Conceptual question

• A rock, initially at rest with respect to Earth and located an infinite distance away is released and accelerates toward Earth. An observation tower is built 3 Earth-radii high to observe the rock as it plummets to Earth. Neglecting friction, the rock’s speed when it hits the ground is
  – a) twice
  – b) three times
  – c) four times
  – d) six times
  – e) eight times

its speed at the top of the tower.

\[ KE_f + PE_f = KE_0 + PE_0 = PE_0 \]

\[ PE = -\frac{GM_Em}{r} \Rightarrow PE_0 = 0 \]

\[ \Rightarrow KE_f = -PE_f = \frac{GM_Em}{r_f} \]

\[
\frac{KE_{\text{ground}}}{KE_{\text{tower}}} = \frac{GM_Em}{r_{\text{ground}}} = \frac{r_{\text{tower}}}{r_E} = \frac{4r_E}{r_E} = 4
\]
Banked Turns (Lon-cap homework)

- An engineer wishes to design a curved exit ramp for a toll road in such a way that a car will not have to rely on friction to round the curve without skidding. She does so by banking the road in such a way that the force causing the centripetal acceleration will be supplied by the component of the normal force toward the center of the circular path.

- Strategy: Use Newton's second law: \( \mathbf{F}_{\text{net}} = m\mathbf{a}_c \)
A car is going down a winding road with a speed of 22 m/s and is going around a curve with a radius \( r \). The coefficient of static friction between road and car is 0.5. What is the minimum radius the bend can have without the car sliding of the road?

\[
f_s = \mu_s mg = \frac{mv^2}{r}
\]

\[
\mu_s g = \frac{v^2}{r}
\]

\[
r = \frac{v^2}{(\mu_s g)}
\]
Archimedes: “GIVE ME A PLACE TO STAND AND I WILL MOVE THE EARTH”
Torques

- The direction that you make an object turn about an axis depends not only on the force, but the point at which you apply the force.
- The relevant quantity that governs rotation is not the force, but is the torque instead:
**Torques**

- The direction that you make an object turn about an axis depends not only on the force, but the point at which you apply the force.
- The relevant quantity that governs rotation is not the force, but is the torque instead:
  \[ \tau_1 = F_1 r_1 \sin(\theta_1) = F_1 d_1 \text{ counterclockwise} \]
  \[ \tau_2 = -F_2 r_2 \sin(\theta_2) = F_2 d_2 \text{ clockwise} \]

Here, I assume positive torque is in the counterclockwise direction. 

- \( d_1 \) and \( d_2 \) are the lever arms for \( \tau_1 \) and \( \tau_2 \).
- \( \sum \tau = I\alpha \) is the rotational analog of \( \sum F = ma \)
- To calculate the torque about an axis:
  1. Choose your axis
  2. Draw your force.
  3. Draw the line, which is perpendicular to the force and goes from the line of force to the axis.
  4. This line is your lever arm \( d \).
  5. Torque has the units N\( \cdot \)m.
Example

- Find the net torque (magnitude and direction) produced by the forces \( F_1 \) and \( F_2 \) about the rotational axis shown in the drawing. The forces are acting on a thin rigid rod, and the axis is perpendicular to the page.

\[
\tau_{\text{net}} = \tau_1 + \tau_2
\]

\[
\tau_1 = -F_1d_1, \quad d_1 = 0.5m \cdot \sin(90^\circ) = 0.50m
\]

\[
\Rightarrow \tau_1 = -(20N)(0.5m) = -10Nm
\]

\[
\tau_2 = F_2d_2, \quad d_2 = 1.10m \cdot \sin(60^\circ) = 0.95m
\]

\[
\Rightarrow \tau_2 = (35N)(0.95m) = 33Nm
\]

\[
\tau_{\text{net}} = -10Nm + 33Nm = 23Nm = 23Nm \text{ counterclockwise}
\]
Conceptual quiz

- You are using a wrench and trying to loosen a rusty nut. Which of the arrangements shown is least effective in loosening the nut?
Rotational Equilibrium I

- Newton’s second law for linear motion states that the net force on an object is equal to its mass times its acceleration.
  \[ \sum F_i = ma \]

- If the net force is zero, the acceleration of the object is zero. It could be at rest or moving with constant non-zero velocity.

- But is the object necessarily in rotational equilibrium? Consider the case of a merry-go-round sitting on a frictionless surface. The only forces parallel to the surface are shown in the Figure. Obviously, the net horizontal force vanishes. What can you say about the angular acceleration around the vertical axis shown?
  - a) It is zero. The two torques cancel out.
  - b) It is non-zero and counterclockwise.
  - c) It is non-zero and clockwise
Rotational equilibrium II:

- Newton’s 2nd law for rotational motion states that the angular acceleration is given by the net torque. It states that the net torque, i.e. the sum of all torques about an axis, on a body is equal to the moment of inertia $I$ times the angular acceleration $\alpha$ about that axis:

$$\sum_i \tau_i = I \alpha$$

- Here, $I$ is the moment of inertia. It plays the same role as the mass in Newton’s law for linear motion. We discuss $I$ later.

- We define equilibrium to be the situation when both the linear acceleration and angular acceleration vanish. This occurs when both the net force and the net torque vanish. In other words:

$$\sum_i \tau_i = 0 \quad \text{and} \quad \sum_i F_i = 0$$

- Here, torques must vanish around any axis. Axes can be chosen so as to make the solutions of problems easier.