Main points of today’s lecture:

- Newton’s law of universal gravitation:
  \[ F = \frac{GMm}{r^2} \]

- Kepler’s laws and the relation between the orbital period and orbital radius.
  \[ T^2 = \left( \frac{4\pi^2}{GM} \right) r^3 \]

- Gravitation potential energy
  \[ PE_{\text{grav}} = -\frac{GM_1M_2}{r_{12}} \]

- Tensile stress and strain
  \[ \frac{\Delta F}{A} = Y \frac{\Delta L}{L_0} \]

- Bulk stress and strain:
  \[ \frac{\Delta F}{A} = \Delta P = B \frac{\Delta V}{V} \]
A uniform 10 kg beam is supported at one end by a hinge and at the other by a cable. The cable from the wall is attached to the beam at an angle of 30°. What is the tension in the cable?

a) 9.8 N  

b) 19.6 N  

c) 29.4 N  

d) 39.2 N  

d) 49 N  

e) 58.8 N  

f) 68.6 N  

e) 98 N  

1. Draw the forces.

2. Choose the axis for calculating the torques so as to avoid calculating torques you don’t want to know.

Don’t want to know $F_{\text{hinge}}$, so put axis at hinge.

3. Set the net torque about this axis to zero and solve for the tension.

$$\sum_i \tau_i = 0 = TL \sin(30^\circ) - m_{\text{bar}} g \frac{L}{2}$$

$$TL \sin(30^\circ) = m_{\text{bar}} g \frac{L}{2}$$

$$T \left(1 - \frac{1}{2}\right) = m_{\text{bar}} g / 2$$

$$T = m_{\text{bar}} g / 2$$

$$T = 98 \text{N}$$

To get $\vec{F}_{\text{hinge}}$, use $\sum \vec{F}_i = 0$. 


Newton’s law of universal gravitation

- All objects (even light photons) feel a gravitational force attracting them to other objects. This force is proportional to the two masses and inversely proportional to the square of the distance between them.

\[ F_{12} = G \frac{m_1 m_2}{r_{12}^2} \]

\[ G = 6.673 \times 10^{-11} \text{ N m}^2 /\text{kg}^2 \]
Example

- A spaceship is on a journey to the moon. The masses of the earth and moon are, respectively, $5.98 \times 10^{24}$ kg and $7.36 \times 10^{22}$ kg. The distance between the centers of the earth and the moon is $3.85 \times 10^6$ m. At what point, as measured from the center of the earth, does the gravitational force exerted on the craft by the earth balance the gravitational force exerted by the moon? This point lies on a line between the centers of the earth and the moon.

\[
\frac{GM_{\text{moon}} m_{\text{ship}}}{r_{\text{moon\_ship}}^2} = \frac{GM_{\text{earth}} m_{\text{ship}}}{r_{\text{earth\_ship}}^2}
\]

\[
\frac{M_{\text{moon}}}{r_{\text{moon\_ship}}^2} = \frac{M_{\text{earth}}}{r_{\text{earth\_ship}}^2} \quad \Rightarrow \quad \frac{r_{\text{earth\_ship}}^2}{r_{\text{moon\_ship}}^2} = \frac{M_{\text{earth}}}{M_{\text{moon}}}
\]

\[
\frac{r_{\text{earth\_ship}}}{r_{\text{moon\_ship}}} = \sqrt{\frac{M_{\text{earth}}}{M_{\text{moon}}}} = \sqrt{\frac{5.98 \times 10^{24}}{7.36 \times 10^{22}}} = 9.01
\]

\[
r_{\text{moon\_ship}} = 3.85 \times 10^5 \text{ m} \quad \Rightarrow \quad r_{\text{earth\_ship}} = 3.47 \times 10^6 \text{ m}
\]
Example

- George weighs 100 N on the Earth. What would his weight be on the surface of another planet that has a planetary mass, which is 3 times the mass of Earth and mass and a planetary radius, which is 2 times the radius of Earth?
  - a) 75 N
  - b) 100 N
  - c) 125 N
  - d) 150 N

$$W_{\text{Earth}} = \frac{GM_{\text{Earth}} m_{\text{George}}}{R_{\text{Earth}}^2}$$

$$W_{\text{planet}} = \frac{GM_{\text{planet}} m_{\text{George}}}{R_{\text{planet}}^2} = \frac{G \left(3M_{\text{Earth}}\right)m_{\text{George}}}{\left(2R_{\text{Earth}}\right)^2}$$

$$= \frac{3GM_{\text{Earth}} m_{\text{George}}}{4R_{\text{Earth}}^2} = \frac{3}{4} \frac{GM_{\text{Earth}} m_{\text{George}}}{R_{\text{Earth}}^2}$$

$$W_{\text{planet}} = \frac{3}{4} W_{\text{Earth}} = \frac{3}{4} 100N = 75N$$

<table>
<thead>
<tr>
<th>$M_p$</th>
<th>$3M_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_p$</td>
<td>$2R_e$</td>
</tr>
<tr>
<td>$W_E$</td>
<td>100 N</td>
</tr>
<tr>
<td>$W_p$</td>
<td>?</td>
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</tbody>
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Hint: This is a ratio problem.
Conceptual question

- An astronaut is on a spacecraft floating “weightlessly” in orbit. While her feet are attached securely to the spacecraft, she shakes a large iron anvil rapidly back and forth. She reports back to Earth that
  - a) the shaking costs her no effort because the anvil has no inertial mass in space.
  - b) the shaking costs her some effort but considerably less than on Earth.
  - c) although weightless, the inertial mass of the anvil is the same as on Earth.
Kepler’s laws

• Johannes Kepler proposed three laws of planetary motion:

1. All planets moved in elliptical orbits with the Sun at one of the focal points.

2. Planetary orbits sweep out equal areas in equal times. (This is a consequence of angular momentum conservation.)

\[
\text{area} = \frac{1}{2} r \cdot r \Delta \theta = \frac{1}{2} r^2 \frac{\Delta \theta}{\Delta t} \Delta t \approx \frac{1}{2} r^2 \omega \Delta t
\]

⇒ \( r^2 \omega = \text{const} \).

3. The square of the orbital period of any planet is proportional to the cube of the average distance from the planet to the Sun. For a circular orbit:

\[
F_{\text{grav}} = \frac{GM_{\text{star}} \cdot m_{\text{planet}} v_{\text{planet}}}{r^2} = m_{\text{planet}} a_c = \frac{m_{\text{planet}} v^2}{r}
\]

⇒ \( \frac{GM_{\text{star}}}{r^3} = \frac{v^2}{r^2} = \omega^2 \)

\( \omega T = 2\pi \Rightarrow \omega = \frac{2\pi}{T} \)

\( \Rightarrow \frac{GM_{\text{star}}}{r^3} = \left(\frac{2\pi}{T}\right)^2 \)

⇒ \( T^2 = \left(\frac{4\pi^2}{GM_{\text{star}}}\right) r^3 \)

4. For an elliptical orbit:

⇒ \( T^2 = \left(\frac{4\pi^2}{GM_{\text{star}}}\right) a^3 \)
Conceptual question

- Two satellites $A$ and $B$ of the same mass are going around Earth in concentric circular orbits. The distance of satellite $B$ from Earth’s center is twice that of satellite $A$. What is the ratio of the centripetal force acting on $B$ to that acting on $A$?

- a) $1/8$
- b) $1/4$
- c) $1/2$
- d) $\sqrt{1/2}$
- e) 1

$F_{c, A} = \frac{G M_E m_s}{r_A^2}$
$F_{c, B} = \frac{G M_E m_s}{r_B^2}$

$F_{c, B} = \frac{G M_E m_s}{(2r_A)^2} = \frac{G M_E m_s}{4r_A^2} = \frac{1}{4} F_{c, A}$

$F_{c, B} = \frac{1}{4} F_{c, A}$

$\frac{F_{c, B}}{F_{c, A}} = \frac{1}{4}$

Hint: This is a ratio problem.
Example

- One satellite is in an orbit about Jupiter of radius $r_1$ and a period of 100 days. If a second satellite is placed in an orbit with 4 times the radius (i.e. $r_2 = 4r_1$), what is the period for the orbit of the second satellite?
  - a) 200 d
  - b) 400 d
  - c) 800 d
  - d) 1600 d

\[
T_1^2 = \left( \frac{4\pi^2}{GM_{\text{Jupiter}}} \right) r_1^3
\]

\[
T_2^2 = \left( \frac{4\pi^2}{GM_{\text{Jupiter}}} \right) r_2^3 = \left( \frac{4\pi^2}{GM_{\text{Jupiter}}} \right) (4r_1)^3
\]

\[
= 64 \left( \frac{4\pi^2}{GM_{\text{Jupiter}}} \right) r_1^3 = 64T_1^2
\]

Hint: this can also be solved as a ratio problem

\[
\Rightarrow T_2^2 = 64T_1^2 \Rightarrow T_2 = 8 \cdot T_1 = 8 \cdot 100 \text{ days} = 800 \text{ days}
\]
Conceptual quiz

Suppose Earth had no atmosphere and a ball were fired from the top of Mt. Everest in a direction tangent to the ground. If the initial speed were high enough to cause the ball to travel in a circular trajectory around Earth, the ball’s acceleration would
  - a) be much less than \( g \) (because the ball doesn’t fall to the ground).
  - b) be approximately \( g \).
  - c) depend on the ball’s speed.

Note: someone in a space ship in the same orbit would feel “weightless” and many would call this a “zero g” environment.

“weightlessness” is a sensation that occurs when one feels no effects of gravity. It happens when \( g=0 \) or when someone is falling freely.
Example

• The Earth orbits the sun in an circular orbit of radius \( r_E = 1.5 \times 10^{11} \) m. What is the mass of the sun?

\[
\Rightarrow T^2 = \left(\frac{4\pi^2}{GM_{\text{sun}}}\right) r^3 \Rightarrow M_{\text{sun}} T^2 = \left(\frac{4\pi^2}{G}\right) r^3
\]

\[
\Rightarrow M_{\text{sun}} = \left(\frac{4\pi^2}{G}\right) \frac{r^3}{T^2}
\]

\[
= \frac{4 \cdot (3.14)^2}{6.673 \times 10^{-11} \text{ N m}^2/\text{kg}^2} \frac{(1.50 \times 10^{11} \text{ m})^3}{\left(1 \text{ y} \left[\frac{365 \text{ d}}{3600 \text{ s}}\right]\left[\frac{24 \text{ h}}{3600 \text{ s}}\right] \left[\frac{3600 \text{ s}}{365 \text{ d}}\right]\right)^2} = 2.0 \times 10^{30} \text{ kg}
\]
Example

- A uniform steel beam of length 5.00 m has a weight of $4.5 \times 10^3$ N. One end of the beam is bolted to a vertical wall. The beam is held in a horizontal position by a cable attached between the other end of the beam and a point on the wall. The cable makes an angle of $25^\circ$ above the horizontal. A load whose weight is $12.0 \times 10^3$ N is hung from the beam at a point that is 3.5 m from the wall. Find (a) the magnitude of the tension in the supporting cable and (b) the magnitude of the force exerted on the end of the beam by the bolt that attaches to the wall.

Choose pivot to be at hinge:

$$\sum \tau_i = 0 = T(5\text{m})\sin(25^\circ) - W_{\text{load}}(3.5\text{m}) - W_{\text{beam}}(2.5\text{m})$$

$$T = W_{\text{load}} \cdot \frac{(3.5\text{m})}{(5\text{m})\sin(25^\circ)} + W_{\text{beam}} \cdot \frac{(2.5\text{m})}{(5\text{m})\sin(25^\circ)}$$

$$\Rightarrow T = 2.54 \times 10^4 \text{ N}$$

Hinge forces: $\sum \vec{F}_i = 0$;  
- x-comp: $R_x - T\cos(25^\circ) = 0$
  $$R_x = T\cos(25^\circ) = 2.29 \times 10^4 \text{ N}$$
- y-comp: $R_y - W_{\text{load}} - W_{\text{beam}} + T\sin(25^\circ) = 0$
  $$R_y = W_{\text{load}} + W_{\text{beam}} - T\sin(25^\circ) = 12.0\times 10^3 \text{ N} + 4.5\times 10^3 \text{ N} - 10.7\times 10^3 \text{ N}$$
  $$R_y = 5.8\times 10^3 \text{ N} \Rightarrow R = \sqrt{R_x^2 + R_y^2} = 23.6\times 10^3 \text{ N}$$
  $$\tan(\theta_R) = \frac{5.8\times 10^3 \text{ N}}{2.3\times 10^4 \text{ N}} = .25 \Rightarrow \theta_R = 14^\circ \text{ above horizontal}$$