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Summary of JB lectures

START WITH MICROSCOPIC PHASE SPACE DENSITY

$$\boxed{N(\underline{x},\underline{v},t) = \sum_{i=1}^{N} \delta(\underline{x} - \underline{X}_i(t))\,\delta(\underline{v} - \underline{V}_i(t))}$$ Klimontovitch Density

$\frac{\partial N}{\partial t}$ + LORENTZ EQUATIONS ⇒ KLIMONTIVICH EQUATION:

$$\boxed{\frac{\partial N}{\partial t} + v \cdot \nabla_x N(\underline{x},\underline{v},t) - \frac{q}{m}(E^m + v \times B^m) \cdot \nabla_v N(\underline{x},\underline{v},t) = 0}$$

$$\boxed{\text{or } \frac{dN(\underline{x},\underline{v},t)}{dt} = 0}$$

Letting $N = f + \delta f$ $\quad f = \langle N \rangle$ $\quad f = \frac{\int N\, d^3x\, d^3v}{\Delta x^3 \Delta V^3}$

$E^m = E + \delta E$ $\quad E = \langle E_m \rangle$

$B^m = B + \delta B$ $\quad B = \langle B_m \rangle$ $\quad n^{-1/3} \ll \Delta x \ll \lambda_D$

PERFORMING LOCAL AVERAGES TO OBTAIN SMOOTH & "SPIKEY" QUANTITIES:

$$\boxed{\frac{\partial f}{\partial t} + \dot{\underline{v}} \cdot \frac{\partial f}{\partial \underline{x}} + \frac{d\underline{v}}{dt} \cdot \frac{\partial f}{\partial \underline{v}} = \frac{\partial f}{\partial t_c}} \sim \frac{f}{\tau_c}$$

We estimated $\dfrac{|\partial f/\partial t_c|}{\left|\frac{q\underline{E}}{m} \cdot \partial f/\partial \underline{v}\right|} \sim \dfrac{1}{16\,\lambda_D^3 n_0} \ll 1$

$\lambda_D = V_{TH}/\omega_p$ $\quad v_{th} \equiv \sqrt{\frac{kT}{m}}$ $\quad \omega_p \equiv \sqrt{\frac{q^2 n}{\varepsilon_0 m}}$

$$\frac{\partial f}{\partial t} + \underline{v} \cdot \frac{\partial f}{\partial \underline{x}} + \frac{d\underline{p}}{dt} \cdot \frac{\partial f}{\partial \underline{p}} = 0 \quad ; \quad \dot{p} = -\frac{\partial H}{\partial \underline{x}} \; ; \; \dot{x} = \frac{\partial H}{\partial \underline{p}}$$

$$\boxed{\frac{df}{dt} = 0}$$ LIOUVILLES EQUATION (INCOMPRESSIBILITY OF PHASE VOLUME)

DEFINE NORMALIZED EMITTANCES PROPORTIONAL TO $\Delta p_z \Delta z \propto \Delta E \Delta t$

$\Delta p_x \Delta x$

$\Delta p_y \Delta y$

SO THAT

$$\varepsilon_{Nx}^2 = \gamma^2\beta^2\left(\langle x^2\rangle\langle x'\rangle - \langle x x'\rangle^2\right)$$

⇒ CONSTANT IF FORCES ARE LINEAR IN X & FILAMENTATION IS ABSENT. (LINEAR WITHOUT COUPLING TO Z, OR Y).

WE DERIVED TWO SETS OF PARTICLE EQUATION OF MOTION:

PARAXIAL EQUATION (FOR AXISYMMETRIC SYSTEMS) $\left(\frac{\partial}{\partial\theta} = 0\right)$

STARTING WITH THE LORENTZ FORCE EQUATION $\frac{dP}{dt} = q(E + \underline{v} \times \underline{B})$ IN cyl. COOR

$$\underset{\substack{\uparrow\\ \text{INERTIAL}}}{\frac{d}{dt}(\gamma m \dot{r})} - \underset{\substack{\uparrow\\ \text{CENTRIFUGAL}}}{\gamma m r \dot{\theta}^2} = q\Big(\underset{\substack{\uparrow\\ \text{Er external}\\ \text{(DIVERGENCE OF}\\ E = 0)}}{\frac{V''}{2} r} + \underset{\substack{\uparrow\\ v_\theta B_z}}{r\dot{\theta} B}\Big) + \underset{\text{SELF-FIELDS}}{q\left(E_r^{self} + v_z B_\theta^{self}\right)}$$

$$\boxed{\dot{r} \equiv \frac{dr}{dt};\ r' = \frac{dr}{ds} = \frac{\dot{r}}{\beta c}}$$

θ-component:

$$p_\theta = \gamma m r^2 \dot{\theta} + \frac{qB(z) r^2}{2} = \text{constant}$$

$$= \gamma m r^2 \beta c \theta' + \frac{qBr^2}{2} = \text{constant}$$

$$\underset{\substack{\uparrow\\ \text{INERTIAL}}}{r''} + \underset{\substack{\uparrow\\ \text{ACCELERATION}\\ \text{(INERTIA)}}}{\frac{(\gamma\beta)'}{\gamma\beta} r'} + \underset{\substack{\uparrow\\ \text{ER}\\ \text{(CONVERGENCE}\\ \text{OF FIELD LINES)}}}{\frac{\gamma''}{2\beta^2\gamma} r} + \underset{\substack{\uparrow\\ v_\theta B_z - \text{CENTRIFUGAL}}}{\left(\frac{\omega_c}{2\gamma\beta c}\right)^2 r} - \underset{\substack{\uparrow\\ \text{CENTRIFUGAL}}}{\left(\frac{p_\theta}{\gamma\beta m c}\right)^2 \frac{1}{r^3}} - \underset{\substack{\uparrow\\ \text{SELF FIELD}}}{\frac{q}{\gamma^3 m v_0^2} \frac{\lambda(r)}{2\pi\epsilon_0 r}} = 0$$

STATISTICAL AVERAGE OF THIS EQUATION $\quad r_b^2 \equiv 2\langle r^2 \rangle$

$$r_b'' + \frac{(\gamma\beta)'}{\gamma\beta} r_b' + \frac{\gamma''}{2\beta^2\gamma} r_b + \left(\frac{\omega_c}{2\gamma\beta c}\right)^2 r_b - \frac{4\langle p_\theta\rangle^2}{(\gamma m\beta c)^2 r_b^3} - \frac{\varepsilon_r^2}{r_b^3} - \frac{Q}{r_b} = 0$$

$$\varepsilon_r^2 \equiv 4\left(\langle r^2\rangle\langle r'^2\rangle - \langle rr'\rangle^2 + \langle r^2\rangle\langle r^2\theta'^2\rangle - \langle r^2\theta'\rangle^2\right); \quad Q = \frac{q\lambda}{2\pi\epsilon_0 \gamma^3 \beta^2 m c^2}$$

$$= \varepsilon_x^2 - 4\langle r^2\theta'\rangle^2 \quad (\text{if } \rho = \rho(r) \text{ only})$$

CARTESIAN EQUATION OF MOTION

J BARNARD

EQUATION OF MOTION

AGAIN STARTING WITH $\frac{d\underline{p}}{dt} = q(\underline{E} + \underline{v} \times \underline{B})$

RETURN TO X, Y COORDINATES

$$x'' + \frac{1}{\gamma v_z}\frac{d}{ds}(\gamma v_z)x' = \frac{-q}{\gamma^3 m v_z^2}\frac{\partial \varphi}{\partial x} \pm \begin{cases} \frac{qB'}{\gamma m v_z} x & \text{for magnetic quads} \\ \frac{qE'}{\gamma m v_z^2} x & \text{for electric quads} \end{cases}$$

$$\text{Let } \frac{\gamma m v_z}{q} = \frac{p}{q} \equiv [B\rho] \equiv \text{RIGIDITY}$$

$$y'' + \frac{1}{\gamma v_z}\frac{d}{ds}(\gamma v_z)y' = \frac{-q}{\gamma^3 m v_z^2}\frac{\partial \varphi}{\partial y} \mp \begin{cases} \frac{B'}{[B\rho]} y & \text{magnetic} \\ \frac{qE'}{\gamma m v_z^2} y & \text{electric} \end{cases}$$

Define r_x, r_y, ε_x, ε_y, in terms of 2nd order moments

ENVELOPE EQUATION

$$\boxed{\begin{aligned} \frac{d\langle x^2\rangle}{ds} &= 2\langle xx'\rangle \\ \frac{d\langle xx'\rangle}{ds} &= \langle xx''\rangle + \langle x'^2\rangle \\ \frac{d\langle x'^2\rangle}{ds} &= 2\langle x'x''\rangle \end{aligned}}$$

$$r_x^2 = 4\langle x^2\rangle; \qquad r_y^2 = 4\langle y^2\rangle$$

$$r_x' = \frac{4\langle xx'\rangle}{r_x}$$

$$r_x'' = \frac{4\langle xx''\rangle}{r_x} + \frac{\varepsilon_x^2}{r_x^3}; \qquad \varepsilon_x^2 = 16(\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2)$$

$$r_y'' = \frac{4\langle yy''\rangle}{r_y} + \frac{\varepsilon_y^2}{r_y^3} \qquad \varepsilon_y^2 = 16(\langle y^2\rangle\langle y'^2\rangle - \langle yy'\rangle^2)$$

for magnetic focusing:

$$r_x'' + \frac{1}{\gamma v_z}\frac{d}{ds}(\gamma v_z)r_x' + \frac{4q}{\gamma^3 m v_z^2}\frac{\langle x\frac{\partial \varphi}{\partial x}\rangle}{r_x} \pm \frac{B'}{[B\rho]}r_x - \frac{\varepsilon_x^2}{r_x^3} = 0$$

$$r_y'' + \frac{1}{\gamma v_z}\frac{d}{ds}(\gamma v_z)r_y' + \frac{4q}{\gamma^3 m v_z^2}\frac{\langle y\frac{\partial \varphi}{\partial y}\rangle}{r_y} \pm \frac{B'}{[B\rho]}r_x - \frac{\varepsilon_y^2}{r_y^3} = 0$$

(for electric focusing $\frac{B'}{[B\rho]} \rightarrow \frac{qE'}{\gamma m v_z^2}$)

We included a SPACE CHARGE TERM WITH ELLIPTICAL SYMMETRY

NOW DEFOCUSING IN ONE DIRECTION AND FOCUSING IN THE OTHER $\Rightarrow$ RADIAL SYMMETRY SHOULD BE REPLACED BY ELLIPTICAL SYMMETRY: $\rho = \rho\left(\frac{x^2}{r_x^2} + \frac{y^2}{r_y^2}\right)$

CAN BE SHOWN THAT

$$\left\langle x \frac{\partial \phi}{\partial x} \right\rangle = \frac{-\lambda}{4\pi\varepsilon_0} \frac{r_x}{r_x + r_y}$$

$$\left\langle y \frac{\partial \phi}{\partial y} \right\rangle = \frac{-\lambda}{4\pi\varepsilon_0} \frac{r_y}{r_x + r_y}$$

USE $\phi(x,y) = \frac{-r_x r_y}{4\varepsilon_0} \int_0^\infty \frac{\eta(\chi)\, ds}{\sqrt{r_x^2+s}\sqrt{r_y^2+s}}$ to prove, where $\hat{\rho}(\chi) = \frac{d\eta}{d\chi}$

$\rho(x,y) = \hat{\rho}(\chi)\big|_{s=0}$

$\chi = \frac{x^2}{r_x^2+s} + \frac{y^2}{r_y^2+s}$

DEFINING $Q = \frac{2\lambda q}{4\pi\varepsilon_0 \gamma^3 m v_z^2}$

$$\Rightarrow \boxed{\begin{aligned} r_x'' + \frac{1}{\gamma v_z}\frac{d}{ds}(\gamma v_z) r_x' - \frac{2Q}{r_x + r_y} \mp \frac{B'}{[B\rho]} r_x - \frac{\varepsilon_x^2}{r_x^3} &= 0 \\ r_y'' + \frac{1}{\gamma v_z}\frac{d}{ds}(\gamma v_z) r_y' - \frac{2Q}{r_x + r_y} \pm \frac{B'}{[B\rho]} r_y - \frac{\varepsilon_y^2}{r_y^3} &= 0 \end{aligned}}$$

(for Electric Focusing $\frac{B'}{[B\rho]} \to \frac{qE'}{\gamma m v_z^2}$).

(ANALOGUE TO CIRCULAR BEAM:

$\left\langle r \frac{\partial \phi}{\partial r} \right\rangle = \frac{-\lambda}{4\pi\varepsilon_0}$ PROVED IN HOMEWORK)

Envelope Equations derived in course:

I. Statistical rms envelope equations (envelopes defined in terms of rms quantities; emittance not guaranteed to be a conserved quantity).

1. Paraxial: r_b; azimuthal symmetry; $\rho(r)$
2. Cartesian; r_x, r_y; elliptical symmetry $\rho(x^2/r_x^2 + y^2/r_y^2)$
3. Longitudinal: r_z for $E_z = -\frac{g}{4\pi\varepsilon_0}\frac{\partial\lambda}{\partial z} \propto z; \quad \lambda \propto (1-4z^2/r_z^2); \quad v \propto z/r_z$
4. Ellipsoidal (rf) bunches: $r_\perp$, r_z(Also r_x, r_y, r_z; cf Wangler sec 9.9)
5. Cartesian with images: r_x, r_y;
6. Larmor frame: periodic solenoids: $\tilde{r}_x$, $\tilde{r}_y$
7. Cartesian including scattering: r_x, r_y; emittance evolves

$$\frac{d\varepsilon_x^2}{ds} = 4C_{sc}r_x^2$$

II. Kinetic envelope equations (constraint equations governing the parameters of the distribution function.
Emittance conserved.)

1. KV distribution elliptical uniform density beam
$f(x,x',y,y') \sim \delta(1-C_x-C_y); \quad E_x \sim x; \quad E_y \sim y;$
(Identical envelope equation to #2 above).
2. Neuffer distribution for 1D (longitudinal projections of phase space) parabolic line charge density profiles
$f(z,z') \sim (1-C_z)^{1/2}$; $E_z \sim z$;
(Identical envelope equation to #3 above).

III. Moment equations

1. Transverse with chromatic effects

$$\langle x^2\rangle, \langle xx'\rangle, \langle x'^2\rangle, \langle x^2\delta\rangle, \langle xx'\delta\rangle, \langle x'^2\,\delta\rangle, \ldots$$

Summary of current limits for different focusing systems

Einzel lens

$$Q_{\max} \cong \frac{3\pi^2}{8}\left(\frac{q\phi_0}{mv_0^2}\right)^2\left(\frac{r_b}{L}\right)^2$$

Solenoid

$$Q_{\max} \cong \left(\frac{\omega_c r_b}{2\gamma\beta c}\right)^2$$

Quadrupole

$$Q_{\max} \cong \frac{\eta\sigma_0}{2\pi}\left(\frac{\sin\frac{\eta\pi}{2}}{\frac{\eta\pi}{2}}\right)\begin{cases}\left(\frac{Br_b}{[B\rho]}\right)\left(\frac{r_b}{r_p}\right) & \textbf{magnetic}\\ \left(\frac{2qV_q}{\gamma m v_z^2}\right)\left(\frac{r_b}{r_p}\right)^2 & \textbf{electric}\end{cases}$$

$$\sigma_0 \sim \begin{cases}\eta L^2 B/(r_p[B\rho]) & \text{electric}\\ 2\eta L^2 qV_q/(r_p^2\gamma m v_z^2) & \text{magnetic}\end{cases}$$

Here 2ϕ_0 = voltage between Einzel lenses;
Vq = quad voltage relative to ground; qV = ion energy

For non-relativistic beams: $\lambda_{\max} \cong 4\pi\varepsilon_0 V Q_{\max}$

$$\lambda_{\max} \propto \frac{\phi_0^2}{V}$$

$$\lambda_{\max} \propto \frac{q}{m}B^2 r_p^2$$

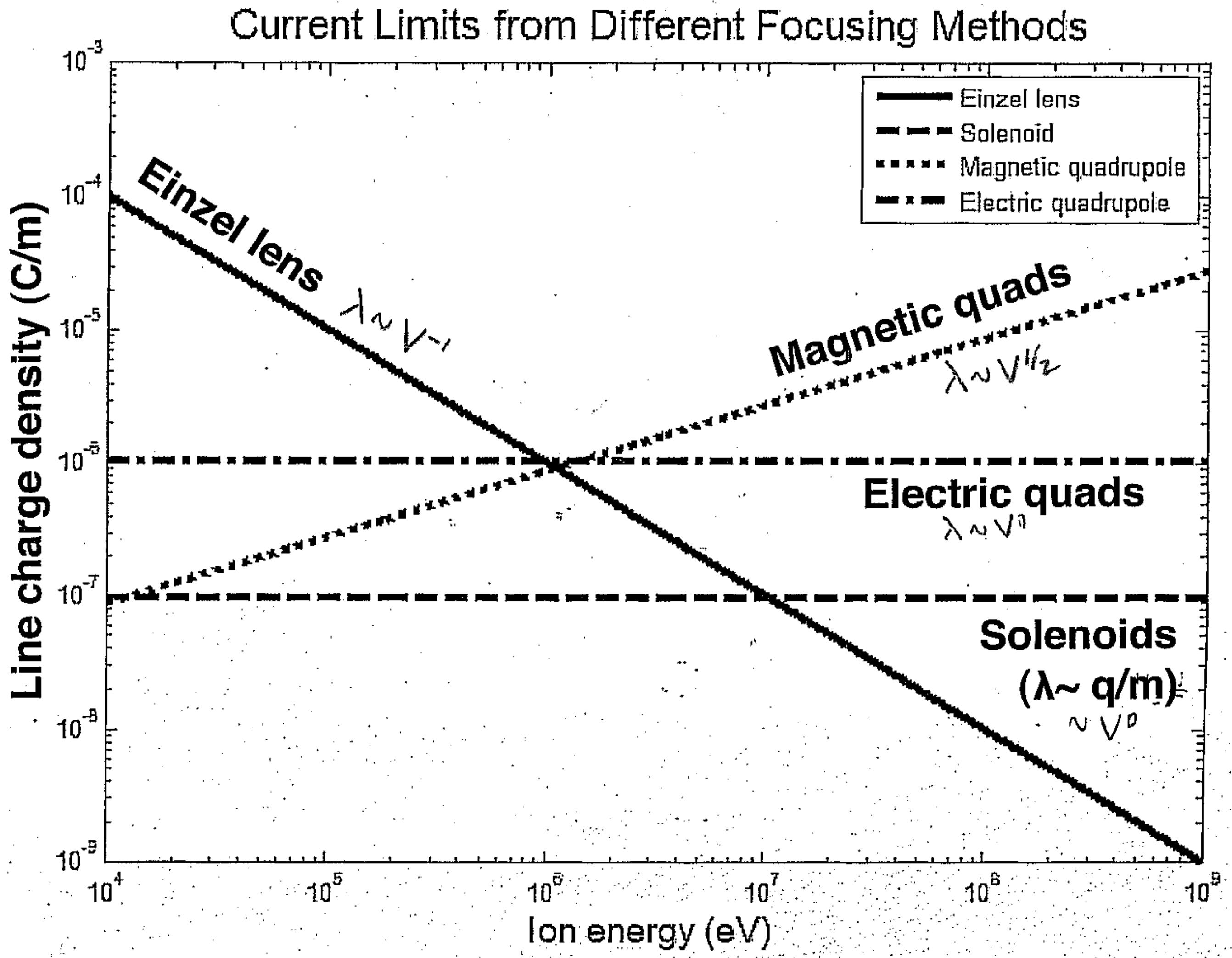
$$\lambda_{\max} \propto \begin{cases}\left(\frac{qV}{m}\right)^{1/2} Br_b & \textbf{magnetic}\\ V_q & \textbf{electric}\end{cases}$$

For non-relativistic beams: $I_{\max} \cong \beta c\lambda_{\max} = \left(\frac{qV}{m}\right)^{1/2}\lambda_{\max}$

$$I_{\max} \propto \left(\frac{q}{m}\right)^{1/2}\frac{\phi_0^2}{V^{1/2}}$$

$$I_{\max} \propto \left(\frac{q}{m}\right)^{3/2} V^{1/2} B^2 r_p^2$$

$$I_{\max} \propto \begin{cases}\left(\frac{qV}{m}\right) Br_b & \textbf{magnetic}\\ \left(\frac{qV}{m}\right)^{1/2} V_q & \textbf{electric}\end{cases}$$


Current Limits from Different Focusing Methods
Einzel lens
Solenoid
Magnetic quadrupole
Electric quadrupole
Einzel lens
λ ~ V⁻¹
Magnetic quads
λ ~ V^{1/2}
Electric quads
λ ~ V⁰
Solenoids
(λ~ q/m)
~ V⁰
Line charge density (C/m)
10⁻³
10⁻⁴
10⁻⁵
10⁻⁶
10⁻⁷
10⁻⁸
10⁻⁹
Ion energy (eV)
10⁴
10⁵
10⁶
10⁷
10⁸
10⁹

Longitudinal Dynamics Summary

1D Vlasov Equation $(\int(Vlasov\ Equation)dxdx'dydy'$

$$\frac{\partial \hat{f}}{\partial s} + z' \frac{\partial \hat{f}}{\partial z} + z'' \frac{\partial \hat{f}}{\partial z'} = 0$$

$$E_z = -\frac{g}{4\pi\varepsilon_0}\frac{\partial\lambda}{\partial z}$$

"g-factor model"

$$z'' = \frac{qE_z}{mv_0^2};$$

$$\frac{\partial^2\phi}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) = \frac{-\rho}{\varepsilon_0}$$

Child-Langmuir in 1-D diode

Leads to fluid equations $(\int(1D\ Vlasov\ Equation)dz'$

$$\frac{\partial\lambda}{\partial s} + \frac{\partial(\lambda\bar{z}')}{\partial z} = 0$$

Continuity Equation

$$\frac{\partial\bar{z}'}{\partial s} + \bar{z}'\frac{\partial\bar{z}'}{\partial z} + \cancelto{0}{\frac{1}{\lambda}\frac{\partial(\lambda\Delta\bar{z}'^2)}{\partial z}} = \frac{qE_z}{mv_0^2}$$

Momentum Equation

1D Ez => Non-linear solution to fluid equations:

Child Langmuir solution;

2D Pierce electrodes; (extends CL for finite radius beam)

Time dependent Lampel-Tiefenback sol. (extends CL for finite Δt)

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g-factor => Space-charge waves -> $\omega = c_s k$, $\Lambda_1 \sim f^+ + f^-$ and $v_1 \sim f+ -f-$

=> Longitudinal resistive instability

=> Space charge rarefaction waves (non-linear solution to fluid eqs.)

=> Parabolic bunch compression and stagnation (non-linear solution to fluid eqs.)

Longitudinal Envelope Equation: $(\int(1D\ Vlasov\ Equation)dzdz'$

$$\frac{\partial^2 r_z}{\partial s^2} - K(s)r_z + \frac{3gqQ_c}{4\pi\varepsilon_0 mv_0^2}\frac{1}{r_z^2} + \frac{\varepsilon_z^2}{r_z^3} = 0$$

Kinetic solution to Vlasov Equation, obeying envelope equation: Neuffer distribution:

$$f(z,z') = \frac{3N}{2\pi\varepsilon_z}\sqrt{1-C_z^2} = \frac{3N}{2\pi\varepsilon_z}\sqrt{1 - \frac{z^2}{r_z^2} - \frac{r_z^2\left(z' - \frac{r_z' z}{r_z}\right)^2}{\varepsilon_z^2}}$$

ESTIMATING SPOT SIZE

$$r_x'' + \frac{(\gamma_b \beta_b)'}{\gamma_b \beta_b} r_x' + K_x r_x - \frac{2Q}{r_x + r_y} - \frac{\epsilon_x^2}{r_x^3} = 0$$

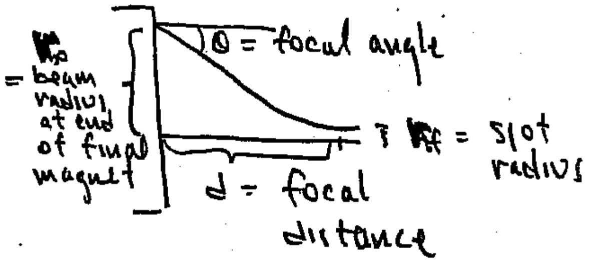
$$r_y'' + \frac{(\gamma_b \beta_b)'}{\gamma_b \beta_b} r_y' + K_y r_y - \frac{2Q}{r_x + r_y} - \frac{\epsilon_y^2}{r_y^3} = 0$$

IN CHAMBER: NO EXTERNAL FOCUSING, NO ACCELERATION AND BEAM IS OFTEN CIRCULAR (BY DESIGN)

$$\Rightarrow \quad K_x = K_y = (\gamma_b \beta_b)' = 0 \quad \& \quad r_x = r_y = r_b$$

$\Rightarrow$ ENVELOPE EQUATION IS:

$$r_b'' = \frac{Q}{r_b} + \frac{\epsilon^2}{r_b^3}$$



MULTIPLYING BY r_b' & INTEGRATING $\Rightarrow$

$$\frac{r_{bf}'^2}{2} - \frac{r_{bo}'^2}{2} = Q \ln \frac{r_{bf}}{r_{bo}} + \frac{\epsilon^2}{2 r_{bo}^2} - \frac{\epsilon^2}{2 r_{bf}^2}$$

NOW $\quad r_{b\theta}' \cong \theta \qquad r_{bf} = \text{spot radius}$

$\quad\quad\;\; r_{bf}' = 0 \qquad r_{bo} \cong d\theta$

$r_{bf} << r_{bo}$

$$\Rightarrow \boxed{\theta^2 \cong 2Q \ln\left(\frac{\theta d}{r_{bf}}\right) + \frac{\epsilon^2}{r_{bf}^2}}$$

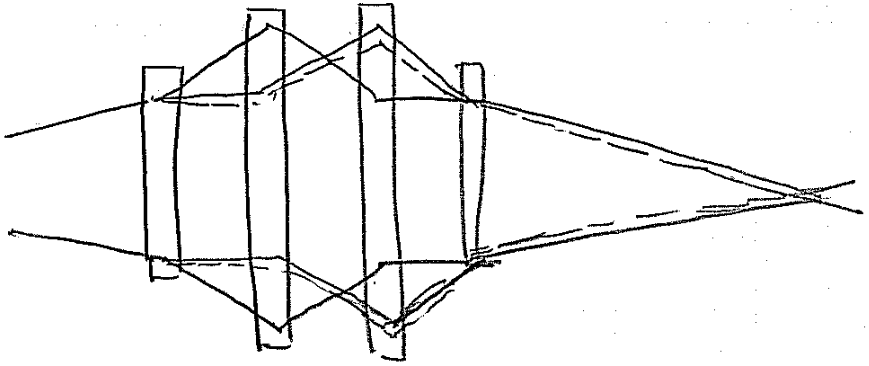
WHEN $Q = 0$

$$r_{spot}^2 = \frac{\epsilon^2}{\theta^2} + r^2_{\substack{\text{CHROMATIC}\\\text{ABERRATION}}} + \dots$$

$$r^2_{\text{CHROMATIC}} = \alpha^2 d^2 \left(\frac{\delta p}{p}\right)^2 \theta^2$$

$\alpha \cong 6$ (system dependent)

"CHROMATIC ABERRATIONS" TEND TO BROADEN SPOT

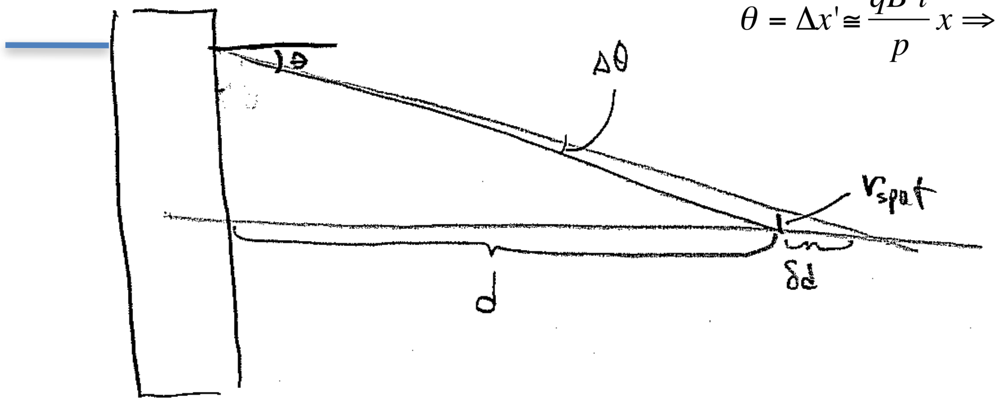


SINCE QUADRUPOLE MAGNET FOCUSING $\propto \frac{1}{V_z}$

(i.e. $x'' = \frac{qB'}{\gamma m v_z} x$) A SPREAD IN LONGITUDINAL VELOCITY GIVES RISE TO A BROADENING OF FINAL SPOT.

$x'' \cong \frac{qB'}{p} x$ For a single B quad:

$$\theta = \Delta x' \cong \frac{qB'l}{p} x \Rightarrow \boxed{\frac{d\theta}{dp} = -\frac{qB'l}{p^2} = -\frac{\theta}{p}}$$



$$\begin{aligned} r_{spot} &= \theta\, \delta d \\ &= \theta \frac{dd}{d\theta} \frac{d\theta}{dp} \delta p \\ &= \alpha\, \theta d \left(\frac{\delta p}{p}\right) \end{aligned}$$

α = some constant depending on focal system

Geometry $\Rightarrow \frac{dd}{d\theta} = \frac{\delta d}{\Delta\theta} \cong \frac{d}{\theta}$

$$r_{spot} = \theta \frac{dd}{d\theta} \left|\frac{d\theta}{dp}\right| \delta p$$

$$= \theta \left(\frac{d}{\theta}\right)\left(\frac{\theta}{p}\right) \delta p = \theta d \frac{\delta p}{p}$$ (for a single magnet)

NORMAL MODES

LONGITUDINAL

SPACE-CHARGE WAVES (FLUID)

$\omega = \pm c_s k$ [IN COMOVING BEAM FRAME]

$$C_s = \sqrt{\frac{q g \lambda_0}{4\pi\epsilon_0 m}}$$ = SPACE CHARGE WAVE SPEED

TRANSVERSE

ENVELOPE MODES

CONTINUOUS FOCUSING (LONG BUNCHES)

BREATHING: $k_B^2 = 2k_{\beta 0}^2 + 2k_\beta^2$

QUADRUPOLE $k_Q^2 = k_{\beta 0}^2 + 3k_\beta^2$

(HERE $k_\beta^2 \equiv k_{\beta 0}^2 - \frac{Q}{r_b^2}$)

(ANALOGOUS MODES IN BUNCHED BEAMS)

STEVE LOOKED AT MODES IN PERIODIC SYSTEMS (+ CONTINUOUS FOCUSING

+ KINETIC MODES (GLUCKSTERN MODES)

+ FLUID MODES

Instabilities

1. Longitudinal (resistive wall) instability
(fluid instability)

2. Electron-ion instability
(centroid instability)

Steve talked about:
3. Envelope instabilities

Steve talked about:
4. Kinetic instabilities
(distribution function dependent)

5. Single particle resonant instabilities
-- Halo
-- Ring resonances (covered by Steve)

Several potential instabilities have been investigated in HIF drivers

Temperature anisotropy instability

After acceleration $T_{\parallel} << T_{\perp}$ internal beam modes are unstable; saturation occurs when $\bar{T}_{\parallel} \sim T_{\perp}/3$. (cf E.A. Startsev, R.C. Davidson, H. Qin, PRSTAB 6 084401(2003) and references therein).

Longitudinal resistive instability

Module impedance interacts with beam, amplifying space charge waves that are backward propagating in beam frame. (cf. Reiser, 2nd ed., chap. 6, K. Takayama and R. J. Briggs,eds., in *Induction Accelerators*, [Springer, NY], (2012), chap. 9 and references therein).

Beam-break up (BBU) instability

High frequency waves in induction module cavities interact transversely with beam (cf., K. Takayama and R. J. Briggs, eds., in *Induction Accelerators*, [Springer, NY], (2012), chap. 7 and references therein).

Beam-plasma instability

Beam interacts with residual gas in the target chamber (cf. R.C. Davidson and H. Qin in *Phys. of Intense Charged Particle Beams in High Energy Accelerators*, [Imperial College Press,London], (2001), chap 10).

HALO:

CORE TEST PARTICLE MODEL:

$$x'' = \begin{cases} -\left[k_{\beta 0}^2 - \dfrac{Q}{r_b^2}\right]x & \text{for } r < r_b \\ -\left[k_{\beta 0}^2 - \dfrac{Q}{r^2}\right]x & \text{fr } r > r_b \end{cases}$$

$$r_b = r_{b0} + \delta r_b \cos(k_B s + \phi)$$

Gluckstern's phase-amplitude analysis:

$$x'' + \overbrace{\left[k_{\beta 0}^2 - \frac{Q}{r_{b0}^2}\right]}^{k_\beta^2} x = \underset{\uparrow}{f(x)}$$

Non linear + forcing part

$x = A\sin\psi \quad x' = k_\beta A\cos\psi$ ← PHASE/AMPLITUDE

$\psi = k_\beta s + \alpha$ If $f = 0$ A & ϕ would be constant

$\Rightarrow A' = \dfrac{1}{k_\beta r_{b0}} f\cos\psi \qquad \alpha' = \dfrac{-1}{k_\beta r_{b0} A} f\sin\psi$

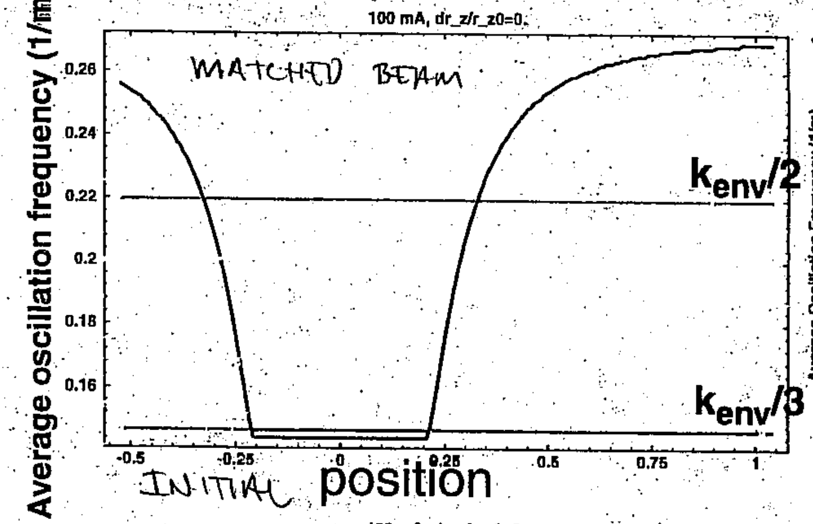
DEFINE RESONANT PHASE $\Psi_r = 2\psi - k_B s$

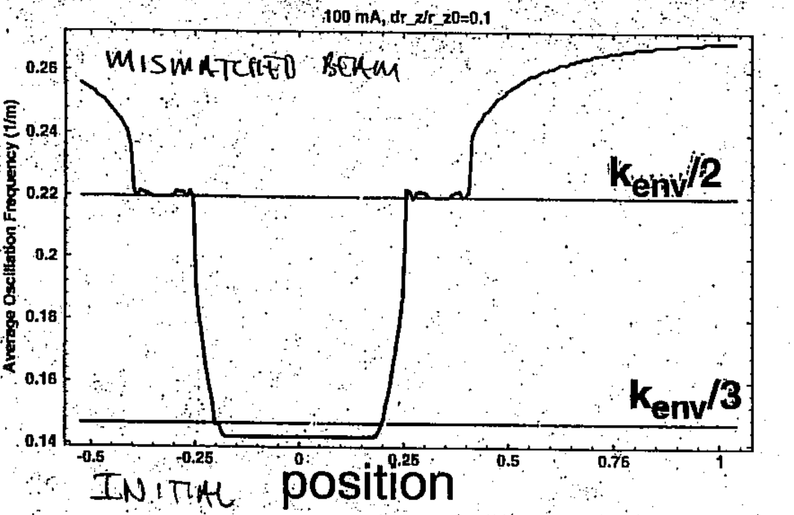
AVERAGE OVER ALL NON-RESONANT FREQUENCIES

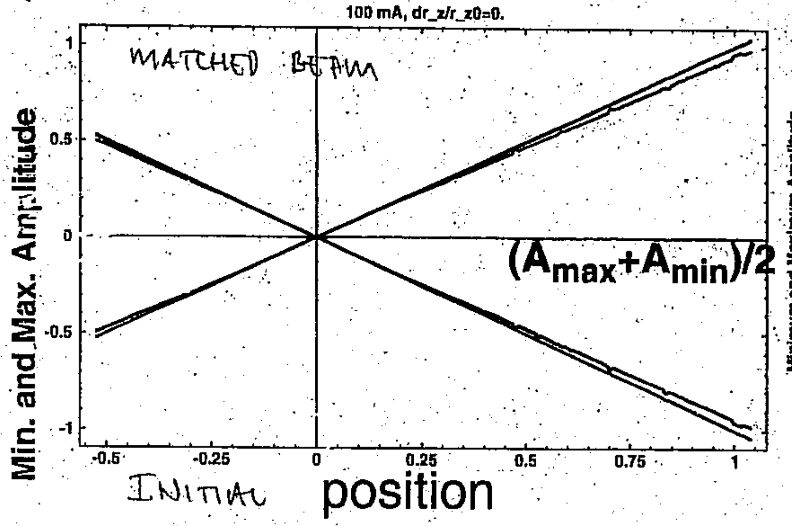
$$A_r' = \frac{1}{k_\beta r_{b0}} \int_{-\pi}^{\pi} f\cos\psi \frac{d\psi}{2\pi}; \quad \alpha_r' = \frac{-1}{k_\beta A_r} \int_{-\pi}^{\pi} \frac{d\psi}{2\pi} f\sin\psi$$

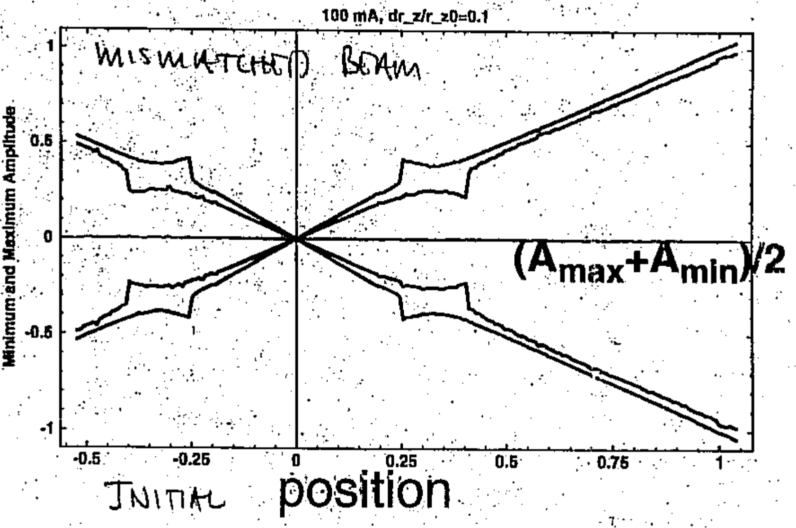
$$\longrightarrow A_r', \Psi_r' \underset{(\omega = A_r^2)}{\longrightarrow} \omega', \Psi_r' \longrightarrow H(\omega, \Psi_r) \longrightarrow$$ GIVE RESONANT PARTICLE TRAJECTORY & SEPARATRIX

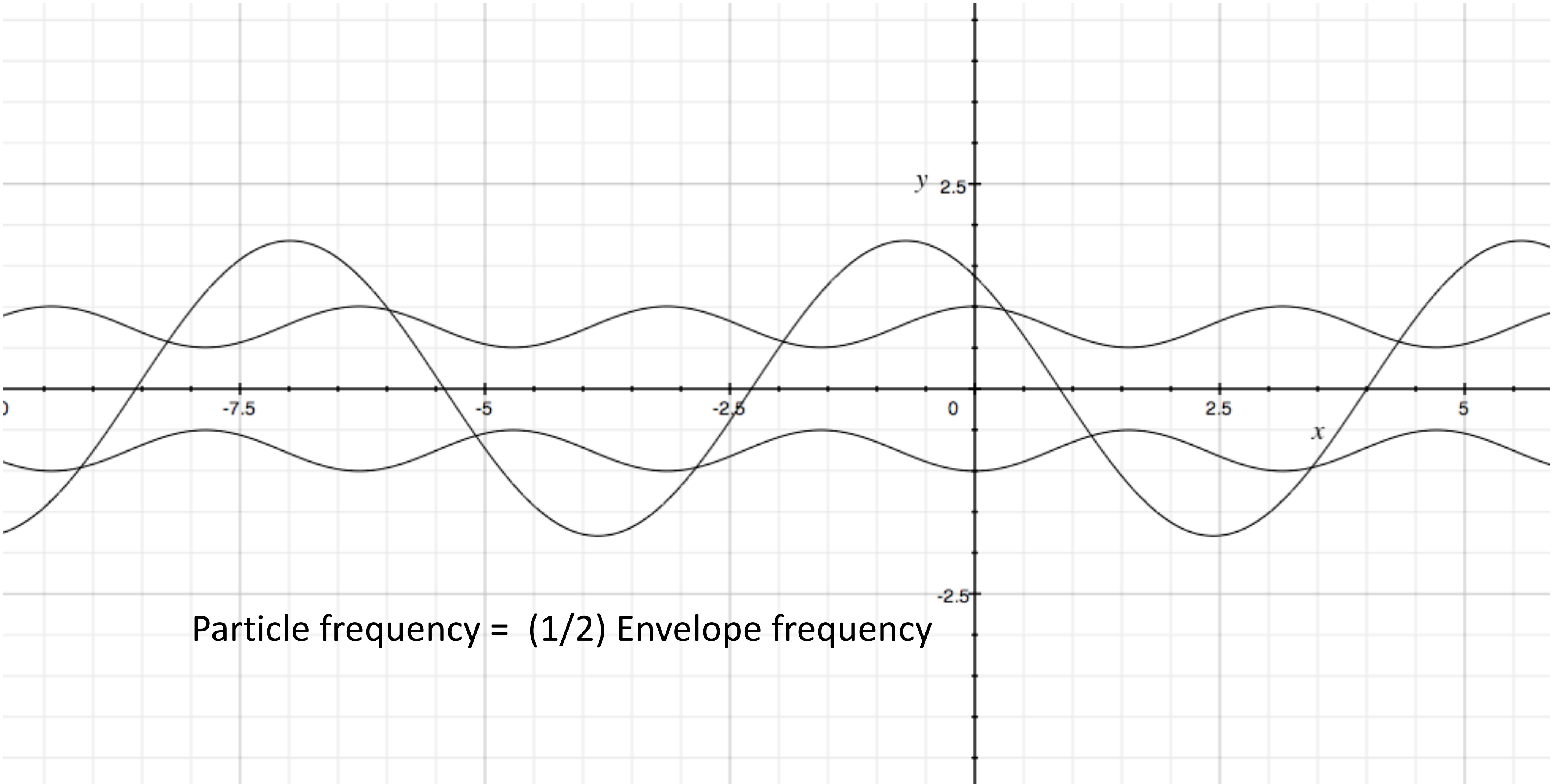
Numerically determined frequency and amplitude of particle oscillations: linear rf focusing







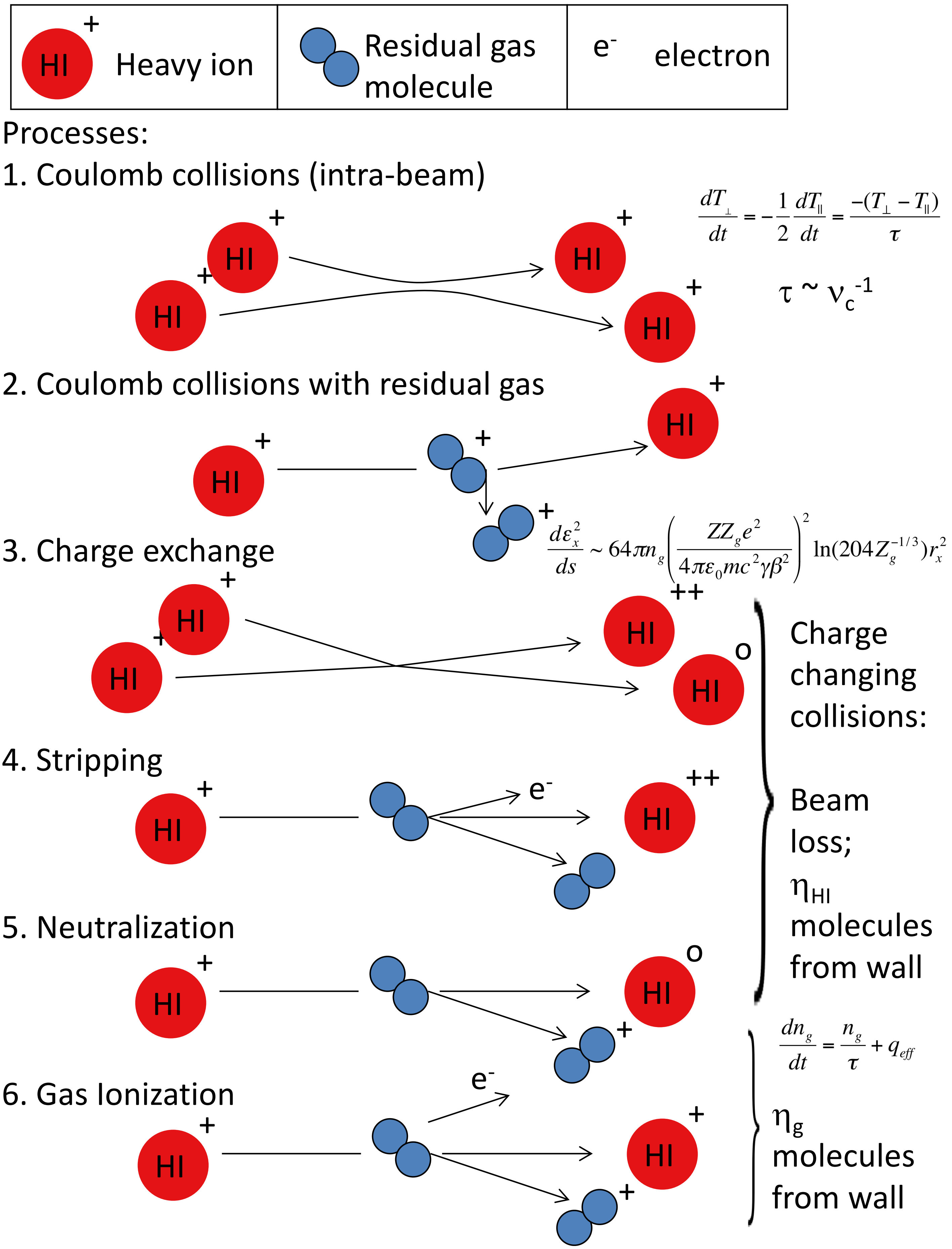




Particle frequency = (1/2) Envelope frequency

As particle dives in and out of envelope, particle is at same phase of envelope oscillation.

Those particles that are exiting the beam when beam radius is small, and entering beam when beam radius is large, get larger "kick" going out and smaller kick coming in, so are driven to large amplitude.



Processes:

1. Coulomb collisions (intra-beam)

$$\frac{dT_{\perp}}{dt} = -\frac{1}{2}\frac{dT_{\parallel}}{dt} = \frac{-(T_{\perp} - T_{\parallel})}{\tau}$$

$$\tau \sim \nu_c^{-1}$$

2. Coulomb collisions with residual gas

$$\frac{d\varepsilon_x^2}{ds} \sim 64\pi n_g \left(\frac{Z Z_g e^2}{4\pi\varepsilon_0 mc^2\gamma\beta^2}\right)^2 \ln(204 Z_g^{-1/3}) r_x^2$$

3. Charge exchange

4. Stripping

5. Neutralization

Charge changing collisions:

Beam loss;

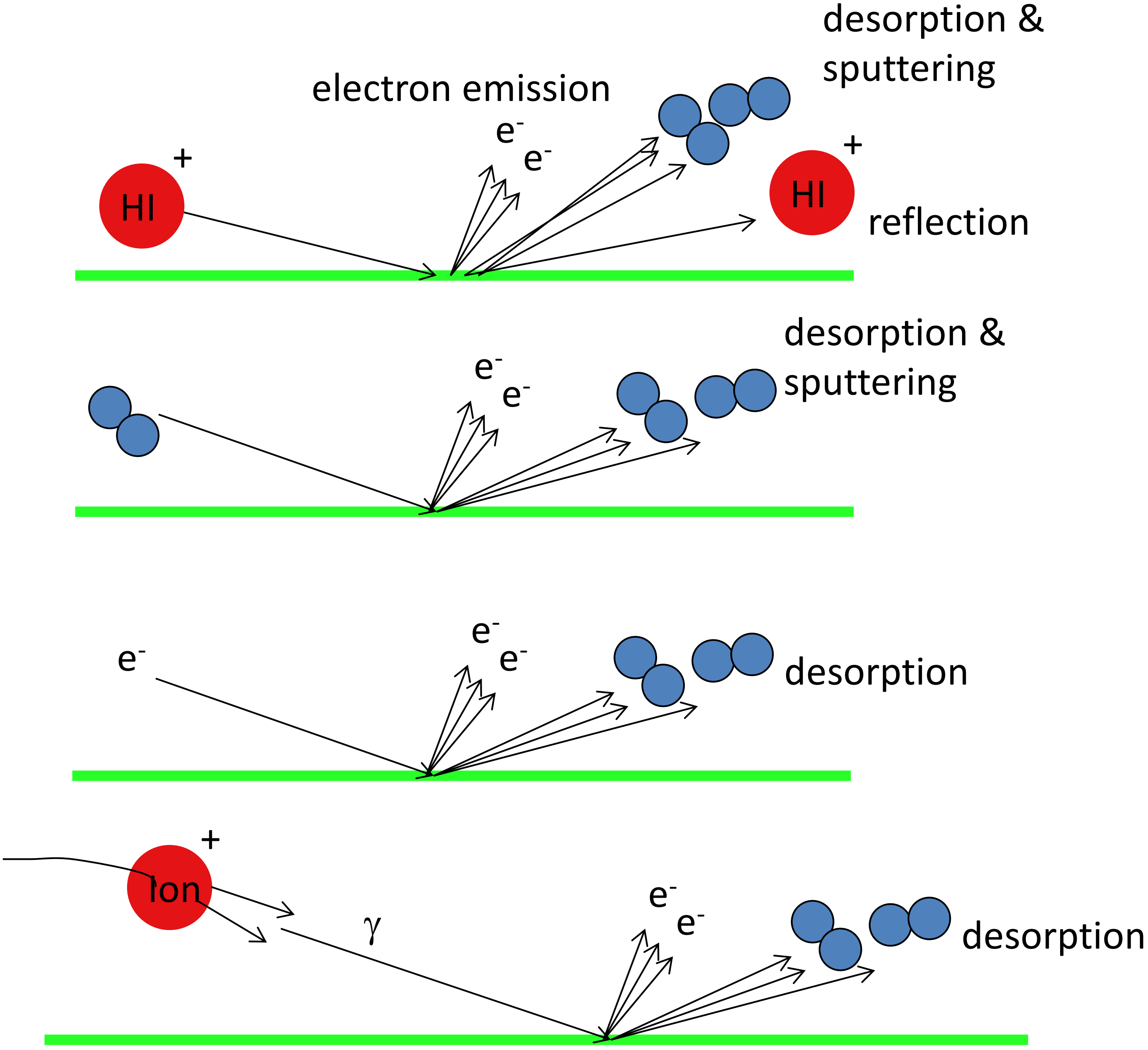
η_{HI} molecules from wall

6. Gas Ionization

$$\frac{dn_g}{dt} = \frac{n_g}{\tau} + q_{eff}$$

η_g molecules from wall

7. Wall interactions



γ	synchrotron photon

Summary of electron, gas, pressure, and scattering effects

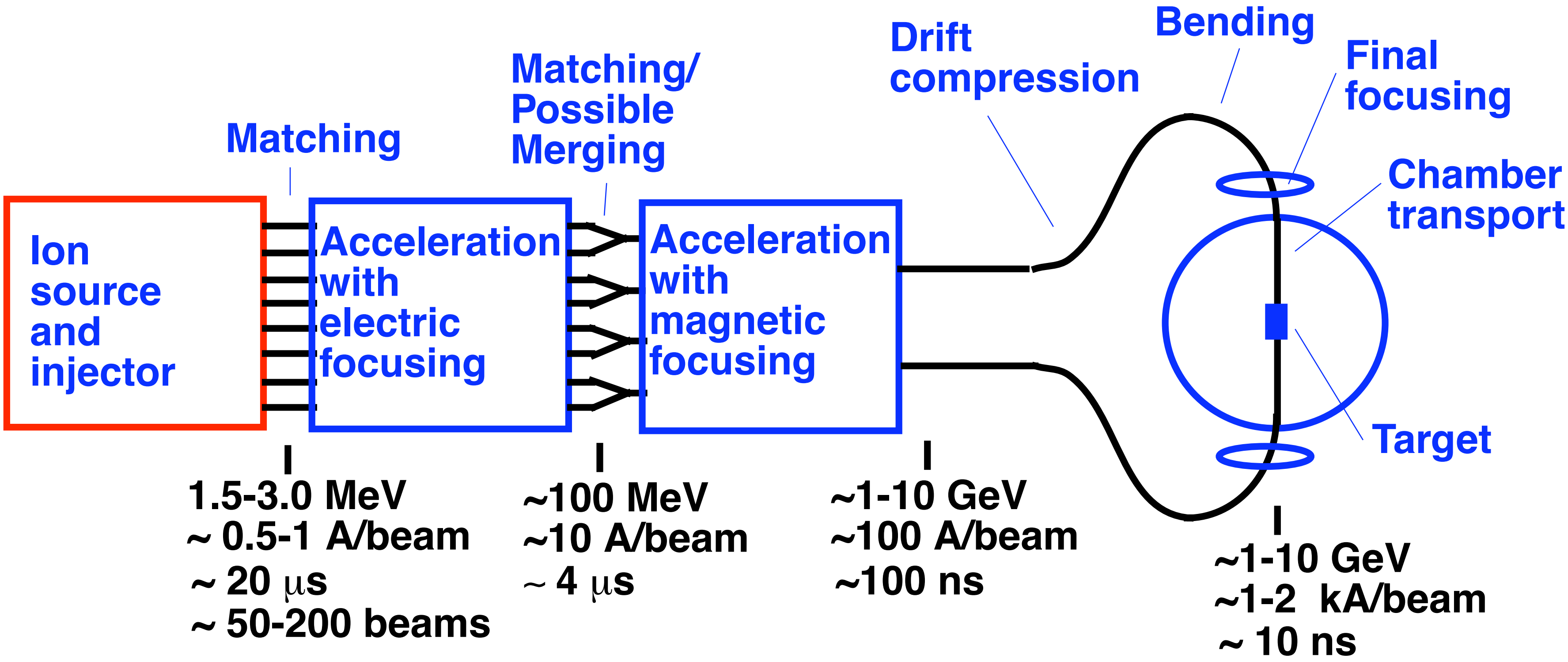
1. Coulomb collisions within beam can transfer energy from $\perp$ to $||$ and provide lower limit on $T_{||}$, higher than from accelerative cooling.

2. Coulomb interactions with residual gas nuclei provide a source of emittance growth (but usually not important for higher mass and linac residence times.)

3. Pressure instability from desorption of residual gas by stripped beam ions hitting wall or beam ionized residual gas atoms, forced to wall by E-field of beam. Limits current in rings or high repetition rate linac.

4. Electron can cascade and reach a "quasi" equilibrium population of similar line charge to the ion beam electron-ion two stream instability is unstable, and can lead to transverse instability, similar to what is observed in some proton rings.

Induction acceleration for HIF consists of several subsystems and a variety of beam manipulations



Ion source and injector

Matching

Acceleration with electric focusing

Matching/ Possible Merging

Acceleration with magnetic focusing

Drift compression

Bending

Final focusing

Chamber transport

Target

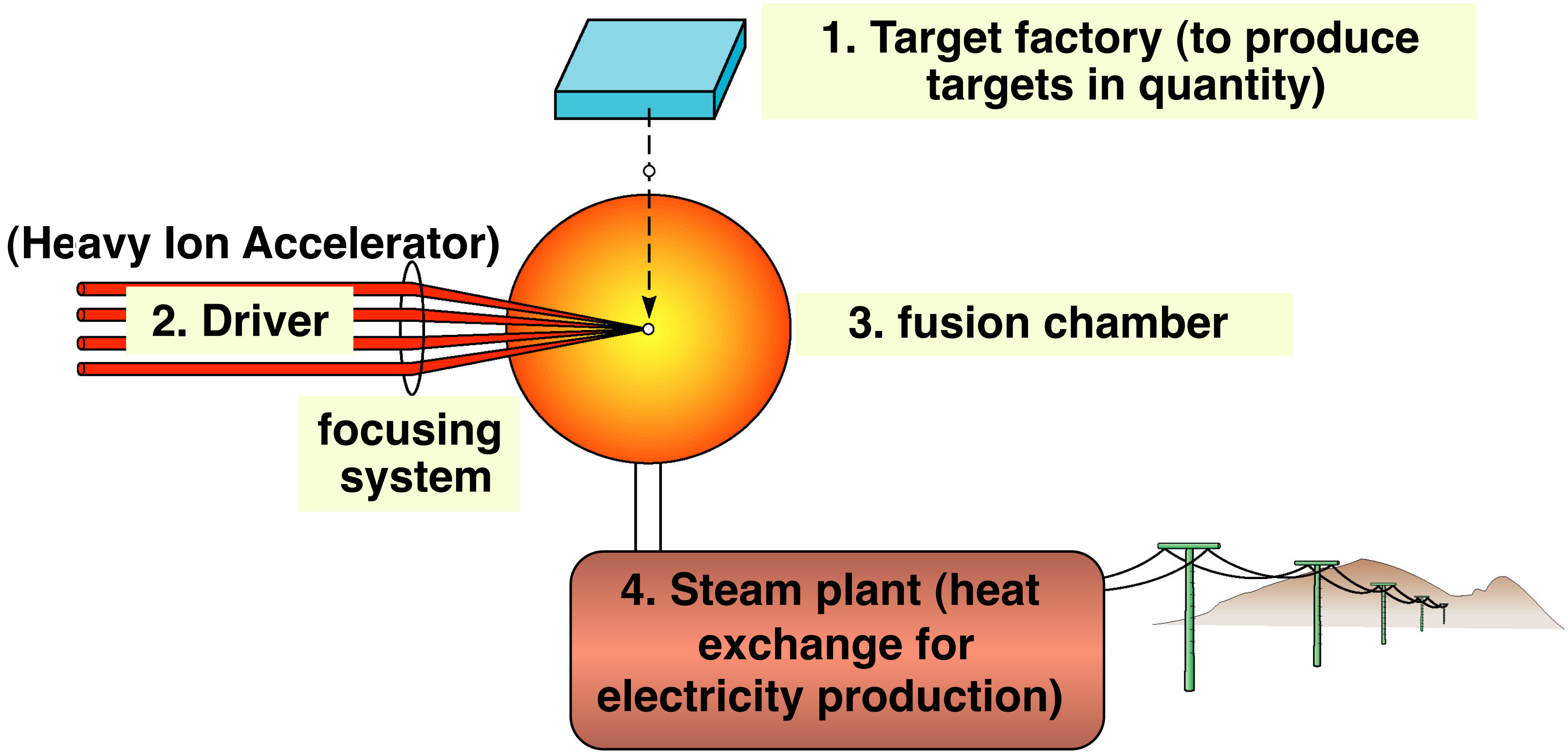
1.5-3.0 MeV
~ 0.5-1 A/beam
~ 20 µs
~ 50-200 beams

~100 MeV
~10 A/beam
~ 4 µs

~1-10 GeV
~100 A/beam
~100 ns

~1-10 GeV
~1-2 kA/beam
~ 10 ns

Inertial fusion energy (IFE) power plants of the future will consist of four parts



1. Target factory (to produce targets in quantity)

(Heavy Ion Accelerator)

2. Driver

3. fusion chamber

focusing system

4. Steam plant (heat exchange for electricity production)

A power plant driver would fire about five targets per second to produce as much electricity as today's 1000 Megawatt power plant

$$f(z,z') = \frac{3N}{2\pi\varepsilon_z}\sqrt{1 - C_z^2} \;= \frac{3N}{2\pi\varepsilon_z}\sqrt{1 - \frac{z^2}{r_z^2} - \frac{r_z^2\left(z' - \frac{r_z' z}{r_z}\right)^2}{\varepsilon_z^2}}$$

LONGITUDINAL DYNAMICS SUMMARY

1D VLASOV EQUATION ($\int$(VLASOV EQUATION) dx dx' dy dy')

$$\frac{\partial \tilde{f}}{\partial s} + z' \frac{\partial \tilde{f}}{\partial z} + z'' \frac{\partial \tilde{f}}{\partial z'} = 0$$

$$E_z = \frac{-g}{4\pi\epsilon_0}\frac{\partial \lambda}{\partial z} \qquad \text{"g-factor" model}$$

$$z'' = \frac{q E_z}{m v_0^2} ; \qquad \frac{\partial^2 \phi}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial \phi}{\partial r}\right) = -\frac{\rho}{\epsilon_0}$$

CHILD-LANGMUIR IN 1-D DIODE

LEADS TO FLUID EQUATIONS ($\int$(1D Vlasov Equation) dz')

$$\frac{\partial \lambda}{\partial s} + \frac{\partial}{\partial z}(\lambda \bar{z}') = 0$$

$$\frac{\partial \bar{z}'}{\partial s} + \bar{z}' \frac{\partial \bar{z}'}{\partial z} + \frac{1}{\lambda}\frac{\partial}{\partial z}\cancel{(\lambda \Delta z'^2)} = \frac{q E_z}{m v_0^2}$$

1D E_z: ⇒ CHILD LANGMUIR SOLUTION ← NON-LINEAR SOLUTION TO FLUID EQUATIONS

g-factor: ⇒ SPACE-CHARGE WAVES

↳ LONGITUDINAL OR RESISTIVE WALL INSTABILITY (IF $E_1 = Z^* I_1$)

{ 2D PIERCE ELECTRODE; TIME DEPENDENT LAMPEL TIEFENBACK SOLUTION

⇒ SPACE-CHARGE RAREFACTION WAVES ← NON-LINEAR SOLUTION TO FLUID EQNS.

Outward expansion at $2c_s$; Inward at c_s

⇒ PARABOLIC BUNCH COMPRESSION ← NON-LINEAR SOLUTION TO FLUID EQUATIONS

⇒ "EAR" FIELDS

VLASOV EQUATION ALSO ⇒ ENVELOPE EQUATION $\iint$ 1D VLASOV Equation dz dz'

$$\frac{d^2 r_z}{ds^2} = \frac{\varepsilon_z^2}{r_z^3} + \frac{3}{2}\frac{g q Q_c}{4\pi\epsilon_0 m v^2}\frac{1}{r_z^2} - K(r) r_z$$

KINETIC SOLUTION TO VLASOV EQUATION & SATISFYING RMS ENVELOPE EQUATION IS NEUFFER DISTRIBUTION

$$f(z,z') = \frac{3N}{2\pi\varepsilon_z}\sqrt{1 - \frac{z^2}{r_z^2} - \frac{r_z^2 (z' - r_z' z/r_z)^2}{\varepsilon_z^2}}$$