Main points of last lecture:
- Impulses: forces that last only a short time
- Momentum
  \[ \mathbf{p} = m \mathbf{v} \]
- Impulse-Momentum theorem
  \[ \mathbf{F} \Delta t = \Delta \mathbf{p} = m \Delta \mathbf{v} = m(\mathbf{v}_f - \mathbf{v}_i) \]
- Impulse-Momentum theorem
  \[ \mathbf{p}_{tot,f} = \mathbf{p}_{1,f} + \mathbf{p}_{2,f} \]
  \[ = \mathbf{p}_{1,i} + \mathbf{p}_{2,i} = \mathbf{p}_{tot,i} \]
- Momentum and external forces
  \[ \mathbf{F}_{ext} \Delta t = \mathbf{p}_{tot,f} - \mathbf{p}_{tot,i} \]

Main points of today’s lecture:
- Rocket propulsion.
- Totally inelastic collisions
  \[ \mathbf{v}_f = \frac{m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2}{m_1 + m_2} \]
- Elastic collisions in one dimension:
  \[ v_{1f} = \frac{(m_1 - m_2)}{m_1 + m_2} v_{10} + \frac{2m_2}{m_1 + m_2} v_{20} \]
  \[ v_{2f} = \frac{(m_2 - m_1)}{m_1 + m_2} v_{20} + \frac{2m_1}{m_1 + m_2} v_{10} \]
Conceptual question

- Consider two carts, of masses $m$ and $2m$, at rest on an air track. If you push first one cart for 3 s and then the other for the same length of time, exerting equal force on each, the momentum of the light cart is
  - a) four times
  - b) twice
  - c) equal to
  - d) one-half
  - e) one-quarter the momentum of the heavy cart.
Conceptual question

- Which of these systems are isolated (i.e. no external forces)?
  - a) While slipping on a patch of ice ($\mu_k=0$), a car collides totally inelastically with another car. *System*: both cars
  - b) Same situation as in a). *System*: the slipping car
  - c) A single car slips on a patch of ice. *System*: car
  - d) A car makes an emergency stop on a road. *System*: car
  - e) A ball drops to Earth. *System*: ball
  - f) A billiard ball collides elastically with another billiard ball on a pool table. *System*: both balls
Example

An astronaut is motionless in outer space. Upon command, his propulsion unit strapped to his back ejects some gas with a velocity of +14 m/s, and the astronaut recoils with a velocity of -0.5m/s. After the gas is ejected, the mass of the astronaut is 160kg. What is the mass of the ejected gas?

\[ p_{\text{ast},0} + p_{\text{gas},0} = 0 = p_{\text{ast},f} + p_{\text{gas},f} \]

\[ p_{\text{gas},f} = -p_{\text{ast},f} \]

\[ p_{\text{ast},f} = m_{\text{ast}} \vec{v}_{\text{ast},f} ; \quad p_{\text{gas},f} = m_{\text{gas}} \vec{v}_{\text{gas},f} \]

take motion to be along x axis

\[ m_{\text{gas}} v_{\text{gas},f,x} = -m_{\text{ast}} v_{\text{ast},f,x} \]

\[ m_{\text{gas}} = -m_{\text{ast}} \frac{v_{\text{ast},f,x}}{v_{\text{gas},f,x}} = -160kg \frac{-0.5m/s}{14m/s} = 5.7kg \]
Rocket propulsion

- The thrust force on a rocket can be computed using the impulse momentum theorem knowing the rate of mass emission $\Delta m/\Delta t$ of propellant and its emission velocity $v_f$ and the velocity of the rocket $v_0$.

\[
F_{\text{propellant}} \Delta t = \Delta m \cdot v_f - \Delta m \cdot v_0
\]

\[
F_{\text{propellant}} = \frac{\Delta m}{\Delta t} (v_f - v_0) = -\text{Thrust}
\]

\[
\text{Thrust} = \frac{\Delta m}{\Delta t} (v_0 - v_f)
\]
Principles of collisions

• If there are no external forces, the total momentum is always conserved during a collision:

\[
\vec{p}_{tot,f} \equiv \vec{p}_{1,f} + \vec{p}_{2,f} = \vec{p}_{1,i} + \vec{p}_{2,i} \equiv \vec{p}_{tot,i}
\]

• In such collisions, however, the mechanical energy may or may not be conserved. We have two useful limits:
  – Totally inelastic collisions where the two objects stick together after the collision. Here, largest energy loss possible for an isolated system occurs.
  – Totally elastic collisions where the two objects bounce off each other and the mechanical energy is the same after the collisions as it is before the collision.

• Inelastic collisions can occur in which the objects do not stick together. The energy loss in such collisions is less than what occurs in totally inelastic collisions where the object do stick together.
Totally inelastic collisions

- In isolated systems (systems without external forces) momentum is conserved. In totally inelastic collisions, the particles stick together after the collision.

\[
\vec{p}_{tot,i} = m_1\vec{v}_{1,0} + m_2\vec{v}_{2,0} = m_1\vec{v}_f + m_2\vec{v}_f = (m_1 + m_2)\vec{v}_f
\]

\[
\vec{v}_f = \frac{m_1\vec{v}_{1,0} + m_2\vec{v}_{2,0}}{m_1 + m_2}
\]

- Example: A 40 kg skater, sliding to the right without friction with a velocity of 1.5 m/s, suffers a head on collisions with a 30 kg skater who is initially at rest.
  - a) 0.17 m/s
  - b) 0.23 m/s
  - c) 0.42 m/s
  - d) 0.86 m/s

\[
\nu_f = \frac{m_1}{m_1 + m_2}\nu_0
\]

\[
= \frac{40\text{kg}}{40\text{kg} + 30\text{kg}}(1.5\text{m/s}) = 0.86\text{m/s}
\]
Conceptual question

• Suppose rain falls vertically into an open cart rolling along a straight horizontal track with negligible friction. As a result of the accumulating water, the speed of the cart
  – a) increases.
  – b) does not change.
  – c) decreases.
Quiz

- A 40 kg skater, sliding to the right without friction with a velocity of 1.5 m/s, suffers a head on collisions with another skater, moving to the left with a velocity of 2 m/s. After the collision, the two skaters come to rest. The mass of the second skater is:
  - a) 120 kg
  - b) 30 kg
  - c) 13.3 kg
  - d) 53.3 kg

\[
v_f = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = 0
\]

\[
\Rightarrow m_1v_1 = -m_2v_2
\]

\[
m_2 = -\frac{m_1v_1}{v_2} = -\frac{40 \text{kg} \cdot 1.5 \text{m/s}}{-2 \text{m/s}} = 30 \text{kg}
\]
Conceptual question

• If all three collisions in the figure shown here are totally inelastic, which bring(s) the car on the left to a halt?

- a) I
- b) II
- c) III
- d) I, II
- e) I, III
- f) II, III
- g) all three
Totally elastic collisions

- To calculate the result of an elastic collision in one dimension, we considered the constraints of total momentum and energy conservation:

\[ \text{Eq. 1: } p_{\text{tot}} = m_1 v_{1,0} + m_2 v_{2,0} = m_1 v_{1,f} + m_2 v_{2,f} \]

\[ \text{Eq. 2: } E_{\text{tot}} = \frac{1}{2} m_1 v_{1,0}^2 + \frac{1}{2} m_2 v_{2,0}^2 = \frac{1}{2} m_1 v_{1,f}^2 + \frac{1}{2} m_2 v_{2,f}^2 \]

- We rearrange both equations to get object 1 on the left and object 2 on the right:

\[
\text{Rearranged Eq. 1: } m_1 (v_{1,0} - v_{1,f}) = m_2 (v_{2,f} - v_{2,0})
\]

\[
\text{Rearranged Eq. 2: } \frac{1}{2} m_1 (v_{1,0}^2 - v_{1,f}^2) = \frac{1}{2} m_2 (v_{2,f}^2 - v_{2,0}^2)
\]

\[
m_1 (v_{1,0} - v_{1,f}) (v_{1,0} + v_{1,f}) = m_2 (v_{2,f} - v_{2,0}) (v_{2,f} + v_{2,0})
\]

\[
\text{New Eq. 2: } (v_{1,0} + v_{1,f}) = (v_{2,f} + v_{2,0})
\]

- Combining the last equation and the rearranged Equation 1, we have two equations and 2 unknowns which we can solve to get \(v_{1,f}\) and \(v_{2,f}\):

\[
v_{1,f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1,0} + \frac{2 m_2}{m_1 + m_2} v_{2,0}
\]

\[
v_{2,f} = \frac{m_2 - m_1}{m_1 + m_2} v_{2,0} + \frac{2 m_1}{m_1 + m_2} v_{1,0}
\]