Physics 492 Homework 4:

1. In the semi-classical limit, the Fermi energy of an ideal gas of $N$ identical spin-$1/2$ particles with mass $m$ in a volume $V$ is:

$$E_F = \frac{\hbar^2}{2m} \left( \frac{3\pi^2 N}{V} \right)^{2/3}$$

Consider a nucleus with $Z$ protons, $N = A - Z$ neutrons and radius $R = r_0 A^{1/3}$, where $r_0 = 1.18$ fm. In the ideal-gas model, the total internal kinetic energy of the nucleus, in terms of Fermi energies for protons and neutrons, is

$$E = \frac{3}{5} Z E_F(Z) + \frac{3}{5} N E_F(N)$$

(a) (3 pts.) Determine $E_F$ and $E$ for $^{16}$O.

(b) (4 pts.) If $|N - Z| \ll A$, then

$$E \approx E_0 + a_A \frac{(N - Z)^2}{A}$$

where $E_0 = 3/5 A E_F(A/2)$ is the energy of a symmetric nucleus with $N = Z = A/2$. Determine the value of $a_A$ in the ideal-gas limit. Hint: Write

$$N = \frac{A}{2} + \delta \quad \text{and} \quad Z = \frac{A}{2} - \delta$$

where

$$N - Z = 2\delta$$

and expand the ideal-gas energy in $\delta$. Be careful in retaining the proper order of expansion. (This give you another shot as solving is a modified Problem 4.3 in Williams.)

2. (3 pts) Problem 4.1 from Williams

3. (4 pts) Problem 4.2 from Williams

4. (4 pts) Williams, Problem 5.1. Note that to maintain the unit consistency, the mass formula in Williams should be actually written as

$$M(Z, A) \ c^2 = Z M_H \ c^2 + N M_N \ c^2 - a_v \ A - a_e \ A^{2/3} - \ldots$$

5. (5 pts) Williams, Problem 5.5. Hint: Calculate $a_C$ from the $Q$ value of the $\beta^+$ decay of $^{35}$Ar. Estimate $a_A$ using the fact that $^{155}$Ba is stable and thus the $Q$ values of $\beta^+$ and $\beta^-$ decays must be negative; $Q (\beta^+) < 0$ and $Q (\beta^-) < 0$ imply upper and lower bounds on $a_A$ after substituting the value of $a_C$. 
