Physic 231 Lecture 13

Main points of last lecture:
- Work, energy and non-conservative forces:
  \[ KE_f - KE_0 + PE_f - PE_0 = W_{\text{nonconservative}} \]
- Power
  \[ \bar{P} = \frac{W}{\Delta t}; \quad \bar{P} = F\bar{v} \]

Main points of today’s lecture:
- Impulses: forces that last only a short time
- Momentum
  \[ \bar{p} = m\bar{v} \]
- Impulse-Momentum theorem
  \[ \bar{F}\Delta t = \Delta \bar{p} = m\Delta \bar{v} = m(\bar{v}_f - \bar{v}_i) \]
- Momentum conservation
  \[ \bar{p}_{\text{tot},f} = \bar{p}_{1,f} + \bar{p}_{2,f} = \bar{p}_{1,i} + \bar{p}_{2,i} = \bar{p}_{\text{tot},i} \]
- Momentum and external forces
  \[ \bar{F}_{\text{ext}}\Delta t = \bar{p}_{\text{tot},f} - \bar{p}_{\text{tot},i} \]
Example

In screeching to a halt, a car leaves skid marks that are 65m long. The coefficient of kinetic friction between the tires and the road is $\mu_k = 0.71$. How fast was the car going before the driver applied the breaks?

<table>
<thead>
<tr>
<th>$\Delta x$</th>
<th>65m</th>
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<tbody>
<tr>
<td>$\mu_k$</td>
<td>0.71</td>
</tr>
<tr>
<td>$v_f$</td>
<td>0</td>
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\[
KE_f - KE_0 = W_{\text{friction}} \\
-KE_0 = W_{\text{friction}} \\
\text{Friction is in the opposite direction of the displacement} \\
W_{\text{friction}} = -f_k \Delta x \\
\frac{1}{2}mv_0^2 = f_k \Delta x = \mu_k N \Delta x = \mu_k mg \Delta x \\
v_0^2 = 2\mu_k g \Delta x \\
v_0 = \sqrt{2\mu_k g \Delta x} = \sqrt{2(0.71)(9.8)(65)} \text{m/s} \\
v_0 = 30 \text{ m/s}
\]
Impulse: forces that last a very short time

- There are many processes in which forces last a very short time and are difficult to mathematically describe. Examples are:
  - Kicking, striking batting, dribbling a ball.
  - Various types of explosions, firearms, etc.

- The typical time dependence of such impulse forces is described below:

\[ \approx 0.01 \text{s} \]
Momentum

• Typically, we are interested in knowing how the velocity of an object is changed by an impulse force.

• Since the impulse force is neither well understood mathematically nor reproducible, it is not the natural quantity with which one describes such events. Linear momentum and the change in linear momentum, i.e. impulse, are more useful for such descriptions:

• The linear momentum of a particle of mass m is:

\[ \vec{p} = m\vec{v} \]

• The change in velocity is related to the change in momentum, i.e. impulse:

\[ \Delta\vec{p} = \vec{p}_f - \vec{p}_i = m(\vec{v}_f - \vec{v}_i) \]

• It is related to the average impulsive force:

\[ \overline{F}\Delta t = m\overline{a}\Delta t = m\frac{(\vec{v}_f - \vec{v}_i)}{\Delta t}\Delta t = m(\vec{v}_f - \vec{v}_i) = \Delta\vec{p} \]

\[ \Rightarrow \vec{v}_f = \vec{v}_i + \frac{\overline{F}\Delta t}{m} \]
Bouncing balls

• Assuming each ball has the same mass, which ball experiences the larger impulse?
  – a) the first ball
  – b) the second ball
You are a passenger in a car and not wearing your seat belt. Without increasing or decreasing its speed, the car makes a sharp left turn, and you find yourself colliding with the right-hand door. Which is the correct analysis of the situation?
- a) Before and after the collision, there is a rightward force pushing you into the door.
- b) Starting at the time of collision, the door exerts a leftward force on you.
- c) both of the above
- d) neither of the above
Example

- A 0.4 kg ball is dropped from rest at a point 1.5 m above the floor. The ball rebounds straight upward to a height of 0.8 m. What is the magnitude and direction of the impulse applied to the ball by the floor? If the ball is in contact with the floor for 0.01 seconds, what is the impulse force?

\[
\Delta p = p_f - p_0 = m(\bar{v}_f - \bar{v}_0)
\]

\[
\frac{1}{2}mv_0^2 = mgh_0 \Rightarrow v_0 = -\sqrt{2gh_0} = -\sqrt{2(9.8)(1.5)} m/s = -5.4 m/s
\]

\[
\frac{1}{2}mv_f^2 = mgh_f \Rightarrow v_f = \sqrt{2gh_f} = \sqrt{2(9.8)(0.8)} m/s = 3.96 m/s
\]

\[
\Delta p = m(\bar{v}_f - \bar{v}_0) = 0.4 kg[3.96 m/s - (-5.4 m/s)] = 3.75 kg \cdot m/s \text{ upwards}
\]

\[
\bar{F} = \frac{\Delta p}{\Delta t} = \frac{3.75 kg \cdot m/s}{0.01 s} = 375 N \text{ upwards}
\]

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<tr>
<td>h₀</td>
<td>1.5 m</td>
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<tr>
<td>h₉</td>
<td>0.8 m</td>
</tr>
<tr>
<td>m</td>
<td>0.4 kg</td>
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Quiz

- Jack swings at a 0.2 kg ball that is moving west with a velocity of 40 m/s and hits a line drive. The leaves his bat with a velocity of 40 m/s due east. Assuming the ball is in contact with the bat for 0.010 s, what is the average impulse force of the bat on the ball?
  - a) 800N east
  - b) 1600 N east
  - c) 1600 N west
  - d) 800 N west

\[
\Delta p = 0.2 \text{ kg}(40 \text{ m/s} - [-40 \text{ m/s}]) = 16 \text{ kg} \cdot \text{m/s} \text{ east}
\]

\[
\overline{F} = \frac{\Delta p}{\Delta t} = \frac{16 \text{ kg} \cdot \text{m/s}}{0.01 \text{ s}} = 1600 \text{ N east}
\]
Example

- A dump truck is being filled with sand. The sand falls straight downward from rest from a height of 2.00 m above the truck bed, and the mass of sand that hits the truck per second is 55.0 kg/s. The truck is parked on the platform of a weight scale. By how much does the scale reading exceed the combined weight of truck and sand?

Force on the sand $F_s$ is given by

$$F_s \Delta t = \Delta m_s (v_{sand,f} - v_{sand,0}) = -\Delta m_s v_{sand,0}$$

$$\Delta m_s = (55 \text{kg} / \text{s}) \Delta t$$

$$\Rightarrow F_s = -(55 \text{kg} / \text{s}) v_{sand,0}$$

$$\frac{1}{2} \Delta m_s v_{sand,0}^2 = \Delta m_s gh \Rightarrow v_{sand,0}^2 = 2gh$$

$$v_{sand,0} = -\sqrt{2gh} = -\sqrt{2(9.8 \text{m} / \text{s}^2)(2 \text{m})} = -6.26 \text{m} / \text{s}$$

$$\Rightarrow F_s = -(55 \text{kg} / \text{s})(-6.26 \text{m} / \text{s}) = 344 \text{N} \text{ upwards}$$

By Newton's third law, the force on the truck is

$$F_{truck} = 344 \text{N} \text{ downward}$$

This is the amount by which the weight of truck is increased.
Conservation of linear momentum

• Consider the collision of objects that interact with each other but whose interactions with the rest of the world can be neglected. As an example, one can consider two hockey pucks (one larger and the other smaller) that are sliding without friction on a frictionless ice surface.

• From Newton’s 3rd law:

\[ \bar{F}_{21} = -\bar{F}_{12} \text{ at all times. On the average} \]
\[ \bar{F}_{21} = -\bar{F}_{12} \Rightarrow \bar{F}_{21}\Delta t = -\bar{F}_{12}\Delta t \]
\[ \Rightarrow \Delta \bar{p}_2 = -\Delta \bar{p}_1 \Rightarrow \bar{p}_{2,f} - \bar{p}_{2,o} = \bar{p}_{1,o} - \bar{p}_{1,f} \]

• If we reorganize terms:

\[ \bar{p}_{2,f} + \bar{p}_{1,f} = \bar{p}_{2,o} + \bar{p}_{1,o} \]
\[ \Rightarrow \bar{p}_{\text{tot,f}} = \bar{p}_{\text{tot,o}} \]

• Thus total momentum is conserved in isolated system. i.e. one without external forces. When there are external forces such as gravity:

\[ \bar{F}_{\text{ext}}\Delta t = \bar{p}_{\text{tot,f}} - \bar{p}_{\text{tot,i}} \]