Physic 231 Lecture 34

- Main points of today’s lecture:
- Cycles
- Reversible and irreversible processes.
- Carnot cycle and Carnot engine.

\[ e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{|T_h| - |T_c|}{|T_h|} = 1 - \frac{|T_c|}{|T_h|} \]

- \( T \) is in Kelvin.

- Engines and refrigerators.
- Entropy:

\[ \Delta S = \frac{Q_{\text{reversible}}}{T} \]
Thermal systems

- In thermal systems, there are a set of “state variables” that are sufficient to completely describe the macroscopic state of the system.
  - example: ideal gas \((P,T,N),(V,T,N),(P,U,N)\) and \((U,V,N)\) are examples of a set of state variables. \(U=\frac{3}{2}Nk_B T\) is the internal energy.

- Zeroeth law of thermodynamics: Two macroscopic systems are in thermal equilibrium if and only if they are at the same temperature.

- In a thermal process, a macroscopic system changes its state variables in a smooth and controlled manner. Examples of some common processes are:
  - Isobaric process: \(\Delta P=0\); the pressure remains the same
  - Isovolumetric process: \(\Delta V=0\); the volume remains the same.
  - Isothermal process: \(\Delta T=0\); the temperature remains the same.
  - Adiabatic process: \(\Delta Q=0\); the system is thermally isolated.
Constant-Pressure Process

- Isobaric process: \( \Delta P = 0 \); the pressure remains the same, work can be done, heat can be exchanged and internal energy can change.

In equilibrium, pressure must be the same on both sides of the piston.

Moveable piston

Because the external pressure doesn’t change, the gas pressure remains constant as the gas expands.

An isobaric process appears on a \( pV \) diagram as a horizontal line.

\[
W = \text{area} = p \left( V_f - V_i \right)
\]
Constant-Volume Process: Isovolumetric

- Isovolumetric process: $\Delta V=0$; the volume remains the same, internal energy changes, heat can be exchanged with environment, but no work is done.

As the temperature increases, so does the pressure.

A constant-volume process appears on a $pV$ diagram as a vertical line.

$W = 0$
Constant-Temperature Process: isothermal

- Isothermal process: $\Delta T = 0$; the temperature remains the same. Internal energy of a gas remains the same, the heat transferred equals the work done.

$$W = \int PdV = nRT \ln \left( \frac{V_f}{V_i} \right)$$
Adiabatic process: no exchange of heat with environment

- Adiabatic process: $\Delta Q = 0$; the system is thermally isolated. Heat cannot come in or out of the system. The work done equals the change in internal energy.

\[\text{The curve of the adiabatic compression moves from a lower-temperature isotherm to a higher-temperature isotherm.}\]

\[\text{PV}^{\gamma} = \text{constant}\]

\[\gamma = \frac{5}{3} \text{ for monatomic ideal gas}\]

\[W = \int PdV = \text{const}(V_3^{1-\gamma} - V_1^{1-\gamma})\]

When a gas expands, it does work, when a gas is compressed, work is done on it. In an adiabatic process, the work done by a gas while expanding decreases its thermal energy and temperature. Conversely, the work done on the gas while compressing it increases its thermal energy and temperature.
Additional Questions

When I do work on a gas in an adiabatic process, compressing it, I add energy to the gas. Where does this energy go?

A. The energy is transferred as heat to the environment.

B. The energy is converted to thermal energy of the gas.

C. The energy converts the phase of the gas.
Cyclic process:

- A thermal path which returns to its initial condition is called a *cycle*.
- The work done by the gas on a *clockwise* cycle is the area contained in the path.
- The work done by the gas on a *counterclockwise* cycle is the *negative* of the area in the path.
- The work done on the gas is the negative of the work done by the gas.
Example of Cyclic process

- The drawing refers to one mole of monatomic ideal gas and shows a process that has four steps, two isobaric (A to B and C to D) and two isovolumetric (B to C and D to A).

- a) Complete a table by calculating $\Delta U$, $\Delta W$ and $\Delta Q$ (including the algebraic signs) for each of the four steps.

- b) What is the net heat/cycle absorbed by the system?

- c) What is the net work/cycle done by the system? $n = 1$

<table>
<thead>
<tr>
<th>Path</th>
<th>$\Delta U$</th>
<th>$\Delta W$</th>
<th>$\Delta Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-B</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>B-C</td>
<td></td>
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<tr>
<td>C-D</td>
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</tr>
<tr>
<td>D-A</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Net</td>
<td>${}$</td>
<td></td>
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</tbody>
</table>

$U_A = \frac{3}{2}RT_A = \frac{3}{2}(8.31)400J = 4986J$  $U_B = \frac{3}{2}RT_B = 9972J$

$U_C = \frac{3}{2}RT_C = 4986J$  $U_D = \frac{3}{2}RT_D = 2493J$

$\Delta U_{A-B} = U_B - U_A$  $\Delta U_{A-B} = 9972 - 4986 \approx 5.0kJ$

Similarly get $\Delta U_{B-C}$, $\Delta U_{C-D}$, & $\Delta U_{D-A}$

$W_{D-A} = W_{B-C} = 0$ because isovolumetric  $PV = nRT$

$W_{A-B} = P_A(V_B - V_A) = P_BV_B - P_AV_A = RT_B - RT_A = 3.3kJ$

$W_{C-D} = RT_D - RT_C = -1662J$  $\Delta Q = \Delta U + \Delta W$
Important Cyclic Processes: Engines

- In a heat engine, thermal energy $Q_h$ is used to do work, $W_{eng}$. Some of the original thermal energy $Q_c$ escapes and ends up heating something else.
- A heat engine involves some working substance in a cyclical process.
- In many cases heat comes from reservoir at $T_H$ and is exhausted to the environment at $T_C$.
- Thermal efficiency is defined as the ratio of the work done by the engine to the energy absorbed at the higher temperature. For simplicity both can be computed over one cycle:

$$ e = \frac{W_{eng}}{|Q_h|} = \frac{|Q_h| - |Q_c|}{|Q_h|} = 1 - \frac{|Q_c|}{|Q_h|} $$

- $e = 1$ (100% efficiency) only if $Q_c = 0$
  - No energy expelled to cold reservoir, which is theoretically possible for $T_c = 0$, but practically impossible.
Example

- The energy absorbed by an engine is three times as large as the work it performs. (a) What is its thermal efficiency? (b) What fraction of the energy absorbed is expelled to the cold reservoir?

\[ a) \quad Q_h = 3W \]

\[ e = \frac{W}{Q_h} = \frac{W}{3W} = \frac{1}{3} \]

\[ b) \quad Q_h = W + Q_c \]

\[ \Rightarrow Q_c = Q_h - W = 2W = \frac{2}{3}Q_h \]

\[ \Rightarrow \frac{Q_c}{Q_h} = \frac{2}{3} \]
Quiz

• A heat engine performs 200 J of work in each cycle and has an efficiency of 30%. For each cycle of operation, (1) how much energy is taken in from the hot reservoir \(Q_h\) and (2) how much energy is expelled to the cold reservoir \(Q_c\)?

• Answers below are in the form \(Q_h, Q_c\).
  - a) 667J, 467J
  - b) 467J, 667J
  - c) 60J, 140J
  - d) 140J, 60J

\[
a) \quad e = \frac{W}{Q_h} \Rightarrow Q_h = \frac{W}{e} = \frac{200J}{0.3} = 667J
\]
\[
b) \quad Q_H = W + Q_c \Rightarrow Q_c = Q_h - W = 667J - 200J = 467J
\]
1. A sample of nitrogen gas is inside a sealed container. The container is slowly compressed, while the temperature is kept constant. This is a ________ process.

   A. constant-volume
   B. isobaric
   C. isothermal
   D. adiabatic
Reversible and Irreversible Processes

• *reversible* process is one in which every state along some path is an equilibrium state.
  – And one for which the system can be returned to its initial state by going along the same path in the \((p,V)\) diagram but in the opposite direction.
  – Volume and pressure changes are “slow”.
  – When objects are brought into thermal contact, they are at the same temperatures.
  – Carnot cycle is an example of a reversible process.

• An *irreversible* process does not meet these requirements
  – Most natural processes are irreversible
    • Burning fueling in an automobile engine
    • Dropping ice into warm water
    • Heating water on a range

• Reversible process are an idealization, but some real processes are good approximations.