Physics 231 Lecture 16

- Main points of today’s lecture:
  - Buoyancy
    \[ F_B = W_{\text{displaced}} = \rho_f V_{\text{displaced}} g \]
  - Bernoulli’s equation
    \[ P_f + \frac{1}{2} \rho v_f^2 + \rho g y_f = P_0 + \frac{1}{2} \rho v_0^2 + \rho g y_0 \]
  - Viscous flow
    \[ F = \eta \frac{A v}{d} \]
  - Poiseuille’s law
    \[ \frac{\Delta V}{\Delta t} = \frac{\pi R^4 (P_1 - P_2)}{8 \eta L} \]
Reading Quiz

3. The buoyant force on an object submerged in a liquid depends on

A. the object’s mass.
B. the object’s volume.
C. the density of the liquid.
D. both B and C.
**Buoyancy**

- Consider the small volume in the vessel at the right. If the volume is filled with the same fluid in the vessel, it will be in equilibrium. Thus: 
  \[ F_B - W = 0 \text{ where } W = \rho_{\text{water}} V_{\text{in\_box}} g, \]
  \[ V_{\text{in\_box}} = V_{\text{displaced}} \Rightarrow \text{therefore } F_B = \rho_{\text{water}} V_{\text{displaced}} g \]
- Now, replace the volume of fluid with the same volume of some other material. There will still be the same buoyant force pushing up on the block of material.
- Example: A kidney weighs 5.7 N in air. If the kidney is completely submerged in water, its apparent weight is 1.6 N. Determine the specific gravity of the kidney. (specific gravity = \( \rho_{\text{kidney}}/\rho_{\text{water}} \))

Apparent weight = \( T = W - F_B = W \left(1 - \frac{F_B}{W}\right) = W \left(1 - \frac{\rho_{\text{water}} V_{\text{kidney}} g}{\rho_{\text{kidney}} V_{\text{kidney}} g}\right) \)

Apparent weight = \( W \left(1 - \frac{\rho_{\text{water}}}{\rho_{\text{kidney}}}\right) \Rightarrow 1 - \frac{\rho_{\text{water}}}{\rho_{\text{kidney}}} = \text{Apparent weight}/W \)

\( \frac{\rho_{\text{water}}}{\rho_{\text{kidney}}} = 1 - \text{Apparent weight}/W = 1 - 1.6/5.7 = .71 \)

Specific gravity = \( \frac{\rho_{\text{kidney}}}{\rho_{\text{water}}} = 1/.71 = 1.39 \)
IT'S OK, THAT'S JUST ARCHIMEDES
Example

- A block of wood floats on the surface of a lake with 60% of its volume below the surface of the water. What is the density of the wood?

\[ F_B = \rho_{\text{water}} V_{\text{displaced}} g \quad W = \rho_{\text{wood}} V_{\text{wood}} g \]

\[ F_B = W \]

\[ \Rightarrow \rho_{\text{water}} V_{\text{displaced}} g = \rho_{\text{wood}} V_{\text{wood}} g \]

also, \[ V_{\text{displaced}} = 0.6 \cdot V_{\text{wood}} \]

\[ \Rightarrow \rho_{\text{water}} (0.6 \cdot V_{\text{wood}}) g = \rho_{\text{wood}} V_{\text{wood}} g \]

\[ \rho_{\text{wood}} = 0.6 \cdot \rho_{\text{water}} = 600 \text{ kg/m}^3 \]
Quiz

- A metal block \((\rho_{\text{steel}} = 7.86 \times 10^3 \text{ kg/m}^3)\) floats on the top of a pool of mercury \((\rho_{\text{mercury}} = 1.35 \times 10^4 \text{ kg/m}^3)\). The fraction of the volume of the block which lies below the surface is
- Hint: mercury plays the role of water. The Buoyancy force comes from displacing the mercury. Equate the weight of the steel to the buoyancy force from the mercury.

\[
V_{\text{displaced}} = f_{\text{below}} V_{\text{steel}}
\]

- a) 0.0
- b) 0.25
- c) 0.58
- d) 0.42
- e) 0.75

\[
W = F_B; \quad W = \rho_{\text{steel}} V_{\text{steel}} g; \quad F_B = \rho_{\text{mercury}} f_{\text{below}} V_{\text{steel}} g
\]

\[
\Rightarrow \rho_{\text{steel}} V_{\text{steel}} g = \rho_{\text{mercury}} f_{\text{below}} V_{\text{steel}} g \Rightarrow \rho_{\text{steel}} / \rho_{\text{mercury}} = f_{\text{below}}
\]

\[
\Rightarrow 7.86 \times 10^3 \text{ kg/m}^3 / 1.35 \times 10^4 \text{ kg/m}^3 = .58 = f_{\text{below}}
\]
Checking Understanding

Two blocks of identical size are submerged in water. One is made of lead (heavy), the other of aluminum (light). Upon which is the buoyant force greater?

A. On the lead block.
B. On the aluminum block.

C. They both experience the same buoyant force.

The buoyant force depends only on the density of the liquid and the volume that is displaced.
Checking Understanding

Two blocks are of identical size. One is made of lead, and sits on the bottom of a pond; the other is of wood and floats on top. Upon which is the buoyant force greater?

A. On the lead block.
B. On the wood block.
C. They both experience the same buoyant force.

The buoyant force depends only on the density of the liquid and the volume that is displaced.
Does the water level go up or down when the anchor is lowered to the bottom of the lake?

**When resting on the bottom**

\[ V_{\text{displaced}} = V_{\text{anchor}} \]

- a) The water level goes up.

**When floating in the boat**

\[ \rho_w V_{\text{displaced}} g = \rho_{\text{anchor}} V_{\text{anchor}} g \]
\[ \Rightarrow V_{\text{displaced}} = \frac{\rho_{\text{anchor}}}{\rho_w} V_{\text{anchor}} \]

- b) The water level goes down.

- c) The water level goes neither up nor down.
Conceptual question

- A 200-ton ship enters the lock of a canal. The fit between the sides of the lock and the ship is tight so that the weight of the water left in the lock after it closes is much less than 200 tons. Can the ship still float if the weight of the quantity of water left in the lock is much less than the ship’s weight?
  
  - a) Yes, as long as the water gets up to the ship’s waterline.
  - b) No, the ship touches bottom because it weighs more than the water in the lock.

It doesn’t matter how deep is the ocean. A boat displaces water until it comes up to the waterline and then it floats.
Conceptual question

- A lead weight is fastened on top of a large solid piece of Styrofoam that floats in a container of water. Because of the weight of the lead, the water line is flush with the top surface of the Styrofoam. If the piece of Styrofoam is turned upside down so that the weight is now suspended underneath it,
  - a) the arrangement sinks.
  - b) the water line is below the top surface of the Styrofoam.
  - c) the water line is still flush with the top surface of the Styrofoam.

Same displaced water in both figures!
A lead weight is fastened to a large solid piece of Styrofoam that floats in a container of water. Because of the weight of the lead, the water line is flush with the top surface of the Styrofoam. If the piece of Styrofoam is turned upside down, so that the weight is now suspended underneath it, the water level in the container
- a) rises.
- b) drops.
- c) remains the same.

If an object floats:
$$F_B = \rho_w g V_{\text{displaced}} = \text{weight of object}.$$  
The weight doesn’t change.
$$V_{\text{displaced}} \text{ doesn’t change.}$$  
The level doesn’t change.
Flow of an incompressible fluid

- Water is so difficult to compress that it is pretty accurate to assume that its density as liquid is constant. That has a consequence for its flow along a pipe.
- The density being constant means that you can neither increase or decrease the amount of water in the tube. The volume of water that goes in must equal the volume that comes out.
- In time $\Delta t$, we have $A_1 \Delta x_1$ as the volume going in and $A_2 \Delta x_2$ as the volume coming out. Thus:

$$A_1 \Delta x_1 = A_2 \Delta x_2 \Rightarrow A_1 v_1 \Delta t = A_2 v_2 \Delta t \Rightarrow A_1 v_1 = A_2 v_2$$
Example

• (a) Calculate the mass flow rate (in grams per second) of blood ($\rho = 1.0 \text{ g/cm}^3$) in an aorta with a cross-sectional area of 2.0 cm$^2$ if the flow speed is 40 cm/s. (b) Assume that the aorta branches to form a large number of capillaries with a combined cross-sectional area of $3.0 \times 10^3$ cm$^2$. What is the flow speed in the capillaries?

\[
\begin{align*}
\text{a)} & \quad \frac{\Delta m}{\Delta t} = \rho \frac{\Delta V}{\Delta t} ; \quad \Delta V = A v \Delta t \\
\Rightarrow & \quad \frac{\Delta m}{\Delta t} = 1 \text{ g/cm}^3 \frac{A v \Delta t}{\Delta t} = 1 \text{ g/cm}^3 \cdot 2 \text{ cm}^2 \cdot 40 \text{ cm/s} = 80 \text{ g/s}
\end{align*}
\]

b) $A_1 v_1 = A_2 v_2$

\[
\Rightarrow v_2 = \frac{A_1}{A_2} v_1 = \frac{2 \text{ cm}^2}{3000 \text{ cm}^2} 40 \text{ cm/s} = .027 \text{ cm/s}
\]
Bernoulli’s equation

- Consider the consequences of the work energy theorem on the tube of fluid to the right. Let’s do this assume no viscosity.
- The pressure on the left does positive work on the fluid and the pressure on the right does negative work. In this absence of viscosity, this is all of the work by external forces.
  \[ W_{\text{ext}} = F_1 \Delta x_1 - F_2 \Delta x_2 \]
  \[ = P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2 = (P_1 - P_2) \Delta V \]
- The work-energy theorem says:
  \[ W_{\text{ext}} = KE_f - KE_0 + PE_f - PE_0 \]
- The only change to the tube is that we replace a section to the lower left by a section to the upper right with an equal mass of fluid. The change in the kinetic and potential energy of the tube is:
  \[ KE_f - KE_0 + PE_f - PE_0 = \frac{1}{2} \Delta m v_2^2 - \frac{1}{2} \Delta m v_1^2 + \Delta mg (y_2 - y_1) \]
- Using \( \Delta m = \rho \Delta V \) and rearranging terms we get:
  \[ (P_1 - P_2) \Delta V = \frac{1}{2} \rho \Delta V v_2^2 - \frac{1}{2} \rho \Delta V v_1^2 + \rho \Delta V g (y_2 - y_1) \]
  \[ \Rightarrow P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2 = P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 \]
Example

- The water in a beaker is about 0.1 m deep. A small hole opens about 0.01 m from the bottom. What is the speed of water emerging from this hole?

\[
P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2 = P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1
\]

\[P_1 = P_2 \Rightarrow \frac{1}{2} \rho v_2^2 + \rho g y_2 = \frac{1}{2} \rho v_1^2 + \rho g y_1\]

\[\Rightarrow v_2^2 = 2g(y_1 - y_2) \Rightarrow v_2 = \sqrt{2g(y_1 - y_2)}\]

\[v_2 = \sqrt{2(9.8)(0.09)} \text{ m/s} = 1.4 \text{ m/s}\]
Example

- A liquid ($\rho = 1.65 \text{ g/cm}^3$) flows through two horizontal sections of tubing joined end to end. In the first section the cross-sectional area is 10.0 cm$^2$, the flow speed is 275 cm/s, and the pressure is $1.20 \times 10^5$ Pa. In the second section the cross-sectional area is 2.50 cm$^2$. Calculate the smaller section’s (1) flow speed and (2) pressure.

- answers to (1)
  - a) 11 m/s
  - b) 3 m/s
  - c) 0.7 m/s
  - d) 15 m/s

1) $A_1v_1 = A_2v_2 \Rightarrow v_2 = \frac{A_1}{A_2}v_1 = \frac{10\text{ cm}^2}{2.5\text{ cm}^2} \cdot 275\text{ cm/s} = 1100\text{ cm/s}$

2) $P_2 + \frac{1}{2}\rho v_2^2 = P_1 + \frac{1}{2}\rho v_1^2 \Rightarrow P_2 = P_1 + \frac{1}{2}\rho (v_1^2 - v_2^2)$

$\Rightarrow P_2 = 1.2 \times 10^5 \text{ Pa} + \frac{1}{2} (1650 \text{ kg/m}^3) [ (2.75 \text{ m/s})^2 - (11.00 \text{ m/s})^2 ]$

$\Rightarrow P_2 = 1.2 \times 10^5 \text{ Pa} - 9.36 \times 10^4 \text{ Pa} = 2.64 \times 10^4 \text{ Pa}$