Physics 231 Lecture 33

- Main points of today’s lecture:
- Work in thermodynamic processes:
  \[ W_{\text{system}} = P\Delta V \]
- First Law of Thermodynamics:
  \[ \Delta U = \Delta Q - P\Delta V \]
- Specific heat at constant pressure
- Processes
  - cyclic: \( \Delta P = \Delta V = \Delta T = 0 \)
  - isobaric: \( \Delta P = 0 \)
  - isovolumetric: \( \Delta V = 0 \)
  - isothermal: \( \Delta T = 0 \)
  - adiabatic: \( \Delta Q = 0 \)
Conceptual question

• Which gives the largest average radiation absorbed on a 1 m² area at that distance?
  - 1. a 50-W source at a distance $R$.
  - 2. a 100-W source at a distance $2R$.
  - 3. a 200-W source at a distance $4R$.

$$P_{\text{absorbed}} = \frac{1 \text{ m}^2}{4\pi R^2} \cdot \frac{P}{\text{emitted power}}$$

$$P_1 = \frac{1 \text{ m}^2}{4\pi R^2} \cdot 50$$

$$P_2 = \frac{1 \text{ m}^2}{4\pi (2R)^2} \cdot 100$$

$$P_3 = \frac{1 \text{ m}^2}{4\pi (4R)^2} \cdot 200$$
Work and the 1st Law of thermodynamics

- Consider the ideal gas contained in the volume under the cylinder at right. The piston compresses the gas very slowly, moving downwards with a constant velocity.
- The gas exerts a force $F_{gas} = PA$ upward on the piston and conversely, the piston exerts a force $F_{piston} = -PA$ downward on the gas.
- The work done by the piston on the gas is:
  \[ W_{piston} = |F_{piston}|\Delta y = PA|\Delta y| = -P\Delta V \]
- The work done by the gas on the piston is:
  \[ W_{gas} = -F_{gas}|\Delta y| = -PA|\Delta y| = P\Delta V \]
- If $U$ denotes the internal energy of the system, the conservation of energy dictates:
  \[ \Delta U = \Delta E_{th} = \Delta Q + W_{piston} = \Delta Q - W_{gas} = \Delta Q - P\Delta V \]

Here I have changed notation to $U$, because other systems, besides the ideal gas, can have potential energy.
Why does work increase thermal energy

- Positive work done on the gas increases U because the collisions with the piston moving downward increases the velocity of the molecules.
- Negative work done on the gas decreases U because the collisions with the piston moving upward decreases the velocity of the molecules.
Example

- The work done to compress one mole of a monatomic ideal gas is 6200 J. The temperature of the gas changes from 350 K to 550 K. How much heat flows between the gas and its surroundings? Determine whether the heat flows into or out of the gas.

Work done by the gas is: \( W_{\text{gas}} = -6200 \text{J} \)

Internal energy change in the gas is: \( \Delta U = \frac{3}{2} nR (\Delta T) \)

\[
Q = \Delta U + W_{\text{gas}} = \frac{3}{2} nR (\Delta T) - 6200 \text{J} = \frac{3}{2} (8.31 \text{J/K} \cdot 550 - 8.31 \text{J/K} \cdot 350) - 6200 \text{J}
\]

\( Q = -3700 \text{J} \)
Computation of work:

- What is the work done *by* the gas going from \((P_i, V_i)\) to \((P_f, V_f)\)?
  - a) \(P_i(V_i-V_f)\)
  - b) \(P_i(V_f-V_i)\)
  - c) 0
  - d) \(V_i(P_i-P_f)\)
  - e) \(V_f(P_i-P_f)\)

The work done *by* the gas is the mathematical area under the curve \(P_i(V_f-V_i)\). It is negative if the volume is decreasing. It is pushing to increase the volume, but the volume is being decreased.

- No work is done on the isovolumetric (constant volume) part of the path!
Computation of work:

- What is the work done on the gas going from \((P_i,V_i)\) to \((P_f,V_f)\) ?
  - a) \(P_i(V_i-V_f)\)
  - b) \(P_i(V_f-V_i)\)
  - c) 0
  - d) \(V_i(P_i-P_f)\)
  - e) \(V_f(P_i-P_f)\)

- Note that the work done on the gas is (-1) times the mathematical area under the curve, -\(P_i(V_f-V_i)\). The mathematical area is negative if the volume is decreasing. The work done on the gas is positive, you are compressing it and thereby exerting a force in the direction of that it is being compressed.

- No work is done on the isovolumetric (constant volume) part of the path!
Conceptual quiz

• A gas is taken from an initial state of pressure and volume, $P_A V_A$, to a final, different, state, $P_B V_B$. As it changes work is done on the gas.

  – a) The amount of work depends on the path taken from A to B
  – b) The amount of work does not depend on the path taken from A to B
**quiz:**

- What is the work done on the gas going from \((P_i, V_i)\) to \((P_f, V_f)\)?
  - a) \(P_f(V_i-V_f)\)
  - b) \(P_f(V_f-V_i)\)
  - c) 0
  - d) \(V_i(P_i-P_f)\)
  - e) \(V_f(P_i-P_f)\)

- Note that the work done on the gas is the negative of the mathematical area under the curve. The mathematical area is negative if the volume is decreasing. We are compressing the gas and the volume is decreasing and we are pushing in the direction to decrease it.
- No work is done on the isovolumetric part of the path!
Computation of work:

- What is the work done on the gas going from \((P_i, V_i)\) to \((P_f, V_f)\) ?
  - a) \(P_i(V_i-V_f)\)
  - b) \(P_f(V_i-V_f)\)
  - c) 0
  - d) \((P_i+ P_f)(V_i-V_f)/2\)
  - e) \(-(P_i+ P_f)(V_i-V_f)/2\)
Heat capacity at constant pressure

Consider a cylinder filled with a n=2 moles of monatomic gas and plugged at the top by a frictionless piston. Above the piston is atmospheric pressure. This ensures the piston will be at a constant pressure P=299kPa. The bottom of the cylinder piston is a heat source that can heat the gas. The sides of the cylinder and the piston have zero thermal conductivity. What is the heat Q required to raise the temperature of the gas from initial temperature $T_i = 300 \text{ K}$ to final temperature $T_f = 400 \text{ K}$?

$$Q = \Delta U + W$$

$$\Delta U = n \frac{3}{2} R \Delta T = n \frac{3}{2} R (T_f - T_i)$$

$$W = p \Delta V = p (V_f - V_i) = pV_f - pV_i = nRT_f - nRT_i = nR (T_f - T_i)$$

$$Q = n \frac{3}{2} R (T_f - T_i) + nR (T_f - T_i) = n \frac{5}{2} R (T_f - T_i) = 2 \times 8.31 \times (400 - 300) \text{ J} = 1.66 \text{ kJ}$$

$$c_p = c_v + R = \frac{3}{2} R + R = \frac{5}{2} R$$ is the molar heat capacity at constant pressure
Example

- A person takes in a breath of 0°C air and holds it until it warms to 37.0°C. The air has an initial volume of 0.600 L and a mass of 7.70 x 10^-4 kg. Determine (a) the work done by the air on the lungs if the pressure remains constant at atmospheric pressure, (b) the change in internal energy of the air, and (c) the energy added to the air by heat. Model the air as if it were a diatomic gas: \( c_v = \frac{5}{2}R \), \( c_p = \frac{7}{2}R \).

\[
P_f = P_i = P_{\text{atm}} = 1.01 \times 10^5 \text{ Pa}
\]

\[
W_{\text{gas}} = P_{\text{atm}} (V_f - V_i) = nRT_f - T_i
\]

\[
= P_{\text{atm}} V_i \left( \frac{P_{\text{atm}} V_f}{P_{\text{atm}} V_i} - 1 \right) = P_{\text{atm}} V_i \left( \frac{nRT_f}{nRT_i} - 1 \right) = P_{\text{atm}} V_i \left( \frac{T_f}{T_i} - 1 \right)
\]

\[
W_{\text{gas}} = (1.01 \times 10^5 \text{ Pa}) (0.0006 \text{ m}^3) \left( \frac{310}{273} - 1 \right) = 8.2 \text{ J}
\]

Assume diatomic gas:

\[
\Delta U = \frac{5}{2} nR (T_f - T_i) = \frac{5}{2} nRT_i \left( \frac{T_f}{T_i} - 1 \right) = \frac{5}{2} W_{\text{gas}} = 20.5 \text{ J}
\]

\[
\Delta Q = \Delta U + W_{\text{gas}} = \frac{5}{2} nR (T_f - T_i) = 28.7 \text{ J}
\]