I adopted a different notation in the first posting of this solution. The original posting is on the next page. Here is the solution in the notation that I have used so far. At threshold we have:

\[ g_{\mu\nu}p^\mu_{\text{tot}}p^\nu_{\text{tot}} = (m_{k0} + m_{\Lambda})^2 c^2 = \left( m_{k0} c^2 + m_{\Lambda} c^2 \right) / c^2 \]

In the proton rest frame \( k^\mu = \left( E_\pi / c, \vec{p}_\pi \right) \) and \( p^\mu = \left( m_p c, 0, 0, 0 \right) \).

We have

\[ g_{\mu\nu}p^\mu_{\text{tot}}p^\nu_{\text{tot}} = (m_{k0} + m_{\Lambda})^2 c^2 = g_{\mu\nu} (k^\mu + p^\mu)(k^\nu + p^\nu) = g_{\mu\nu} k^\mu k^\nu + g_{\mu\nu} p^\mu p^\nu + 2 g_{\mu\nu} k^\mu p^\nu \]

\[ = m_{\pi}^2 c^2 + m_p^2 c^2 + 2E_\pi m_p \]

\[ \Rightarrow E_z = \frac{(m_{k0} + m_{\Lambda})^2 c^2 - m_{\pi}^2 c^2 - m_p^2 c^2}{2m_p} = 909.1 \text{MeV} \]

\[ T_\pi = 909.1 - 139.6 = 769 \text{MeV} \]
1. a) \[ Q = m_\pi c^2 + m_\rho c^2 - (m_\pi + m_\rho c^2)^2 = 139.6 + 938.3 - 998 \text{ MeV} = 536 \text{ MeV} \]

b) \[ p_{tot}^\mu = (m_\pi + m) c^2 = \frac{(m_\pi c^2 + m_\rho c^2)}{c^2} \]

At the proton rest frame, \[ k^\mu = (E_\pi/c, \vec{p}_\pi) \quad p^\mu = (m_\pi c, 0, 0, 0) \]

We have \[ p_{tot}^\mu = (m_\pi + m) c^2 = (k^\mu + p^\mu)(k^{\mu'} + p^{\mu'}) \]

\[ = k^\mu k^{\mu'} + p^\mu p^{\mu'} + 2k^{\mu'}p^\mu \]

\[ \Rightarrow E_\pi = \frac{(m_\pi + m)^2 c^2 - m^2 c^2 - m_\rho^2 c^2}{2m_\rho} \]

\[ = \frac{(m_\pi c^2 + m_\rho c^2)^2 - m_\pi^2 c^2 - m_\rho^2 c^2}{2m_\rho} \]

\[ = \frac{(498 \text{ MeV} + 116 \text{ MeV})^2 - (140 \text{ MeV})^2 - (938)^2}{2(938)} \]

\[ = 909.1 \text{ MeV} \quad \Rightarrow T = 769.1 \text{ MeV} \]

c) \[ \pi^- : 1\bar{u}d > \rho : 1uud \quad k^0 : 1\bar{s}d > \Lambda \quad Z = \frac{1}{2}[(u\bar{d} - d\bar{u})\Lambda] \]
2a) \[ P(e) + P(e') = P(z) = (m_{e0}c^2, 0, 0, 0) \]
\[ P^u(e^-) = (E_e/c, \vec{p}) \quad P^u(e^+) = (E_e/c, -\vec{p}) \]
\[ \Rightarrow 2E_e/c = m_{e0}c \quad \Rightarrow E_e = \frac{1}{2} m_{e0}c^2 = 4.6 \text{ GeV} \]
(this includes electron rest energy)

b) In lab frame
\[ P_{e(1)}^u = (E_e/c, \vec{p}) \quad P_{e(2)}^u = (m_e c, 0) \]
\[ P_{z(1)} = (P_{e(1)}^u + P_{e(2)}^u) \]
\[ P_{z(2)} = (P_{e(1)}^u + P_{e(2)}^u) \]
\[ m_{e0}^2 c^2 = 2m_e^2 c^2 + 2 P_{e(1)}^u \cdot P_{e(2)}^u = 2m_e^2 c^2 + 2E_e m_e \]
\[ \Rightarrow \bar{E} = m_{e0}^2 c^4 - 2m_e^2 c^4 \approx \frac{(m_{e0} c^2)^2}{2(m_e c^2)} = 8.3 \times 10^6 \text{ GeV} \]

(c) The problem is that the velocity of the CM system in part b) is so high. The corresponding energy of the z in the lab is
\[ E_{z(c)} = 0.92 \text{ GeV} \sim E_e + m_e c^2 \]
when \[ \gamma = (1 - (\frac{V}{c})^2)^{-1/2} \] is the velocity of the CM.
Most of the electron energy goes to make the kinetic energy of the z.
Very large \[ T(z) = (\gamma_{cm} - 1) \cdot 92 \text{ GeV} \gg 92 \text{ GeV} \]
You might regard \( T(z) \) as "wasted" energy.
3. We have part of the table:

<table>
<thead>
<tr>
<th>Particle</th>
<th>$M_e^2$ (MeV)</th>
<th>$S$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta^{++}$</td>
<td>1230</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\Delta^+$</td>
<td>1231</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\Delta^0$</td>
<td>1282</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\Delta^-$</td>
<td>1284</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\Xi^+$</td>
<td>1383</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>$\Xi^0$</td>
<td>1384</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>$\Xi^-$</td>
<td>1387</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>$\Omega^-$</td>
<td>1532</td>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>1672</td>
<td>-3</td>
<td>3</td>
</tr>
</tbody>
</table>

a) Assume Mass of $\Sigma = \text{mass of } \Xi$. Plot $M_e^2$ vs $151$ (Plot is on attached sheet)

b) Assume Mass = Mass ($s = 0$) + $151 \cdot \overline{\xi} M_e^2 - \frac{[M_o^2 + M_e^2]}{2}$

$c^2 = \frac{M_o + M_d}{2} c^2 + 148.47 \pm 1.2 \text{ MeV}$

4. $K^- : 1u\bar{s}\bar{d} \quad K^+ : 1u\bar{s}\bar{d} \quad \Xi^- : 1s\bar{s}d \quad p : 1uud$

Can not do a quark flow for the decay processes because flow is not conserved.

- In $\Xi^- \rightarrow \Lambda^0 + \pi^0$ we have $1s\bar{d}s \rightarrow 1s\bar{d}u + \frac{1}{2} [1u\bar{s}\bar{d} - 1d\bar{d}]$
- which does not conserve the strangeness flow.

- Also in $\Lambda_0 \rightarrow p + \pi^-$ we have $1uds \rightarrow 1uud + 1d\bar{d}$
- which also does not conserve strangeness.

To accomplish such decays we need weak interactions which are described in the Quark Flow diagrams with $W$ bosons.
Strange $S=3/2$ Baryon Masses

\[ y = m_1 + m_2 \cdot M_0 \]

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>m1</td>
<td>1233.6</td>
<td>1.7493</td>
</tr>
<tr>
<td>m2</td>
<td>148.4</td>
<td>1.2369</td>
</tr>
<tr>
<td>Chisq</td>
<td>122.4</td>
<td>NA</td>
</tr>
<tr>
<td>R</td>
<td>0.99972</td>
<td>NA</td>
</tr>
</tbody>
</table>

$M_c^2$ (MeV) vs $|S|$
Charge symmetry means \( V_{mn} = V_{pp} \) where the \( V_{mn}, V_{pp} \) represent the strong interaction part of the \( mn \) and \( pp \) interaction potential.

Charge independence means \( V_{np} = V_{pn} = V_{nm} \) when the involved nucleons are in the same spatial and spin states.

Considering Fig. 9.10, we have two "mirror nuclei" where \( N_1 = Z_2 \) and \( Z_1 = N_2 \). The equality of the level diagrams can be expected if \( m_p = m_n \) and \( m_{np} = m_{pn} \). There is only a common shift of all levels due to the additional constant change in the same nucleus.

Fig. 9.11 illustrates charge independence. The are identical levels in \( ^{6}\text{Li}, ^{6}\text{Be} \) and \( ^{6}\text{He} \) that have the same \( E^* \) and \( J^\pi \). The additional levels in \( ^{6}\text{Li} \) are ones that are excluded from \( ^{6}\text{Be} \) and \( ^{6}\text{He} \) due to the Pauli exclusion principle.

i) Generally, charge independence (or isospin symmetry) is to say that \( V_{uu} = V_{dd} = V_{uu} \) when the quarks are in the same state. Then if \( m_u = m_d \), we have

\[
\begin{align*}
\bar{m}_u (u \bar{d}) &= m_p (d \bar{u}) \\
p^{+} &\Rightarrow \bar{p}^{-} &= \pi^+ = \frac{1}{2} f^+ f^-
\end{align*}
\]

at incident momenta \( \approx 300 \text{ MeV} \) but cross sections are dominated by the \( \Delta \) baryons.

![Diagram of \( \Delta \) baryons and quarks](image)

The \( \Delta \) baryon is in a quark state \( \Delta^+ \)(1) \( t = \frac{3}{2}, t_3 = \frac{1}{2} \) \( \Delta^0 \)(2) \( t = \frac{1}{2}, t_3 = -\frac{1}{2} \) \( \Delta^+ \)(2) \( t = \frac{3}{2}, t_3 = -\frac{1}{2} \)

\( t_3 = \frac{1}{2}(\text{number of } u \text{ quarks} - \text{number of } d \text{ quarks}) - \text{number of } \bar{u} \text{ quarks} + \text{number of } \bar{d} \text{ quarks} \)
\[ k^+ : (u \bar{s}) \text{ has } t = \frac{1}{2}, \ t_3 = \frac{1}{2}, \ s = +1 \]
\[ k^- : (\bar{u} s) \text{ has } t = \frac{1}{2}, \ t_3 = -\frac{1}{2}, \ s = -1 \]
\[ p + k^- \text{ has } t_3 = 0, \ t = 0, \ s = -1 \quad u u d + \bar{u} s = \text{uds quark} \]
\[ m + k^+ \text{ has } t_3 = 0, \ t = 0, \ s = 1 \quad u d d + u \bar{s} = \text{uuds quark} \]

The two channels differ in their strangeness so cannot be the same. The \( p + n^- \) can have contributions from \( \Lambda \) and \( \Sigma^0 \) as resonances which are built from \( u d s \) quark states.

\( m + k^+ \) is not compatible with any baryonic resonances.

\( \rho^0 \) decay by strong interaction to \( \pi^+ \pi^- \)

\( \rho \quad \text{need to state that flavor change is not required} \)

so its lifetime is short, \( 10^{-23} \text{s} \)

\( \kappa^0 \) must decay by weak interaction because it requires a flavor change of the quark

\( \kappa^0 \) must decay by weak interaction

because it requires a flavor change of the quark

\( \pi^0 \) can decay by E.M.

\( \pi^0 \stackrel{1}{\rightarrow} \left\{ \frac{1}{2} u \bar{u} - 1 \bar{d} d \right\} \)
(10.3) a) $\pi^- + \nu^0 \rightarrow K^+ + \Sigma^+$

does not conserve strangeness $\Sigma^+ : S = 3/2$
$\Sigma^+ : S = -1$

cannot go by strong force

f) does not conserve charge.
not allowed.

b) does not conserve baryon number.

The rest appears to be allowed

7.0 a) $p \uparrow 1 u u d >$
$n \uparrow l u d d >$
$\Delta^- \uparrow l u d d >$
$\Delta^0 \uparrow l u d d >$
$\Delta^+ \uparrow l u d d >$
$\Delta^+ \uparrow l u u d >$

b) There is an obvious problem in that:
$\Delta^+ \uparrow l u u d >$, all $j = 3/2$ and $m_j = 3/2$

all the quarks have $m_j = 3/2$ and all
have spatial wavefunction as no
radial node and $l = 0$. Thus they all are
in the same spatial and spin state.

Something occurs for the $\Delta^-$ which a l u d d > state with
all quarks in the same space $l = 0$ and spin $m_j = 3/2$

This problem is resolve by saying that quarks have "color"
and putting quark in the "green" state, one in the "blue" state
and the last of the three quarks in the "red" state.
c) Higher states of the nucleon have higher values of the orbital angular momentum or of the radial quantum number. Something occurs for higher states of the N.

d) N and N* have positive parity. The parity of an N* would be \((-1)^l\) where \(l\) is the orbital angular momentum of the N*. Something applies to the higher \(l\) states of a \(N^*\).

\[\Delta^0\]

\[\Lambda(\bar{s}u)\]

The quark flow diagram shows that this followed by the strong interactions.

need to state that it does not require flavor changing diagram shown but diagram is not required.

because the flavor change in switching goes to a \(u\) quark is needed, goes by a weak decay, which is much slower.