Physics 231 Lecture 27

- Concepts for today’s lecture”
- Wave speed for a string
  \[ v = \sqrt{\frac{F}{\mu}} : F = \text{tension}; \mu = \text{m/L}. \]
- Sound intensity
  \[ \beta = 10 \log_{10} \left( \frac{I}{I_0} \right) \]
- I=P/A, \( I_0 = 1 \times 10^{-12} \text{ W/m}^2 \)
- Spherical waves
  \[ I = \frac{P}{4\pi r^2} \]
- Dopper shift
  \[ f' = f \left( \frac{v + v_o}{v - v_s} \right) \]
Wave velocity on string

- Waves travel faster on strings with higher tension and lower mass/unit length. The exact relationship is

\[ v = \sqrt{\frac{F}{\mu}} \]

Here \( F \) is the string tension and \( \mu \) is the mass/length.

- Example: A 0.5 m string is stretched so the tension is 1.7 N. A transverse wave of frequency 120 Hz and wavelength 0.31 m travels on the string. What is the mass of the string? (Hint: use wave velocity to get mass.)

\[ \mu = \frac{F}{v^2} \]

\[ v = \lambda f = (0.31 \text{m})(120 \text{Hz}) = 37.3 \text{m/s} \]

\[ \Rightarrow \mu = \frac{F}{v^2} = \frac{1.7 \text{N}}{(37.3 \text{m/s})^2} = 1.22 \times 10^{-3} \text{kg/m} \]

\[ \Rightarrow m = \mu L = (1.22 \times 10^{-3} \text{kg/m})(0.5 \text{m}) = 0.61 \text{g} \]
Example Problem

A particular species of spider spins a web with silk threads of density 1300 kg/m³ and diameter 3.0 µm. A typical tension in the radial threads of such a web is 0.007 N. What is the speed of the waves traveling on the web. If a fly lands in this web, which will reach the spider first, the sound or the wave on the web silk? (sound velocity=343 m/s)

a) 172 m/s, no
b) 215 m/s, no
c) 286 m/s, no
d) 430 m/s, yes
e) 873 m/s, yes

\[ \rho = 1300 \text{ kg/m}^3 \quad M = \rho V = \rho A_{\text{silk}} L \]

\[ \mu = \frac{M}{L} = \rho \frac{A_{\text{silk}}}{L} = \left(1300 \text{ kg/m}^3\right) \times 3.14159 \left(3 \times 10^{-6} \text{ m}^2\right) = 9.19 \times 10^{-9} \text{ kg/m} \]

\[ v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{0.007 \text{ kg/m/s}^2}{9.19 \times 10^{-9} \text{ kg/m}}} = 873 \text{ m/s} \]

This is faster than the speed of sound in air.
Conceptual question

- A weight is hung over a pulley and attached to a string composed of two parts, each made of the same material but one having four times the diameter of the other. The string is plucked so that a pulse moves along it, moving at speed $v_1$ in the thick part and at speed $v_2$ in the thin part. What is $v_1/v_2$?

$$M = V\rho = \pi R^2 L \rho \Rightarrow \mu = M/L = \pi R^2 \rho$$

$$\Rightarrow v_1 = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{F}{\pi R^2 \rho}} \quad \Rightarrow \quad \frac{v_1}{v_2} = \sqrt{\frac{1}{R_1^2}} \frac{1}{R_2^2} \frac{1}{\sqrt{R_2}} = \sqrt{\frac{R_2^2}{R_1^2}} = \frac{R_2}{R_1} = 1/4$$
Using a Tuning Fork to Produce a Sound Wave

- A tuning fork will produce a pure musical note of frequency $f$.
- As it’s tine swings to the right, it forces the air molecules near it closer together.
- This produces a high density area in the air.
  - This is an area of compression.
- As the tine moves toward the left, the air molecules to the right of the tine spread out.
- This produces an area of low density.
  - This area is called a rarefaction.
- As the tuning fork continues to vibrate, a sinusoidal succession of compressions and rarefactions spread out from the fork.

- Crests correspond to compressions and troughs to rarefactions.

$$v = \lambda f$$
Sound Waves

- Audible range: between 20 Hz to 20,000 Hz
- Infrasonic waves: f < 20 Hz; Ultrasonic: f > 20,000 Hz

- Sound velocity: \( v \approx \sqrt{\frac{\text{elastic property}}{\text{inertial property}}} \)

- Sound velocity in solid: \( v \approx \sqrt{\frac{Y}{\rho}} \); \( Y \) is Young's modulus

- Sound velocity in liquids or gasses: \( v \approx \sqrt{\frac{B}{\rho}} \); \( B \) is the bulk modulus

- Sound velocity in air: \( v \approx (331 \text{ m/s})\sqrt{\frac{T}{273K}} \); \( T \) is in Kelvin
  \[ v \approx 331 \text{ m/s} + (0.60 \text{ m/s/°C})T \]
  Here, \( T \) is in Celsius.

\[ T \text{ (in Kelvin)} = T \text{ (in Celsius)} + 273 \]
Example

• You are watching a pier being constructed on the far shore of a saltwater inlet when some blasting occurs. You hear the sound in the water 4.50 s before it reaches you through the air. How wide is the inlet? (Assume the air temperature is 20°C and sound velocity in water is 1530 m/s at its temperature (25°C). )

\[ v_{s,\text{water}} = 1530 \text{ m/s}; \quad v_{s,\text{air}} = \left(331 \text{ m/s}\right)\sqrt{\frac{T}{273\text{K}}} = \left(331 \text{ m/s}\right)\sqrt{\frac{293}{273}} = 343 \text{ m/s} \]

\[ t_{\text{air}} = \frac{d}{v_{s,\text{air}}}; \quad t_{\text{water}} = \frac{d}{v_{s,\text{water}}}; \quad t_{\text{air}} - t_{\text{water}} = d \left(\frac{1}{v_{s,\text{air}}} - \frac{1}{v_{s,\text{water}}}\right) \]

\[ \Rightarrow d = \left(\frac{t_{\text{air}} - t_{\text{water}}}{1/v_{s,\text{air}} - 1/v_{s,\text{water}}}\right) = \frac{4.5\text{ s}}{\left(\frac{1}{343 \text{ m/s}} - \frac{1}{1530 \text{ m/s}}\right)} = 1989 \text{ m} \]
A dolphin located in sea water at a temperature of 25°C emits a sound directed toward the bottom of the ocean 150 m below. How much time passes before it hears an echo? (The sound velocity in sea water = 1530 m/s at 25°C.)

\[
\Delta t = \frac{2d}{v} = \frac{300\text{m}}{1530\text{m/s}} = 0.196\text{s}
\]

- a) 0.2 s
- b) 0.4 s
- c) 0.6 s
- d) 0.8 s
- e) 0.1 s
Sound amplitude and intensity

- The amplitude of the sound wave is proportional to the maximum velocity of the air as it moves from the high pressure to the low pressure domains.

- The energy and the power of the sound wave is proportional to the square of amplitude:

\[ \langle E \rangle \propto \frac{1}{2} \rho v^2 \propto \text{Amplitude}^2 \propto v_{\text{max}}^2 \propto p^2 \propto x_{\text{max}}^2 \]

- More useful than the energy of a sound wave is the intensity, I, which is the power P that the sound wave transmits per unit area.

\[ I = \frac{P}{A} = \frac{\Delta E}{(A\Delta T)} \]

- The ear responds logarithmically to the intensity of sound waves striking the eardrum.

\[ I_{\text{threshold}} = I_0 \approx 10^{-12} \text{ W/m}^2, \quad I_{\text{pain}} \approx 1 \text{ W/m}^2 \]

- This logarithmic behavior motivates the decibel measure of sound wave intensity.

\[ \beta = 10 \log_{10}(I/ I_0) \leftrightarrow \beta/10 = \log_{10}(I/ I_0) \leftrightarrow 10^{\beta/10} = I/I_0 \]

### Table 14.2: Intensity Levels in Decibels for Different Sources

<table>
<thead>
<tr>
<th>Source of Sound</th>
<th>( \beta ) (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nearby jet airplane</td>
<td>150</td>
</tr>
<tr>
<td>Jackhammer, machine gun</td>
<td>130</td>
</tr>
<tr>
<td>Siren, rock concert</td>
<td>120</td>
</tr>
<tr>
<td>Subway, power mower</td>
<td>100</td>
</tr>
<tr>
<td>Busy traffic</td>
<td>80</td>
</tr>
<tr>
<td>Vacuum cleaner</td>
<td>70</td>
</tr>
<tr>
<td>Normal conversation</td>
<td>50</td>
</tr>
<tr>
<td>Mosquito buzzing</td>
<td>40</td>
</tr>
<tr>
<td>Whisper</td>
<td>30</td>
</tr>
<tr>
<td>Rustling leaves</td>
<td>10</td>
</tr>
<tr>
<td>Threshold of hearing</td>
<td>0</td>
</tr>
</tbody>
</table>
50 decibels normal conversation

70 decibels vacuum cleaner

\[ \log_{10}(AB) = \log_{10}(A) + \log_{10}(B) \]

one person \[ \beta = 10 \log_{10} \left( \frac{I}{I_0} \right) = 50 \text{ decibels} \]

10 people \[ \beta = 10 \log_{10} \left( \frac{100I}{I_0} \right) \]

\[ = 10 \log_{10}(10) + 10 \log_{10} \left( \frac{I}{I_0} \right) \]

\[ = 10 + 10 \log_{10} \left( \frac{I}{I_0} \right) = 60 \text{ decibels} \]

100 people

\[ \beta = 10 \log_{10}(100) + 10 \log_{10} \left( \frac{I}{I_0} \right) = 70 \text{ decibels} \]
Example

- The intensity level of sound A is 4.0 dB greater than that of sound B. Determine the ratio \( \frac{I_A}{I_B} \) of the intensity of sound A to the intensity of sound B.

\[
\log_{10} \left( \frac{X}{Y} \right) = \log_{10} (X) - \log_{10} (Y)
\]

\[
\beta = 10 \log_{10} \left( \frac{I}{I_0} \right) = 10 \log_{10} (I) - 10 \log_{10} (I_0)
\]

\[
4 = \beta_A - \beta_B = 10 \log_{10} \left( \frac{I_A}{I_0} \right) - 10 \log_{10} \left( \frac{I_B}{I_0} \right) = 10 \log_{10} (I_A) - 10 \log_{10} (I_0)
\]

\[
= \left( -10 \log_{10} (I_0) \right) - \left( 10 \log_{10} (I_B) - 10 \log_{10} (I_0) \right)
\]

\[
\Rightarrow \beta_A - \beta_B = 10 \log_{10} (I_A) - 10 \log_{10} (I_B) = 10 \log_{10} \left( \frac{I_A}{I_B} \right) = 4
\]

\[
\Rightarrow \log_{10} \left( \frac{I_A}{I_B} \right) = 0.4 \quad \Rightarrow \frac{I_A}{I_B} = 10^{0.4} = 2.5
\]