• Main points of last lecture:
  – Isotope shifts
  – Hadronic scattering
  – Summary of nuclear sizes and shapes
  – Nuclei as liquid drops-Semi-empirical mass formula
    • Bulk
    • Surface
    • Coulomb

• Main points of today’s lecture:
  – Nuclei as liquid drops-Semi-empirical mass formula
    • Asymmetry
    • Pairing
  – Discussion and examples
    • Fission
    • Shell effects
  – Nuclear instability
    • Gamma decay
    • Alpha decay
    • Beta decay
Liquid drop formula

\[ \beta = -\sum M(A,Z)c^2 - N M_{\text{nucl}} - Z M_{\text{nucl}} \}

- Stable masses of light nuclei have \( N \approx Z \). Why?
  \[ B = a_v A - q_s A^{\frac{2}{3}} - q_a \frac{(N-Z)^2}{A} \quad q_a = 23 \text{ MeV} \]

Symmetry energy

Asymmetry energy

- Minimum is shifted to \( N > Z \) for heavier nuclei, Why?

\[ B_{\text{coul}} = -\langle V_{\text{coul}} \rangle \]

\[ V_{\text{coul}} = \frac{1}{4 \pi \varepsilon_0} \frac{e^2}{r} \]

\[ \beta = a_v A - q_s A^{\frac{2}{3}} - q_a \frac{(N-Z)^2}{A} \]

\[ R = 1.2 A^{\frac{1}{3}} - q_c \frac{\varepsilon_2}{A^{\frac{1}{3}}} \]

Nuclear Landscape

Less than 300 stable

Known nuclei

Terra incognita 

Odd A

N = 126
Partial explanation of symmetry energy.

\[ \rho = \frac{k^2}{2m} \]

\[ \frac{\rho}{\bar{m}} \psi = \varepsilon \psi \]

- Neutrons and protons in nuclear interior can be approximated by Fermi gas
  - For \(N \approx Z\)

\[ B = -E = -\left< kE \right> - \left< pE \right> \]

\[ \frac{k^2}{2m} \left( k_x^2 + k_y^2 + k_z^2 \right) = \varepsilon \]

\[ \frac{2}{h^2} \frac{\partial^2}{\partial x^2} \frac{dn}{dx dx dx} \]

\[ N = \int d^3 r d^3 \rho \frac{dn}{d^3 r d^3 \rho} \]

\[ \left< \frac{kE}{A} \right> = \left< \frac{p^2}{2m} \right> \]

\[ \left< \frac{kE}{A} \right> = \frac{1}{N} \cdot 2 \int d^3 r d^3 \rho \frac{1}{h^3} \frac{p^2}{2m} \]

\[ \left< \frac{kE}{A} \right> = 3/5 \frac{p^2}{2m} \left( \frac{3 N}{V} \right) \]

\[ \left< \frac{kE}{A} \right> = 3/5 \frac{p^2}{2m} = 3/5 \varepsilon_F = 22 \text{ MeV} \]

\[ p \approx 0.16 \text{ nucleon} / f_m^3 \]

\[ e_F = (3.5 \text{ MeV}) \left( \frac{3}{5} \right) \]
Partial explanation of symmetry energy, continued.

- Neutrons and protons in nuclear interior can be approximated by Fermi gas
  
  For $N \neq Z$

  $$\frac{\langle kE \rangle}{A} = \frac{1}{A} \left\{ N^{3/5} \epsilon_{F,\text{protons}} + Z^{3/5} \epsilon_{F,\text{neutrons}} \right\}$$

  $$\epsilon_{F,\text{protons}} = \frac{k^2}{2m} \left( 3 \pi^2 \frac{Z}{V} \right)^{2/3}$$

  $$\epsilon_{F,\text{neutrons}} = \frac{k^2}{2m} \left( 3 \pi^2 \frac{N}{V} \right)^{2/3}$$

  $$\frac{kE}{A} = \frac{q \alpha}{V} \left( \frac{Z-N}{V} \right)^2$$

  $q_q \approx 13$ MeV

  Real value $\approx 24$ MeV
In homework:

- In homework, you will Taylor expand $\langle KE/A \rangle$ to second order in N-Z to get an estimate of the symmetry energy coefficient $a_s$. It will lead to a value $\sim 12-13$ MeV that is much smaller than the empirical value. (The difference reflects the enhanced binding of neutrons to protons, compared to the binding of protons to protons or neutrons to neutrons.)
Pairing term

- Compare the binding energies of even-A and odd-A isobars:

- This binding energy difference can be parameterized by a pairing term.

\[-\delta(q, z)\] 

\[
\begin{align*}
\delta(q, z) &= -\frac{q^2}{A+n} \quad \text{for even-even nucleon} \\
&= 0 \quad \text{for } A \text{ odd} \\
&= \frac{q^2}{A+n_2} \quad \text{for } Z \text{ odd, } N \text{-odd}
\end{align*}
\]
Origin of pairing term

Least bound neutrons $N$

These will occupy the same orbit.

C Atom

$\uparrow \uparrow \downarrow$

Ps 2 occupied

$\uparrow \uparrow \downarrow$

2s 6

$\uparrow \downarrow$

1s

- $j \leq m_j \leq j$

$m_j = \frac{3}{2}$

$m_j = \frac{3}{2}$

$m_j = -\frac{3}{2}$

$^{12}\text{C}$

$^{16}\text{O}$ 6 states

$^{16}\text{O}$ 1s

m substates of nuclear state of $^9\text{Be}$
Main points of last lecture:
- Nuclei as liquid drops-Semi-empirical mass formula
  - Coulomb
  - Pairing
- Mass Excess

Main points of today’s lecture:
- Discussion and examples
  - Fission
- Deformation
- Shell effects
- Nuclear instability
  - Gamma decay
  - Alpha decay
  - Beta decay
Full Liquid-Drop formula (Semi-Empirical Mass Formula)

\[-E = - \langle \text{ke}^{2} \text{pe}^{-2} \rangle = B = q_{v} A - q_{s} A^{2/3} - q_{a} \frac{(N-Z)^{2}}{A} - q_{c} \frac{Z^{2}}{A^{1/3}} + S(A, Z)\]

\[q_{v} = 15.56 \text{ MeV} \quad q_{s} = 17.13 \text{ MeV} \quad q_{c} = 0.697 \text{ MeV}\]

\[q_{a} = 23.285 \text{ MeV} \quad q_{p} = 12 \text{ MeV}\]

\[S(A, Z) = \pm \frac{2 \text{ MeV}}{A^{1/2}}\]

\[\text{Pos} \quad \text{Even - Even} \quad \text{Neg} \quad \text{Odd - Odd.}\]
Quick question

• What is the surface tension $T$ for a nucleus? Give an formal expression for it in terms of the nuclear binding energy and compute its typical value.

\[ B_{\text{surface}} = -a_s \left( \frac{A^{2/3}}{R} \right) = -4\pi R^2 T \]

\[ R = 1.2 \sqrt[3]{A} \quad a_s = 17.23 \text{ MeV} \]

\[ T = 0.95 \text{ MeV/fm}^2 \]
Example:

- Calculate the various terms in the binding energy of $^{16}\text{O}$.

\[
B_{\text{vol}} = q_v A = 15.56 \text{ MeV} \cdot 16 = 249 \text{ MeV}
\]

\[
B_{\text{surf}} = -q_s A^{2/3} = -17.23 \frac{16^{2/3}}{16} = -109.4 \text{ MeV}
\]

\[
B_{\text{symmetry}} = -q_s \frac{(N-Z)^2}{A} = 0
\]

\[
B_{\text{coul}} = -q_c \frac{Z^2}{A^{1/3}} = -6.97 \frac{8^2}{16^{1/3}} = -17.7 \text{ MeV}
\]

\[
B_{\text{pair}} = \frac{12 \text{ MeV}}{16^{1/2}} = 3 \text{ MeV}
\]

\[
B = B_{\text{vol}} + B_{\text{surf}} + B_{\text{coul}} + B_{\text{symmetry}} + B_{\text{pair}} = 124.86 \text{ MeV}
\]

Shell effects

\[
B_{\text{exp}} = 127.62 \text{ MeV}
\]
Fission example

- The nucleus $^{238}\text{U}$ can undergo spontaneous fission. One of the many fission channels is
  
  $$^{235}_{92}\text{U} \rightarrow ^{87}_{35}\text{Br} + ^{145}_{57}\text{La} + 3n$$

- Estimate the energy released in this channel.

\[
M(92,235)c^2 = M(35,87)c^2 + M(57,145)c^2 + 3m_nc^2 + Q
\]

\[
-B(92,235) = -B(35,87) - B(57,145) + Q
\]

using LDM

\[
Q = 160\text{MeV}
\]
What doesn’t the LDM predict?

- Quantum effects in the nucleus:
  - Nucleon orbits and closed shells.
  - Deformation of the nucleus.
  - Excited nuclear states

- The values for the saturation density and the density dependence of the various coefficients of the LDM expansion.
B and the instability of nuclear ground states

- Neutron and proton decay
  - drip lines
  \[ S_P = B(z, A) - B(z-1, A-1) \]
  \[ S_n = B(z, A) - B(z, A-1) \]
  \[ \uparrow \text{ governs the upper and lower boundaries} \]

- Alpha decay
  \[ (z, A) \Rightarrow (2, 4) + (z-2, A-4) \]

- Beta decay
  \[ (z, A) \Rightarrow (z+1, A) + e^- + \bar{\nu}_e \]
Stability of nuclear excited states

- Excited states of any nucleus will decay:
  - particle decay
  - gamma decay
  - isomers

\[ (Z,A)^* \rightarrow (Z,A) + \gamma \]
\[ E^* \sim E_\gamma \]

\[ \text{rate} \propto E_\gamma^{2(J_\gamma+1)} \]

Very low \( E_\gamma \), large \( J_\gamma \), long lifetimes
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Main points of last lecture:
- Deformation
- Shell effects
- Drip lines
- Nuclear instability
  - Gamma decay
  - Alpha decay
  - Beta decay

Main points of today’s lecture:
- Nuclear instability
  - Beta decay
- Consequences of nuclear instability.
  - Number of stable isobars
  - Scarcity of odd-odd nuclei.
- Nuclei with more than one decay mode.
- Long decay chains secular equilibrium
- Evolution of alpha decay q value.
- Fission.
Alpha decay

• Alpha decay Q-value
\[ M(Z,A)c^2 \Rightarrow M(2,4)c^2 + M(Z-2, A-4)c^2 + Q \]
\[ B(Z-2, A-4) + B(2,4) - B(Z, A) = Q \]

• Relationship between lifetime and Q value
Many “stable” nuclei can alpha decay

- The lifetimes of many alpha emitters can be very long:
Weak decay

• Beta decay conserves A

\[ \text{^4He} \rightarrow \text{^6Li} + e^- + \bar{\nu}_e \]

\[ \beta^- \text{decay} \]

- changes proton number
- creates or destroys electron and neutrinos
Some weak decay processes

- $\beta^-$ decay
  \[ (Z,A) \rightarrow (Z+1,A) + e^- + \bar{V}_e \]
- $\beta^+$ decay
  \[ (Z,A) \rightarrow (Z-1,A) + e^+ + V_e \]
- Electron capture
  \[ e^- + (Z,A) \rightarrow (Z-1,A) + V_e \]
- Muon capture
  \[ \mu^- + (Z,A) \rightarrow (Z-1,A) + V_\mu \]
- Angular momentum conservation
  \[ ^{14}_7C \rightarrow ^{14}_6N + e^- + \bar{V}_e \]
  \[ \text{J}=0 \quad \text{J}=1 \quad \text{J}=\frac{1}{2} \quad \text{J}=\frac{1}{2} \]
Energetics of weak decay

- Electron capture
  \[ M_e c^2 + M(Z,A) c^2 = M(Z-1,A) c^2 + Q \]

- Nuclear mass

- Atomic mass

- β⁺ decay
  \[(Z,A) \rightarrow (Z-1,A) + e^+ + \nu_e\]

  \[ M(Z,A) c^2 = M(Z-1,A) c^2 + M_e c^2 + Q \]

  \[ M(Z,A) c^2 - Z M_e c^2 = M(Z-1,A) c^2 - (Z-1) M_e c^2 + M_e c^2 + Q \]

  \[ \Rightarrow M(Z,A) c^2 = M(Z-1,A) c^2 + 2 M_e c^2 + Q \]
Energetics of weak decay

- $\beta^-$ decay

\[ M(Z,A)c^2 = M(Z+1,A)c^2 + M_e c^2 + Q \]

\[ M(Z,A)c^2 - Z M_e c^2 = M(Z+1,A)c^2 - (Z+1) M_e c^2 + M_e c^2 + Q \]

\[ M(Z,A)c^2 = M(Z+1,A)c^2 + Q \]
Odd-A decays

\[ e^- + ^{254}_{92}Es \rightarrow ^{254}_{90}Cf + \nu_e \]

\[ ^{254}_{92}Es \rightarrow ^{250}_{90}Fm + e^- + \bar{\nu}_e \]

\[ ^{254}_{92}Es \rightarrow \alpha + Bk \]

\[ ^{254}_{92}Cf \rightarrow ^{250}_{90}Ca + \alpha \]

\[ \rightarrow ^{104}_{40}N_0 + ^{145}_{79}La \]

\[ ^{254}_{92}Fm \rightarrow ^{250}_{90}F + \alpha \]

\[ \rightarrow ^{104}_{40}N_0 + ^{145}_{79}Ce \]

\[ \frac{1}{\tau} = \omega = \sum w \]

1. Odd-odd nuclei have multiple decay branches

2. Can have several different decay pathways

3. Large \( Z \) \( \alpha \)-decays, fission
Actinide decays

- Actinides are unstable with respect to alpha decay and fission
Nuclei decay

\[ \beta \xrightarrow{2.8 \text{ min}} \ \text{Po}^{218}_n \]
\[ \beta \xrightarrow{2.8 \text{ min}} \ \text{Kr}^{216}_n \]

\[ A = 4m + 1 \]

\[ A = 4m + 2 \]

\[ A = 4m + 3 \]

\[ 232 \text{ Th} \]

\[ 238 \text{ U} \]