Physic 231 Lecture 9

• Main points of last lecture:
  • Atwood’s machine.
  • Normal force
  • Newton’s 3\textsuperscript{d} Law:
    – “When a body exerts a force on another, the second body exerts an equal oppositely directed force on the first body.”

• Main points of today’s lecture:
  • Examples of 3\textsuperscript{d} Law
  • Frictional forces:
    – kinetic friction:
      \[ f_k = \mu_k N \]
    – static friction
      \[ f_s < \mu_s N \]
  • More examples.
Your code is: EJDGDF

Put your name here:

Keep this exam \textbf{CLOSED} until advised by the instructor.

50 minute long closed book exam.

Fill out the bubble sheet: last name, first initial, student number, section number and code.

When done, hand in your test and your bubble sheet.

Thank you and good luck!

Possibly useful constant:

\begin{itemize}
  \item $g = 9.81 \text{ m/s}^2$
\end{itemize}
A block with mass 5 kg and a second block with mass 10 kg are supported by a frictionless surface. A force of 60 N is applied to the 10 kg mass. What is the force of the 5 kg block on the 10 kg block?

- **10 kg**
- **5 kg**
- **60 N**

**Example**

\[ \vec{N}_{12} \quad \vec{N}_{21} \]

\[ 60 \text{ N} \]

\[ \text{10 kg} \quad \text{5 kg} \]

x components: (drop subscript x)

body 2: \( N_{21} = m_2 a; \) \( N_{21} = -N_{12} \)

body 1: \( F + N_{12} = m_1 a; \) \( F = -N_{12} + m_1 a \)

\[ F = N_{21} + m_1 a = m_2 a + m_1 a = (m_2 + m_1) a \]

\[ a = \frac{F}{(m_2 + m_1)} \]

\[ N_{12} = -N_{21} = -m_2 F / (m_2 + m_1) = -20N \]

\[ N_{12} = 20N \text{ to the left} \]

If objects move together, the relevant mass is their total mass.
Example

Two skaters, an 82 kg man and a 48 kg woman, are standing on ice. Neglect any friction between the skate blades and the ice. The woman pushes the man with a force of 45 N due east. Determine the accelerations (magnitude and direction) of the man and the woman.

\[
x \text{ components (West is positive)}
\]

\[
a_{\text{man}} = \frac{-45 \text{ N}}{82 \text{ kg}} = 0.55 \text{ m/s}^2 \text{ east}
\]

\[
a_{\text{woman}} = \frac{45 \text{ N}}{48 \text{ kg}} = 0.94 \text{ m/s}^2 \text{ west}
\]
Two skaters, an 100 kg man and a 50 kg woman, are standing on ice. Neglect any friction between the skate blades and the ice. By pushing the man, the woman is accelerated at 2 m/s² in the direction of due west. What is the corresponding acceleration of the man?

- a) 4 m/s² due east
- b) 1 m/s² due east
- c) 2 m/s² due east
- d) 1.5 m/s² due east
Friction

- Friction impedes the motion of one object along the surfaces of another. It occurs because the surfaces of the two objects temporarily stick together via “microwelds”. The frictional force can be larger if the two surfaces are at rest with respect to each other.

- Experimentally we have two cases:
  - kinetic friction:
    \[ f_k = \mu_k N \]
  - static friction
    \[ f_s < \mu_s N \]

- The coefficient of static friction exceeds that for kinetic friction:
  \[ \mu_s > \mu_k \]

- Frictional forces always oppose the motion of one surface with respect to the other.
Example

Consider the figure below, with $M_1 = 105$ kg and $M_2 = 44.1$ kg. What is the minimum static coefficient of friction necessary to keep the block from slipping.

\[ T = M_2 g \]
\[ T \leq \mu_s N = \mu_s M_1 g \]
\[ M_2 g \leq \mu_s M_1 g \]
\[ \frac{M_2}{M_1} \leq \mu_s \]
Example

- Consider the figure below, with $M_1 = 105$ kg. If the two mass move with constant speed and the coefficient of kinetic friction is 0.2, what is $M_2$?

$$T = M_2 g$$
$$T = \mu_k N = \mu_k M_1 g$$
$$M_2 g = \mu_k M_1 g$$
$$M_2 = \mu_k M_1 = 21 \text{ kg}$$
Example

- Chucky puts a block on an incline plane as shown below. He increases the angle to 40°, at which point the block begins to slide. What is the static coefficient of friction?

1. Draw the forces.
2. Choose an appropriate coordinate system.
3. Calculate the components.
4. Use Newton’s 2nd law to get \( \mu_s \)

\( W, N, f_s \) are magnitudes of forces

\[ x : \sum_i F_{i,x} = 0 = f_s - W \sin(\theta) \]

\[ \Rightarrow \mu_s N > f_s = W \sin(\theta) \]

\[ y : \sum_i F_{i,y} = 0 = N - W \cos(\theta) \]

\[ \Rightarrow N = W \cos(\theta) \]

\[ \Rightarrow \mu_s W \cos(\theta) > W \sin(\theta) \]

\[ \Rightarrow \mu_s > \tan(\theta) \]

at max, \( \mu_s = \tan(40^\circ) = .84 \)
Example

- The block shown below starts sliding down the ramp. Assuming the coefficient of kinetic friction $\mu_k = 0.3$, how long does it take for the block to travel 2m to the bottom of the ramp?

1. Draw the forces.
2. Choose an appropriate coordinate system.
3. Calculate the components.
4. Use Newton’s 2nd to get $t$.

$W, f_k, N$ are magnitude of forces

$x: \sum_i F_{i,x} = ma_x = f_k - W \sin(\theta)$

$\Rightarrow \mu_k N - mg \sin(\theta) = ma_x$

$y: \sum_i F_{i,y} = 0 = N - W \cos(\theta)$

$\Rightarrow N = mg \cos(\theta)$

$\Rightarrow \mu_k g \cos(40^\circ) - g \sin(40^\circ) = a_x$

$\Rightarrow a_x = -4.05 m / s^2$

$\Delta x = \frac{1}{2} a_x t^2 \Rightarrow t = \sqrt{\frac{2\Delta x}{a_x}} = \sqrt{\frac{-4m}{-4.05m/s^2}} = 1s$
Example

Two packing crates of masses 10.0 kg and 5.00 kg are connected by a light string that passes over a frictionless pulley as in the figure below. The 5.00-kg crate lies on a smooth incline of angle 40.0°. Find the acceleration of the 5.00-kg crate and the tension in the string.

\[
\begin{align*}
T - M_1 g &= M_1 a_1 \\
T - M_2 g \sin(40^\circ) &= M_2 a_2 \\
a_1 &= -a_2 \\
T &= M_1 (a_1 + g) = M_2 (a_2 + g \sin(40^\circ)) \\
- M_1a_2 + M_1g &= M_2a_2 + M_2g \sin(40^\circ) \\
a_2 &= \frac{M_1g - M_2g \sin(40^\circ)}{M_1 + M_2} = 4.4 \text{m/s}^2 \\
T &= M_1 \left( g - 4.4 \text{m/s}^2 \right) = 54 \text{N}
\end{align*}
\]