Physic 231 Lecture 23

Main points of last lecture:
- Tensile stress: $F/A$
- Tensile strain: $\Delta L/L$
- Relation between stress and strain:
  \[ \frac{\Delta F}{A} = Y \frac{\Delta L}{L} \]
  \[ \frac{\Delta F}{A} = \Delta P = B \frac{\Delta V}{V} \]
- Pressure in fluids:
  \[ P = \frac{F}{A}; P_{\text{bot}} = P_{\text{top}} + \rho gh \]
- Buoyancy:
  \[ F_B = W_{\text{displaced}} = \rho_f V_{\text{displaced}} g \]

Main points of today’s lecture:
- Buoyancy:
  \[ F_B = W_{\text{displaced}} = \rho_f V_{\text{displaced}} g \]
- Bernoulli’s equation
  \[ P_f + \frac{1}{2} \rho v_f^2 + \rho g y_f = P_0 + \frac{1}{2} \rho v_0^2 + \rho g y_0 \]
- Viscous flow
  \[ F = \eta \frac{Av}{d} \]
- Poiseuille’s law
  \[ \frac{\Delta V}{\Delta t} = \frac{\pi R^4 (P_1 - P_2)}{8\eta L} \]
Flow of an incompressible fluid

- Water is so difficult to compress that it is pretty accurate to assume that its density as liquid is constant. That has a consequence for its flow along a pipe.

- The density being constant means that you can neither increase or decrease the amount of water in the tube. The volume of water that goes in must equal the volume that comes out.

- In time $\Delta t$, we have $A_1 \Delta x_1$ as the volume going in and $A_2 \Delta x_2$ as the volume coming out. Thus:

\[
A_1 \Delta x_1 = A_2 \Delta x_2 \Rightarrow A_1 v_1 \Delta t = A_2 v_2 \Delta t \Rightarrow A_1 v_1 = A_2 v_2
\]
Example

(a) Calculate the mass flow rate (in grams per second) of blood ($\rho = 1.0 \text{ g/cm}^3$) in an aorta with a cross-sectional area of 2.0 cm$^2$ if the flow speed is 40 cm/s. (b) Assume that the aorta branches to form a large number of capillaries with a combined cross-sectional area of 3.0 x 10$^3$ cm$^2$. What is the flow speed in the capillaries?

\[
\begin{align*}
\text{a) } & \quad \frac{\Delta m}{\Delta t} = \rho \frac{\Delta V}{\Delta t} = 1 \text{ g/cm}^3 \frac{A \nu \Delta t}{\Delta t} = 1 \text{ g/cm}^3 \cdot 2 \text{ cm}^2 \cdot 40 \text{ cm/s} = 80 \text{ g/s} \\
\text{b) } & \quad A_1 \nu_1 = A_2 \nu_2 \Rightarrow \nu_2 = \frac{A_1}{A_2} \nu_1 = \frac{2 \text{ cm}^2}{3000 \text{ cm}^2} \cdot 40 \text{ cm/s} = 0.027 \text{ cm/s}
\end{align*}
\]
Conceptual quiz

• Blood flows through a coronary artery that is partially blocked by deposits along the artery wall. Through which part of the artery is the flux (volume of blood per unit time) largest?

-- a) The narrow part.
-- b) The wide part.
-- c) The flux is the same in both parts.
Bernoulli’s equation

- Consider the consequences of the work energy theorem on the tube of fluid to the right. Let’s do this assume no viscosity.
- The pressure on the left does positive work on the fluid and the pressure on the right does negative work. In this absence of viscosity, this is all of the work.
  \[ W = F_1 \Delta x_1 - F_2 \Delta x_2 \]
  \[ = P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2 = (P_1 - P_2) \Delta V \]
- The work-energy theorem says:
  \[ W = KE_f - KE_0 + PE_f - PE_0 \]
- The only change to the tube is that we replace a section to the lower left and add a section to the upper right. The change in the kinetic and potential energy of the tube is:
  \[ KE_f - KE_0 + PE_f - PE_0 = \frac{1}{2} \Delta mv_2^2 - \frac{1}{2} \Delta mv_1^2 + \Delta mg(y_2 - y_1) \]
- Using \( \Delta m = \rho \Delta V \) and rearranging terms we get:
  \[ (P_1 - P_2) \Delta V = \frac{1}{2} \rho \Delta V v_2^2 - \frac{1}{2} \rho \Delta V v_1^2 + \rho \Delta V g(y_2 - y_1) \]
  \[ \Rightarrow P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2 = P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 \]
Example

- A liquid ($\rho = 1.65 \ \text{g/cm}^3$) flows through two horizontal sections of tubing joined end to end. In the first section the cross-sectional area is 10.0 cm$^2$, the flow speed is 275 cm/s, and the pressure is $1.20 \times 10^5$ Pa. In the second section the cross-sectional area is 2.50 cm$^2$. Calculate the smaller section’s (1) flow speed and (2) pressure.

- answers to (1)

  a) 11 m/s

  1) $A_1v_1 = A_2v_2 \Rightarrow v_2 = \frac{A_1}{A_2}v_1 = \frac{10\text{cm}^2}{2.5\text{cm}^2} \times 275\text{cm/s} = 1100\text{cm/s}$

  2) $P_2 + \frac{1}{2} \rho v_2^2 = P_1 + \frac{1}{2} \rho v_1^2 \Rightarrow P_2 = P_1 + \frac{1}{2} \rho (v_1^2 - v_2^2)$

  $\Rightarrow P_2 = 1.2 \times 10^5 \text{Pa} + \frac{1}{2} (1650 \text{kg/m}^3) [(2.75 \text{m/s})^2 - (11.00 \text{m/s})^2]$

  $\Rightarrow P_2 = 1.2 \times 10^5 \text{Pa} - 9.36 \times 10^4 \text{Pa} = 2.64 \times 10^4 \text{Pa}$
Example

The water in a beaker is about 0.1 m deep. A small hole opens about 0.01 m from the bottom. What is the speed of water emerging from this hole?

\[
\Rightarrow P_2 + \frac{1}{2} \rho v_2^2 + \rho gy_2 = P_1 + \frac{1}{2} \rho v_1^2 + \rho gy_1
\]

\[
P_1 = P_2 \Rightarrow \frac{1}{2} \rho v_2^2 + \rho gy_2 = \frac{1}{2} \rho v_1^2 + \rho gy_1
\]

\[
\Rightarrow v_2^2 = 2g(y_1 - y_2) \Rightarrow v_2 = \sqrt{2g(y_1 - y_2)}
\]

\[
v_2 = \sqrt{2(9.8)(0.09)} \text{ m/s} = 1.4 \text{ m/s}
\]