds} = 4r_x^2 C_{sc}$$

(Envelope equation unchanged, but $\dfrac{d\varepsilon_x^2}{ds} \neq 0$)

For a beam undergoing acceleration or deceleration or if both stopping and scattering are not negligible:

$$r_x'' + \frac{(\gamma\beta)'}{\gamma\beta} r_x' + \frac{2Q}{r_x + r_y} + K(s) r_x + \frac{\varepsilon_{nx}^2}{\gamma^2\beta^2 r_x^3} = 0$$

$$\frac{d\varepsilon_{nx}^2}{ds} = 4\gamma^2\beta^2 r_x^2 C_{sx}$$

$$mc^2 \frac{d\gamma}{ds} = qE_z(s) - \frac{dE_{stopping}}{ds}$$

Example:

$$\frac{d\varepsilon_{nx}^2}{ds} = 4\gamma^2\beta^2 r_x^2 C_{sx} = 32\pi n_g r_x^2 \left(\frac{ZZ_g e^2}{4\pi\varepsilon_0 mc^2\beta} \right)^2 \ln\left(204 Z_g^{-1/3}\right)$$

$$n_g = 10^{-7} \text{ torr} = 3.5 \times 10^9 \text{ cm}^{-3} = 3.5 \times 10^{15} \text{ m}^{-3}$$

$$r_x = 0.01 \text{ m}; \; Z_g = 7; \; Z = 19; \; A = 39; \; \beta = 0.01; \; \varepsilon_N = 1 \times 10^{-6} \text{ m-rad}$$

$$\frac{d\varepsilon_{nx}^2}{ds} = 4.6 \times 10^{-17} \text{ m}^2\text{-rad}^2/\text{m}$$

$$\Rightarrow \varepsilon_{nx}^2 \Big/ \frac{d\varepsilon_{nx}^2}{ds} = 22{,}000 \text{ m}$$

So 22 km needed to equal original emittance! (So more important for rings and/or low mass particles).

PRESSURE "BUMPS" AND

BEAM LOSS FROM CHARGE CHANGING COLLISIONS

REFERENCE: WORKSHOP ON BEAM INDUCED PRESSURE RISE IN RINGS, BNL, Dec. 2003,

σ_s = STRIPPING CROSS SECTION

σ_{ce} = CHARGE EXCHANGE CROSS SECTION

σ_i = IONIZATION CROSS SECTION

V_{cm} = mean ion velocity in ion beam frame

(1) BEAM LOSS

$$\frac{dn_b}{dt} = -\sigma_s v_i n_b \bar{n} - \sigma_{ce} v_{cm} n_b^2 - \left.\frac{\partial n_b}{\partial t}\right|_{HALO}$$

(2) GAS EVOLUTION

$\bar{n}$ = average gas density

$$\frac{d}{dt}\bar{n} = \underbrace{\eta_G \sigma_i v_i n_b \bar{n}}_{\text{IONIZATION}} \left(\frac{V_{beam}}{V_{pipe}}\right) + \underbrace{\eta_{HI} \sigma_s v_i n_b \bar{n}}_{\text{STRIPPING}} \left(\frac{V_{beam}}{V_{pipe}}\right)$$

$$+ \underbrace{\eta_{HI} \sigma_{ce} v_{cm} n_b^2}_{\text{CHARGE EXCHANGE}} \left(\frac{V_{beam}}{V_{pipe}}\right) + \underbrace{q}_{\text{OUTGASSING}} - \underbrace{(S/A_p)\bar{n}}_{\text{PUMPING}}$$

V_{beam}: VOLUME OF BEAM; V_{pipe}: VOLUME OF PIPE

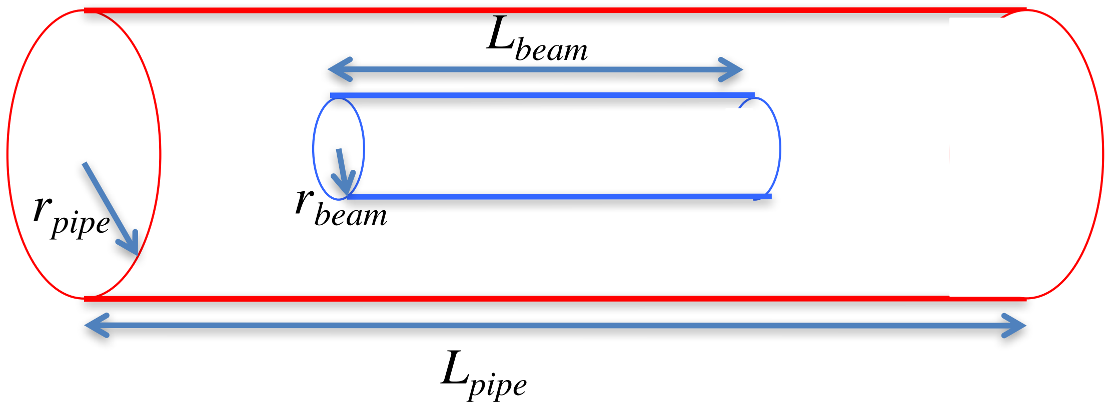
S = Effective linear pumping rate [$m^3 s^{-1}/m$] $\quad A_p = \pi r_p^2$ = AREA OF PIPE

q = outgassing rate $= \frac{2\pi r_p Q}{\pi r_p^2} = \frac{2Q}{r_p}$; $\quad Q = \frac{\#}{m^2 s}$

η_G = GAS MOLECULES DESORBED PER INCIDENT RESIDUAL GAS ION

η_{HI} = GAS MOLECULES DESORBED PER INCIDENT HEAVY ION STRIKING WALL

($V_{beam}/V_{pipe} \rightarrow \left(\frac{r_b^2}{r_{pipe}^2}\right) v_{rep} \Delta t_{beam}$ for a rep rated linac)



$$V_{beam} = \pi r_{beam}^2 L_{beam} \qquad V_{pipe} = \pi r_{pipe}^2 L_{pipe} \qquad A_p = \pi r_{pipe}^2$$

<u>Gas evolution equation:</u>

ionization stripping charge exchange

$$\frac{d\bar{n}}{dt} = \eta_G \sigma_i v_i n_b \bar{n} \left(\frac{V_{beam}}{V_{pipe}}\right) + \eta_{HI} \sigma_s v_i n_b \bar{n} \left(\frac{V_{beam}}{V_{pipe}}\right) + \eta_G \sigma_{ce} v_{cm} n_b^2 \left(\frac{V_{beam}}{V_{pipe}}\right) +$$

$$+ q - \left(\frac{S}{A_p}\right)\bar{n}$$

outgassing pumping

Here S = effective linear pumping rate m^3/s/m = m^2/s
q = effective linear outgassing rate
  = $2\pi r_{pipe} Q_{outgassing}/(\pi r_{pipe}^2) = 2Q_{outgassing}/r_{pipe}$
  where $Q_{outgassing}$ = #/cm^2/s
η_G = gas molecules desorbed per incident ionized gas molecule
η_{HI} = gas molecules desorbed per incident ionized heavy ion
$V_{beam}/V_{pipe} \rightarrow (r_{beam}^2/r_{pipe}^2)(\nu_{rep}\Delta t)$ for a rep rated linac
Δt = pulse duration; ν_{rep} = repetition rate

If we take $n_b \simeq$ constant

then we may express gas evolution equation as:

$$\frac{d\bar{n}}{dt} = \frac{\bar{n}}{\tau} + q_{eff}$$

with solution:

$$\bar{n} = (\bar{n}_0 + \tau q_{eff}) \exp[t/\tau] - \tau q_{eff}$$

HERE $\tau = \dfrac{1}{(\eta_g \sigma_i + \eta_{HI} \sigma_s)\left(\frac{V_{beam}}{V_{pipe}}\right) n_b V_i - S/A_p}$

$$q_{eff} = q + \eta_{HI} \sigma_{CE} V_{cm} n_b^2 \left(\frac{V_{beam}}{V_{pipe}}\right)$$

EQUILIBRIUM REACHED IF $\tau < 0$ (i.e. pumping exceeds desorption).

$$\Rightarrow \quad \bar{n} = -\tau q_{eff} = \frac{q + \eta_{HI} \sigma_{CE} V_{cm} n_b^2 (V_{beam}/V_{pipe})}{S/A_p - (\eta_g \sigma_i + \eta_{HI} \sigma_s)\left(\frac{V_{beam}}{V_{pipe}}\right) n_b V_i}$$

INSTABILITY IF $n_b V_i \gtrsim \dfrac{\frac{S}{A_p}\left(\frac{V_{pipe}}{V_{beam}}\right)}{\eta_g \sigma_i + \eta_{HI} \sigma_s}$

Instability first observed on the ISR proton storage ring, limiting current in rings in 1970's.

If $l_{beam} = l_{pipe}$

INSTABILITY CRITERION MAY BE WRITTEN

$$I > \frac{ZeS}{\eta_g \sigma_i + \eta_{HI} \sigma_s}$$

EXAMPLE: If $S = 100 \, \ell \, s^{-1} m^{-1} = 0.1 \, m^3 \, s^{-1} \, m^{-1}$

ISR

$\eta_g = 4$

$\sigma_i = 10^{-22} \, m^2 = 10^{-16} \, cm^2$; $\sigma_s = 0$

$Z = 1$ (protons)

$\Rightarrow I \leq 40$ Amperes

(PRESSURE RUNAWAYS WERE OBSERVED ON the ISR AT 14-18A, (BENVENUTI et al, IEEE Trans. on Nuc. Sci. NS-24, 1773, 1977)

SEE "BEAM INDUCED PRESSURE RISE IN RINGS"
13th ICFA BEAM DYNAMICS MINI WORKSHOP, BNL, Dec. 9-12, 2003.
WEBSITE: http://www.c-ad.bnl.gov/icfa

"ELECTRON CLOUD EFFECTS"

REFERENCE: CERN e-CLOUD WORKSHOP
http://wwwslap.cern.ch/collective/ecloud02/proceedings.html

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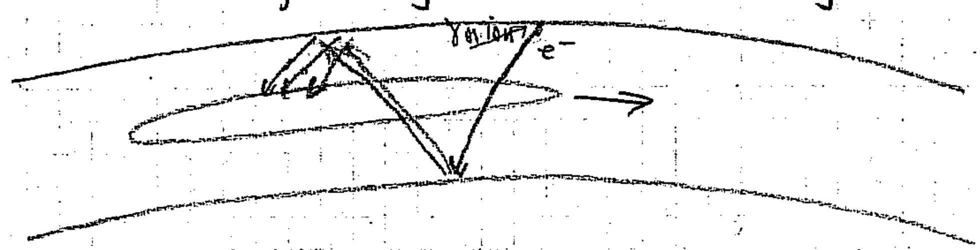
BASIC IDEA

IN ION storage rings or collider rings:



ELECTRONS ARE ATTRACTED TO POSITIVE POTENTIAL OF BEAM & ACCUMULATE

SOME SYMPTOMS:

1. BEAM LOSS & pressure rise
2. HIGH FREQUENCY CENTROID oscillations

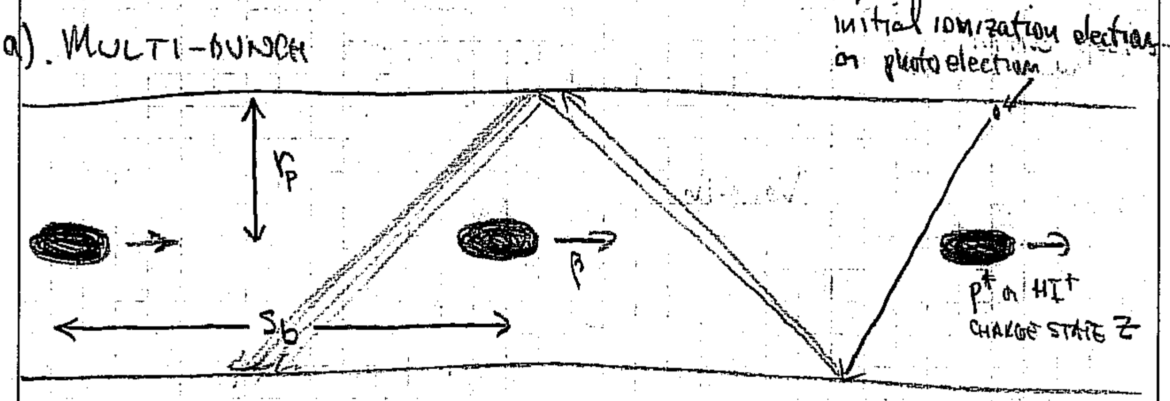
SOME ACCELERATORS WHICH SHOW EVIDENCE OF e^- EFFECTS

1. LANL PSR
2. CERN PS & SPS
3. BNL RHIC

cf. "Electron-cloud effects in High Intensity Proton Accelerators" J. Wei & R. Macek, CERN, 2002

BEAM INDUCED MULTIPACTING

a). MULTI-BUNCH



USING COULOMB COLLISION FORMULA FROM PAGE 9:

N_b = number of ions of charge Z

$$\Delta p_x \cong \frac{2ZN_b e^2}{4\pi\epsilon_0 v r_p} = 2ZN_b\left(\frac{m_e c^2}{v}\right)\left(\frac{r_e}{r_p}\right)$$

$$\Delta E_e = m_e c^2\left[\sqrt{\frac{\Delta p_x^2}{m^2c^2}+1} - 1\right] = m_e c^2\left[\sqrt{\left(\frac{2r_e Z N_b}{\beta r_p}\right)^2 + 1} - 1\right]$$

$$\cong 2 r_e^2 m_e c^2 \frac{Z^2 N_b^2}{\beta^2 r_p^2} \quad \text{for } \Delta E_e \ll m_e c^2 \quad \left(\text{or } \frac{2r_e Z N_b}{\beta r_p} \ll 1\right)$$

$\left(\text{where } r_e = \frac{e^2}{4\pi\epsilon_0 m_e c^2} \simeq 2.8\times10^{-15}\,\text{m}\right)$

DEFINE A MULTIPACTING PARAMETER ζ_m

$$\zeta_m = \frac{\text{TIME FOR ELECTRON TO CROSS PIPE}}{\text{TIME BETWEEN BUNCHES}} = \frac{2r_p}{s_b}\frac{\beta}{\beta_e}$$

$$\cong \frac{\beta^2 r_p^2}{Z N_b r_e s_b}$$

RESONANCE CONDITION: $\zeta_m = 1$

s_b = distance between bunches

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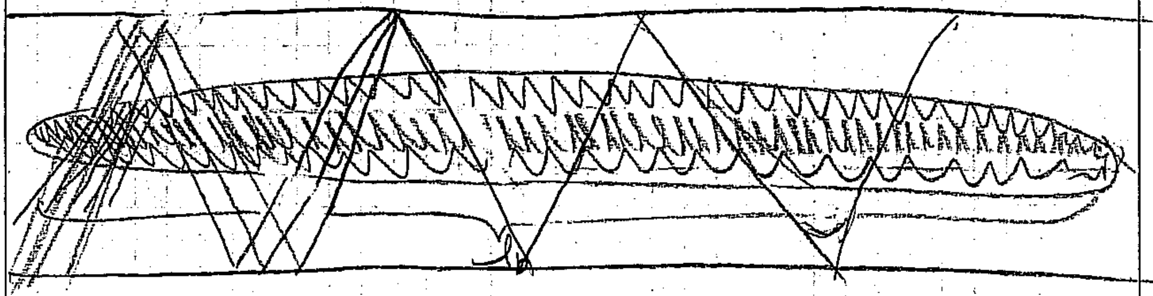
b). SINGLE-BUNCH BEAM-INDUCED MULTIPACTING



$$\xi_s = \frac{r_p \, \beta}{l_b \, \beta_e} = \frac{\text{time for electrons to cross pipe}}{\text{passage time for half of the bunch}}$$

Recall:

$$\varphi = \begin{cases} \frac{\lambda}{2\pi\varepsilon_0}\left[\frac{1}{2}\left(1-\frac{r^2}{r_b^2}\right) + \ln\frac{r_p}{r_b}\right] & 0 < r < r_b \\ \frac{\lambda}{2\pi\varepsilon_0}\left[\ln\frac{r_p}{r}\right] & r_b < r < r_p \end{cases}$$

$$\varphi \sim \frac{\lambda}{2\pi\varepsilon_0} \sim \frac{ZeN_o}{2\pi\varepsilon_0 l_b}$$

$$\frac{1}{2} m_e v_e^2 + q\varphi \simeq \text{const} \simeq 0$$

$$\beta_e \sim \frac{1}{2}\sqrt{\frac{2q\varphi}{m_e c^2}} \sim \sqrt{\frac{N_0 Z e^2}{l_b 4\pi\varepsilon_0 m_e c^2}} \sim \sqrt{\frac{Z r_e N_0}{l_b}}$$

(AVERAGE e⁻ VELOCITY)

$$\Rightarrow \xi_s = \frac{\beta r_p}{\sqrt{r_e l_b N_0 Z}} = \frac{\text{pipe crossing time}}{.5\text{*pulse duration}}$$

$$\left(r_e = \frac{e^2}{4\pi\varepsilon_0 m_e c^2} \right)$$

THE ENERGY GAIN OF THE ELECTRON, RELIES ON THE DENSITY CHANGING OVER THE COURSE OF THE BUNCH.

$$\Delta E_e \sim \frac{m_e c^2}{2}\left[\frac{Z r_e N_0(s)}{l_b}\right] - \frac{m_e c^2}{2}\left[\frac{Z r_e N_0(s+\Delta s)}{l_b}\right]$$

$$\sim \frac{m_e c^2}{2}\left(\frac{\partial N_0}{\partial z}\Delta z\right)\left(\frac{Z r_e}{l_b}\right)$$

$$\Delta E_e \sim \frac{m_e c^2}{Z}\left(\frac{\partial N_0}{\partial z}\Delta z\right)\left(\frac{Z r_e}{l_b}\right)$$

$$\Delta z=(r_p/v_e)\beta c= \frac{r_p}{v_e}\beta c = \beta r_p\sqrt{\frac{l_b}{Z r_e N_0}}\;; \qquad \frac{\partial N_0}{\partial z} \sim \frac{N_0}{l_b} \qquad \left(r_e = \frac{e^2}{4\pi\varepsilon_0 m_e c^2}\right)$$

So $$\Delta E_e \approx m_e c^2\left(\frac{Z N_0 r_e}{l_b^3}\right)^{1/2}\beta r_p \sim e\varphi\,\zeta_s \qquad v_e = c\sqrt{\frac{2\Delta E_e}{m_e c^2}}$$

$\zeta_s \leq 1$ Electron build up possible within bunch where $\zeta_s = \frac{r_p\,\beta}{l_b\,\beta_e}$

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WHAT IS STEADY STATE ELECTRON DENSITY?

Electrons can build up until E_r at pipe ~ 0.

$$\Rightarrow \lambda_e = \lambda_I$$

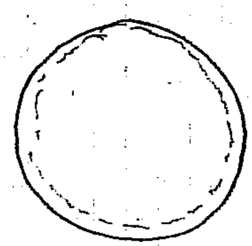
$$\pi r_p^2 n_e = \pi r_b^2 Z n_i$$

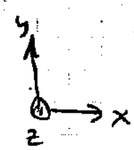
$$n_e = \left(\frac{r_b}{r_p}\right)^2 Z n_i$$

ELECTRON - ION INSTABILITY

(SEE ALSO R.C. DAVIDSON & H. QIN, Physics of Intense Charged Particle Beams in High Energy Accelerators, p. 503 FOR KINETIC TREATMENT).

CONSIDER A UNIFORM DISTRIBUTION OF ELECTRONS (AT REST) WHICH HAS THE SAME RADIUS (OR SLIGHTLY SMALLER RADIUS) AS A UNIFORM DENSITY ION BEAM, THAT IS MOVING AT VELOCITY V_z (OUT OF THE PLANE OF THE PAPER).





Electric field from ions:

$$E_x = \frac{\lambda}{2\pi\epsilon_0}\frac{(x-x_i)}{r_b^2} = \frac{\varrho_i}{2\varepsilon_0}(x - x_i)$$

THE EQUATION OF MOTION FOR THE CENTROID OF THE ELECTRONS IS OBTAINED FROM the equation of motion for single electron:

$$m_e\frac{d^2x}{dt^2} = -\frac{e\varrho_i}{2\varepsilon_0}(x-x_i) + \frac{e\varrho_e}{2\varepsilon_0}(x-x_e)$$

Taking statistical average:

$$\frac{d^2 x_e}{dt^2} = -\frac{\omega_{pi}^2}{2}\left(\frac{m_i}{q}\frac{e}{m_e}\right)(x_e - x_i)$$

here $\omega_{pi}^2 = \frac{q^2 n_i}{\epsilon_0 m_i} = \frac{q\rho_i}{\epsilon_0 m_i}$

(THE CENTER OF oscillation for the electrons is the center of the ion beam).

x_e = centroid of electron beam =<x> for electrons

x_i = centroid of ion beam =<x> for ion

Similarly the single particle equation for the ion is:

$$m_i \frac{d^2x}{dt^2} = -m_i\omega_{\beta 0}^2 x + \frac{q\lambda_i}{2\pi\varepsilon_0 r_b^2}(x - x_i) - \frac{q\lambda_e}{2\pi\varepsilon_0 r_b^2}(x - x_e)$$

$$= -m_i\omega_{\beta 0}^2 x + \frac{q\varrho_i}{2\varepsilon_0}(x - x_i) - \frac{q\varrho_e}{2\varepsilon_0}(x - x_e)$$

Here $\omega_{\beta 0} \equiv v_z k_{\beta 0}$

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The equation of motion for the centroid of the ions is found by taking statistical averages:

$$\Rightarrow \quad \frac{d^2 X_i}{dt^2} = -\omega_{\beta 0}^2 X_i - f\,\frac{\omega_{\rho i}^2}{2}(X_i - X_e)$$

HERE $f \equiv \dfrac{e N_e}{q N_i}$ = fractional neutralization

Now $\dfrac{d}{dt}$ = total derivative $= \dfrac{\partial}{\partial t} + v_z \dfrac{\partial}{\partial z}$

$\Rightarrow$ THE ION & ELECTRON EQUATIONS MAY BE WRITTEN

$$\left(\frac{\partial}{\partial t} + v_z\frac{\partial}{\partial z}\right)^2 X_i = -\omega_{\beta 0}^2 X_i - f\,\frac{\omega_{\rho i}^2}{2}(X_i - X_e)$$

$$\frac{\partial^2}{\partial t^2} X_e = -\frac{\omega_{\rho i}^2}{2}\left(\frac{m_i}{q}\frac{e}{m_e}\right)(X_e - X_i)$$

Now let $X_e = X_e \exp[i(\omega t - kz)]$; $\quad X_i = X_i \exp[i(\omega t - kz)]$

$$\Rightarrow (-\omega^2 + 2\omega k V_z - k^2 V_z^2) X_i = -\omega_{\beta 0}^2 X_i - f \frac{\omega_{pi}^2}{2}(X_i - X_e)$$

$$-\omega^2 X_e = -\frac{\omega_{pi}^2}{2}\left(\frac{m_i}{m_e}\frac{e}{q}\right)(X_e - X_i)$$

$$\Rightarrow \left[(\omega - kV_z)^2 - \omega_{\beta 0}^2 - f\frac{\omega_{pi}^2}{2}\right] X_i = -\frac{f\omega_{pi}^2}{2} X_e$$

$$\left[\omega^2 - \frac{\omega_{pi}^2}{2}\left(\frac{m_i}{m_e}\frac{e}{q}\right)\right] X_e = -\frac{\omega_{pi}^2}{2}\left(\frac{m_i}{m_e}\frac{e}{q}\right) X_i$$

Muliplying the above equations and dividing by $X_e X_i$, yields the dispersion relation:

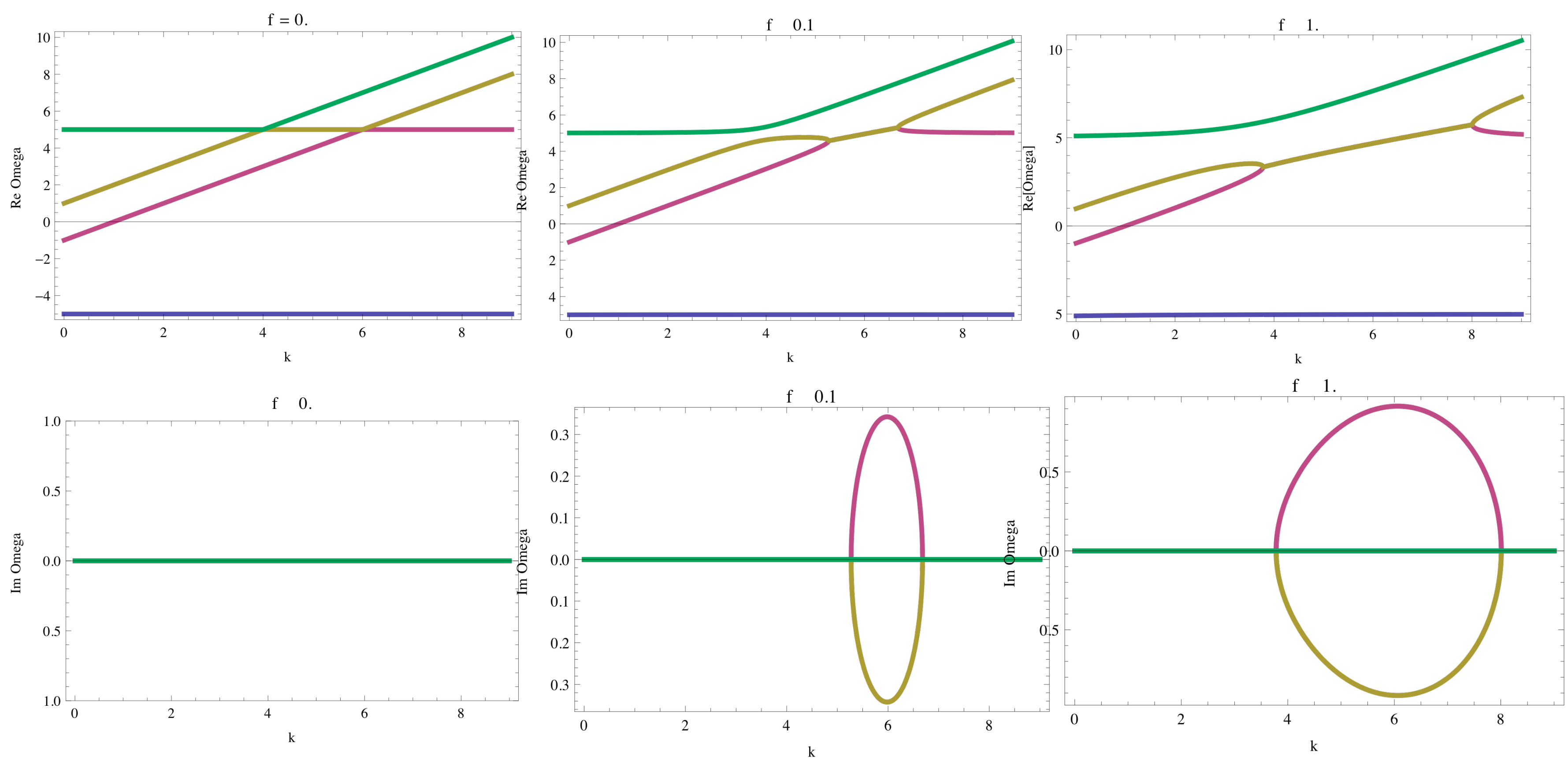
$$\underbrace{\left[(\omega - kV_z)^2 - \omega_{\beta 0}^2 - f\frac{\omega_{pi}^2}{2}\right]}_{\text{ION BETATRON FREQUENCY (INCREASED BY SPACE CHARGE OF ELECTRONS)}} \underbrace{\left[\omega^2 - \frac{\omega_{pi}^2}{2}\left(\frac{m_i}{m_e}\frac{e}{q}\right)\right]}_{\text{ELECTRON oscillating in POTENTIAL WELL OF ION}} = \underbrace{+\frac{f\omega_{pi}^4}{4}\left(\frac{m_i}{m_e}\frac{e}{q}\right)}_{\text{COUPLING}}$$

with high spatial frequency undergoing betatron oscillations in the comoving frame, $\therefore kV_z - \omega \cong \sqrt{\omega_{\beta 0}^2 + \frac{f\omega_{pi}^2}{2}}$ will resonate with electrons oscillating in the ion well if

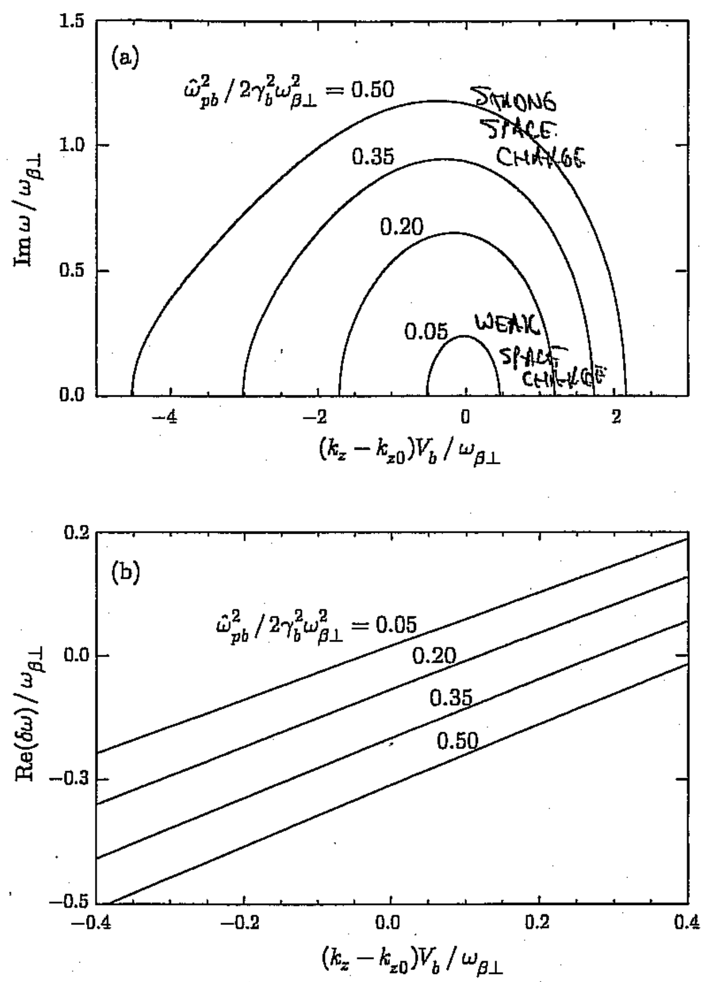
$$\omega \cong \frac{\omega_{pi}}{\sqrt{2}}\sqrt{\frac{m_i}{m_e}\frac{e}{q}}$$

Giving rise to instability!

Dispersion relation for two stream instability
(m_e/m_i=0.04; $\omega_{\beta 0} = \omega_{pi}/2^{1/2}$=1; v=1)



From Davidson & Qin (22)
Physics of Intense Charged Particle Beams, 2001.



$\omega_{\beta\perp}^2 \equiv \omega_{\beta 0}^2$

$\frac{\hat{\omega}_{pb}^2}{2\gamma_b^2 \omega_{\beta\perp}^2} = \left(1 - \frac{\Omega^2}{\Omega_0^2}\right)$

Figure 10.11. Plots of (a) normalized growth rate ($Im\omega/\omega_{\beta\perp}$), and (b) normalized real frequency ($(Re\omega - \omega_e)/\omega_{\beta b}$) versus shifted axial wavenumber $(k_z - k_{z0})V_b/\omega_{\beta\perp}$ obtained from the dispersion relation (10.103) for the unstable branch with positive real frequency. System parameters correspond to $v_{T\parallel b} = 0 = v_{T\parallel e}$, $m_b/m_e = 1836$ (protons), $(\gamma_b - 1)m_b c^2 = 800$ MeV, $r_b/r_w = 0.5$, and $f = 0.1$. Curves are shown for several values of normalized beam intensity $\hat{\omega}_{pb}^2/2\gamma_b^2\omega_{\beta\perp}^2$ ranging from 0.05 to 0.5.

$k_{z0} V_z = \omega \mp \sqrt{\omega_{\beta 0}^2 + f\omega_i^2/2}$; $\omega = \frac{\omega_{pi}}{2}\sqrt{\frac{m_i e}{m_e q}}$

From Davidson & Qin, 2001 (123)

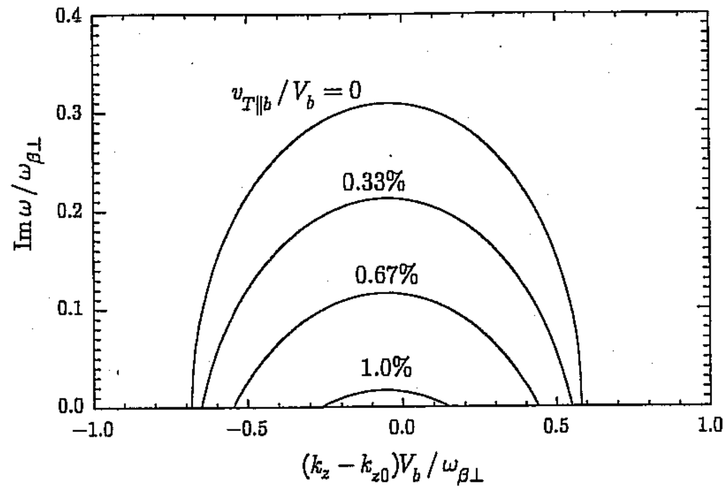


Figure 10.12. Plot of normalized growth rate ($Im\omega/\omega_{\beta\perp}$), and normalized real frequency $(Re\omega - \omega_e)/\omega_{\beta\perp}$ versus positive real frequency. System parameters correspond to $\hat{\omega}_{pb}^2/2\gamma_b^2\omega_{\beta\perp}^2 = 0.07$, $v_{T\|e} = v_{T\|b}$, $m_b/m_e = 1836$ (protons), $(\gamma_b - 1)m_bc^2 = 800$ MeV, $r_b/r_w = 0.5$, and $f = 0.1$. Curves are shown for several values of normalized ion thermal spread $v_{T\|b}/V_b$ ranging from 0 to 0.01.

velocity V_b [Eq. (10.105)], it is expected that Landau damping by parallel ion kinetic effects can have a strong stabilizing influence at modest values of $v_{T\|b}/V_b$. That this is indeed the case is evident from Fig. 10.12, which shows a substantial reduction in maximum growth rate and eradication of the instability over the instability bandwidth as $v_{T\|b}/V_b$ is increased from 0 to 0.01.

We now make use of the nonlinear particle perturbative simulation method [60, 61] described in Sec. 8.5 to illustrate several important properties of the two-stream instability in intense beam systems (Sec. 10.4.2).

PREVENTIVE MEASURES (from J. Wei & R. Macek, CERN electron cloud workshop 2003).

- SUPPRESS ELECTRON GENERATION
  - SURFACE TREATMENT OF THE VACUUM PIPE
  - KICKER MAGNETS IN GAPS
  - VACUUM PORTS SCREENED TO REDUCE E-field
  - CLEARING ELECTRODES
  - HIGH VACUUM
  - SOLENOIDS - TO REDUCE MULTIPACTING

SUMMARY OF ELECTRON, GAS, PRESSURE, & SCATTERING EFFECTS

1. COULOMB COLLISIONS WITHIN BEAM CAN TRANSFER ENERGY FROM ⊥ TO ∥ AND PROVIDE LOWER LIMIT ON $T_{\parallel}$, HIGHER THAN FROM ACCELERATIVE COOLING.

2. COULOMB INTERACTIONS WITH RESIDUAL GAS NUCLEI PROVIDE A SOURCE OF EMITTANCE GROWTH (BUT NOT IMPORTANT FOR HIGHER MASS AND LINAC RESIDENCE TIMES).

3. PRESSURE INSTABILITY FROM DESORPTION OF RESIDUAL GAS BY STRIPPED BEAM IONS HITTING WALL OR BEAM-IONIZED RESIDUAL GAS ATOMS, FORCED TO WALL BY E-field of BEAM. LIMITS CURRENT IN RINGS OR HIGH REP RATE LINAC.

4. ELECTRONS CAN CASCADE AND REACH A "QUASI" EQUILIBRIUM POPULATION OF SIMILAR LINE CHARGE TO THE ION BEAM. ELECTRON-ION TWO STREAM INSTABILITY IS UNSTABLE, AND CAN LEAD TO TRANSVERSE INSTABILITY, SIMILAR TO WHAT IS OBSERVED IN SOME PROTON RINGS.

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