Main points of last lecture:
- Angular momentum dependence.
- Structure dependence.
- Nuclear reactions
- Q-values
- Kinematics for two body reactions.

Main points of today’s lecture:
- Measured quantities – types of reactions
- Kinematics
- Final state properties: inelastic scattering and transfer.
  - Direct reactions
- Compound nuclei and their properties.
Reactions

• Classify by final state
  - $a + b \rightarrow a + b$ (elastic scattering)
  - $a + b \rightarrow a + b^*$ (inelastic scattering)
  - $a + b \rightarrow c + d$
  - $p + ^{12}C \rightarrow p + ^{12}C$
  - $d + ^{12}C \rightarrow p + m + ^{12}C$ (deuteron breakup)

• Inclusive and exclusive reactions
  - Inclusive reaction: $\alpha + ^{12}C \rightarrow (p) + m + ^{12}C$
    - Only thing measured
  - Exclusive reaction: $\rightarrow p + m + ^{12}C$
    - All things measured

Kinematics

• What is it?

  Kinematics tells us the excitation of the final \( nC \) — distinguishes elastic from inelastic.

Ex.: Elastic and inelastic differ by Q-value.

• Determination of reaction Q-value, center of mass angle, etc are important for comparison to theory.

  – What is the Q-value?

  \[
  \text{Q-value} = \sum_{\text{initial}} \gamma c^2 - \sum_{\text{final}} \gamma c^2
  \]

  \[
  M^2 c^2, E^* = 4.44 \text{ MeV}
  \]

  \[
  M^2 c^2, g.s.
  \]

  \[
  M^2 c^2 + 4.44 \text{ MeV}
  \]
Kinematics continued

- In many cases, the physics is simpler in the Center of Mass system.

\[
\begin{align*}
\mathbf{a} + \mathbf{b} & \rightarrow \mathbf{c} + \mathbf{d} \\
\mathbf{a}_a, \mathbf{v}_{a,\text{lab}} & \quad \mathbf{b}_b, \mathbf{v}_{b,\text{lab}} \quad \Rightarrow \quad \mathbf{M}, \mathbf{v}_c, \mathbf{M}_d, \mathbf{v}_d \\
\dot{\mathbf{R}} : \frac{\mathbf{a}_a \dot{\mathbf{r}}_a + \mathbf{b}_b \dot{\mathbf{r}}_b}{\mathbf{m}_a + \mathbf{m}_b} & \Rightarrow \quad \mathbf{v}_{\text{cm}} = \frac{\mathbf{m}_a \mathbf{v}_{a,\text{lab}}}{\mathbf{m}_a + \mathbf{m}_b} - \frac{\mathbf{p}_{\text{tot}}}{\mathbf{M}_t} \\
\mathbf{u}_a, \mathbf{v}_{a,\text{cm}} & = \mathbf{v}_{a,\text{lab}} - \mathbf{v}_{\text{cm}} \\
\mathbf{v}_{b,\text{cm}} & = \mathbf{v}_{b,\text{lab}} - \mathbf{v}_{\text{cm}} \\
\mathbf{p}_{a,\text{cm}} & = \mathbf{m}_a \mathbf{v}_{a,\text{cm}} = \mathbf{m}_a \mathbf{v}_{a,\text{lab}} - \mathbf{m}_a \mathbf{v}_{\text{cm}} \\
& = \mathbf{m}_a \left(1 - \frac{\mathbf{u}_a, \mathbf{v}_{a,\text{lab}}}{\mathbf{m}_a + \mathbf{m}_b} \right) \mathbf{v}_{a,\text{lab}} = \frac{\mathbf{m}_b \mathbf{m}_g}{\mathbf{m}_a + \mathbf{m}_b} \mathbf{v}_{a,\text{lab}} \\
\mathbf{p}_{b,\text{cm}} & = -\mathbf{u}_a, \mathbf{v}_{\text{lab}} = -\mathbf{p}_{\text{cm},a} \\
\mathbf{p}_{\text{cm} \text{ initial}} & = \mathbf{u}_a, \mathbf{v}_{\text{lab}} = \mathbf{u}_a, \mathbf{v}_{\text{lab}} \\
\end{align*}
\]
Kinematics continued

- CM momenta and velocities, etc.
  \[
  \vec{\nu}_{a,\text{cm}} = \frac{\vec{p}_{\text{cm},a}}{m_a}, \quad \vec{\nu}_{b,\text{cm}} = \frac{-\vec{p}_{\text{cm},b}}{m_b}, \quad \vec{\nu}_{a,\text{cm}} = \frac{-m_b \vec{\nu}_{b,\text{cm}}}{m_a}
  \]

- Transformation back to lab frame, determination of $Q$

  \[
  T_{\text{cm},i} = \frac{\vec{p}_{\text{cm},i}^2}{2m_a} + \frac{\vec{p}_{\text{cm},i}^2}{2m_b}
  \]

  \[
  T_{\text{cm},f} = \frac{\vec{p}_{\text{cm},f}^2}{2m_c} + \frac{\vec{p}_{\text{cm},f}^2}{2m_d} - \frac{\vec{p}_{\text{cm},i}^2}{2m_a} - \frac{\vec{p}_{\text{cm},i}^2}{2m_b}
  \]

  \[
  \Phi = m_a c^2 + m_b c^2 - m_c c^2 - m_d c^2
  \]

Choose \[
\vec{p}_{\text{cm}} = -\vec{p}_{a,\text{cm}} \]

\[
\vec{\nu}_{c,\text{lab}} = \vec{\nu}_{c,\text{cm}} + \vec{V}_{\text{cm}} = \frac{\vec{p}_{c,\text{cm}}}{m_c} + \vec{V}_{\text{cm}}
\]

\[
\vec{\nu}_{b,\text{lab}} = \vec{\nu}_{b,\text{cm}} + \vec{V}_{\text{cm}}
\]
Example of Measurement – transfer reaction

• Some notation:

\[ p + {}^{12}\text{C} \Rightarrow d + {}^{11}\text{C} \quad \leftrightarrow \quad {}^{12}\text{C} \ (p,d)_{\text{C}} \]

• Using magnets to get velocities and Q-value

\[ d + {}^{45}\text{Sc} \Rightarrow p + {}^{46}\text{Sc} \]

\[ \vec{F} = q \vec{v} \times \vec{B} = -\hat{r} \frac{mv^2}{r} \]

\[ r = \frac{mv}{qB} \]
A real example

Excitation Energy in $^{46}\text{Sc}$, MeV

$T_d = 7\text{ MeV}$

$^{45}\text{Sc}(d,p)^{46}\text{Sc}$

$\theta = 37.5^\circ$
Calibrate spectrometer to obtain $B \cdot \rho$ as function of $B$ and distance along focal plane (bend angle).

$$r = \frac{mv}{qB}$$

Reaction: $^{45}\text{Sc}(d,p)^{46}\text{Sc}$

- $E^* = 0.0000$ MEV, Q-VALUE = 6.5433 MEV
- ECMI = 6.700 MEV, ECMF = 13.243 MEV
- ANGLE ENERGY ANGLE $B \cdot \rho$
  - 37.5, 13.418, 38.28, 5.331

- $E^* = 0.0520$ MEV, Q-VALUE = 6.4913 MEV
- ECMI = 6.700 MEV, ECMF = 13.191 MEV
- ANGLE ENERGY ANGLE $B \cdot \rho$
  - 37.5, 13.367, 38.28, 5.320

- $E^* = 0.1430$ MEV, Q-VALUE = 6.4003 MEV
- ECMI = 6.700 MEV, ECMF = 13.100 MEV
- ANGLE ENERGY ANGLE $B \cdot \rho$
  - 37.5, 13.276, 38.29, 5.302

- $E^* = 0.2280$ MEV, Q-VALUE = 6.3153 MEV
- ECMI = 6.700 MEV, ECMF = 13.015 MEV
- ANGLE ENERGY ANGLE $B \cdot \rho$
  - 37.5, 13.191, 38.29, 5.285
What is the physics here?

\[ d + ^{45}\text{Sc} \rightarrow p + ^{46}\text{Sc} \]

Neutron is transferred to Quantum state in $^{45}\text{Sc}$ potential well

$\Rightarrow$ measurement teaches about neutron orbits in $^{46}\text{Sc}$
Same technique can be used for inelastic scattering

- Measurement of inelastic scattering on $^{26}\text{Mg}$

$^{26}\text{Mg}(p,p')$ at $E_p = 295$ MeV

V$_{np}$ depends strongly on spin
Physics 492 Lecture 17

• Main points of last lecture:
  – Final state properties: inelastic scattering and transfer, breakup, etc.
  – Kinematics
  – Direct Reactions

• Main points of today’s lecture:
  – review
review

• Closed Book.
• There will be some formulae on the exam itself.
• Homework, in class examples and the review are good study sources.
Chapter 1

- Coulomb scattering, Rutherford formula
- Relationship between $b(\theta)$ and $\frac{d\sigma}{d\Omega}$
- Turning point radius vs. angle for Coulomb scattering.

\[
\frac{d\sigma}{d\Omega} = \frac{b}{\sin(\theta)} \left| \frac{db}{d\theta} \right|
\]

\[
\frac{d\sigma}{d\Omega} = \left( \frac{\kappa}{4T_{cm}} \frac{1}{\sin^2\left(\frac{\theta}{2}\right)} \right)^2
\]

\[
r_{tp}(\theta) = \frac{\kappa}{2T_{cm}} \left[1 + \csc(\theta/2)\right]; \quad \kappa = \frac{Z_{\text{proj}} Z_{\text{targ}} e^2}{4\pi\varepsilon_0}
\]
Example 1

• If \( b(\theta ) = C \cdot \sin^2(\theta) \), what is \( \frac{d\sigma}{d\Omega} \)?

\[
\frac{d\sigma}{d\Omega} = 2\pi \int b d\theta
\]

\[
d\Omega = 2\pi d\omega d\theta = 2\pi \sin \theta d\theta
\]

\[
\frac{d\sigma}{d\Omega} = 2\pi \sin^2 \theta d\theta
\]
Example 2

- Consider the elastic scattering cross section for a $^{20}\text{Ne}$ beam incident on a $^{208}\text{Pb}$ target at 300 MeV. Assuming the nuclear radius can be given by $R = 1.2 \cdot A^{1/3}$, estimate the center of mass scattering angle where $d\sigma/d\Omega$ decreases below about $0.25 \times d\sigma/d\Omega_{\text{Ruth}}$?

\[
R_{tp} = \frac{K}{2T_{cm}} \left[ 1 + \csc(\Theta_{cm/2}) \right]
\]

\[
K = \frac{Z_{\text{proj}} \cdot Z_{\text{tgt}} \cdot e^2}{4\pi\varepsilon_0} = 1.41 \text{ MeV} \cdot \text{fm} \cdot \frac{Z_t Z_p^2}{e^2}
\]

\[
R_{tp} = R_{\text{tgt}} + R_{\text{proj}} = 1.2 (A_{\text{proj}}^{1/3} + A_{\text{tgt}}^{1/3})
\]

\[
T_{cm} = T_{\text{LAB}} \frac{A_{\text{tgt}}^{1/3}}{A_{\text{cm}}^{1/3}}
\]

\[\Theta_{cm} = 30.4^\circ\]
Chapter 2

- Content: Radioactive decay
  - Relationships between $\omega$, $\tau$, and $t_{1/2}$
  - Exponential decay formula
  - Branching ratios
  - Sequential decay
Example 3

• problem 2.4 Williams

– calculate the decay rates for the $K^+$ into the various decay channels; i.e. fill in the last column of the table:

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>B.Ratio</th>
<th>$\omega_i = B.R. \cdot \tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^+ \rightarrow \mu^+ + \nu_\mu$</td>
<td>.635</td>
<td>$5.13 \times 10^{-8}$ s</td>
</tr>
<tr>
<td>$K^+ \rightarrow \pi^+ + \pi^0$</td>
<td>.212</td>
<td></td>
</tr>
<tr>
<td>$K^+ \rightarrow \pi^+ + \pi^+ + \pi^-$</td>
<td>.056</td>
<td></td>
</tr>
<tr>
<td>$K^+ \rightarrow \pi^+ + \pi^0 + \pi^0$</td>
<td>.017</td>
<td></td>
</tr>
<tr>
<td>$K^+ \rightarrow \pi^0 + \mu^+ + \nu_\mu$</td>
<td>.032</td>
<td></td>
</tr>
<tr>
<td>$K^+ \rightarrow \pi^0 + e^+ + \nu_e$</td>
<td>.048</td>
<td></td>
</tr>
</tbody>
</table>

– The mean life of the $K^+$ meson is $1.24 \times 10^{-8}$ s

$\omega = \frac{1}{\tau} = 8.08 \times 10^{-7}$ s
Example 4

- $^{241}\text{Am}$ decays by alpha emission to $^{237}\text{Np}$ with a mean lifetime of $\tau_{241}$. A source has $N_{241}(t)$ $^{241}\text{Am}$ nuclei in it. Write down a differential equation for $N_{241}(t)$ and solve it in terms of its initial value $N_{241}(0)$.

\[
\frac{dN_{241}(t)}{dt} = -\omega_{241} N_{241}(t) \quad \omega_{241} = \frac{1}{\tau_{241}}
\]

Answer:

\[
N_{241}(t) = N_{241}(0) e^{-t/\tau_{241}}
\]

- $^{237}\text{Np}$ decays by alpha emission to $^{233}\text{Pa}$ with a mean lifetime of $\tau_{237}$. Write down a differential equation for the number $N_{237}(t)$ of $^{237}\text{Np}$ in the $^{241}\text{Am}$ source.

\[
\frac{dN_{237}}{dt} = \omega_{241} N_{241}(t) - \omega_{237} N_{237}(t)
\]
Chapter 3

- Content
  - Nuclear charge distributions
  - Nuclear sizes
  - Electron scattering
  - Muonic atoms
  - Nuclear Scattering
Example 5

- Williams 3.6: Estimate the radius of the first Bohr orbit of a $\mu^-$ around a $^{12}\text{C}$ nucleus.

\[ R = \frac{4\pi\varepsilon_0 e^2}{Z m_u c^2} \quad n^2 = \frac{m^2}{Z} \frac{m_e}{m_u} \quad r_8 \]

\[ n = 1 \quad m_u/m_e = 205 \quad a_\infty = 43 \text{ fm} \]

\[ Z = 6 \]

- What is the principal quantum number for a Bohr orbit that lies just outside a lead nucleus ($Z=82, A=208$), if the nuclear radius is $1.2A^{1/3}$ fm?

\[ L_n = \frac{n^2}{82} \cdot \frac{1}{205} \quad 5.29 \times 10^{-4} \text{ fm} = 3.2 \quad n^2 \text{ fm} \]

\[ n = 2 \quad 12.5 \text{ fm} \quad R = 1.2 \quad 208^{1/3} \approx 7.1 \text{ fm} \]
Example 6

- Using the diffraction formula for a black sphere, calculate the nuclear strong absorption radius and $r_0$ from the differential cross section below:

$$R = r_0 A^{\frac{1}{3}}$$

$$\lambda = \frac{\hbar}{\rho} = \sqrt{\frac{\hbar c}{2 M m c^2 \cdot 14 MeV}}$$

$$R = 5 \text{ fm}$$

$$r_0 = 1.29 \text{ fm}$$

14 MeV neutrons
58 Ni target.
Chapter 4

• Content
  – Liquid drop model
  – Alpha decay energetics
  – Fission energetics
  – Neutron and proton separation energies
Example 7

- Provide expressions for the proton separation energy for $^{20}$F in terms of the nuclear masses, the atomic masses, the binding energies and the semi empirical mass formula (LDM)

- $M(Z,A)c^2 = M(Z-1,A-1)c^2 + M_p c^2 - S_p$

- $M(Z,A)c^2 = M(Z,A)c^2 + Z M_{e}c^2$

- $S_p = -M(Z,A)c^2 + M(Z-1,A-1)c^2 + M_p c^2$

- Binding energies: $M(Z,A)c^2 = Z M_{p}c^2 + N M_{n}c^2 - B(Z,A)$

- $-B(Z,A) = -B(Z-1,A-1) - S_p$

- $S_p = B(Z,A) - B(Z-1,A-1)$
Solution continued
Chapter 5

- Contents
  - Beta decay energetics for $\beta^+$, $\beta^-$ and electron capture.
  - Alpha decay energetics
  - Fission energetics
Example 8

- What is the Q value for $\beta^-$ decay of $^{65}$Ni in terms of nuclear masses, atomic masses and binding energies?

$$^{65}_{28}\text{Ni} \rightarrow ^{65}_{29}\text{Cu} + e^- + \bar{\nu}_e$$

Nuclear

$$M(28,65)c^2 = M(29,65)c^2 + M_e c^2 + Q$$

Atomic

$$M(28,A)c^2 = M(29,A)c^2 + Z_m m_e c^2$$

$$M(28,65)c^2 = M(29,65)c^2 + Q$$

$\beta^-$

$$Q = M_n c^2 - M_e c^2 + B(29,65) - B(28,65) - m_e c^2$$
Chapter 6

- Content: Alpha Decay
  - WKB formula for alpha decay rates and lifetimes.
  - Dependence on Q value
  - Dependence on angular momentum
  - Dependence on deformation
  - Sharing of energy between alpha particle and daughter nucleus.
  - Long alpha decay chains.
Example 9

- Describe briefly the physical processes occurring in α-decay. Without detailed calculation, give a qualitative explanation of the dependence of the transition rate on the Z of the daughter nucleus and on the energy released in the transition, Q.

Barrier penetration

- Barrier is Coulomb barrier

\[ \text{height} = V_{\text{coul}} - Q = \frac{Z_1 e^2}{r_{\text{out}}} - Q \]

Barrier increases with Z. Barrier height decreases with Q

\[ \text{Q depends on } r_{\text{out}}, R \text{ decrease with Q} \]

\[ w \propto \frac{4\pi Z_1^2 e^2}{\sqrt{4\pi \beta^2}} \sim \exp\left(-\frac{2Z_1 e^2}{\beta}\right) \]

\[ w \text{ decreases exponentially with } Z \text{ increases with Q} \]
Example

The figure shows the α-decay scheme of $^{244}_{96}\text{Cm}$ to $^{240}_{94}\text{Pu}$. The transitions are marked with the branching fractions (branching ratios) in percent. According to a simple formula the transition rate, $\lambda$, for the ground state to ground state transition is given by $\ln \lambda = C - DZ/Q^{1/2}$, where $C=132.8$ and $D=3.97$ (MeV)$^{1/2}$ when $\lambda$ is in s$^{-1}$. Calculate the mean life of $^{244}_{96}\text{Cm}$.

\[ \lambda = \omega_{g.s} = BR_{g.s}, \quad \omega = BR_{g.s} \]

\[ T = \frac{BR_{g.s}}{\lambda} = BR_{g.s} \exp \left( \frac{DZ}{Q^{1/2} - C} \right) \]

\[ = 8.4 \times 10^8 \text{ s} \]