• Newton’s 3\textsuperscript{rd} Law:
  – “When a body exerts a force on another, the second body exerts an equal oppositely directed force on the first body.”
• Frictional forces:
  – kinetic friction: \( f_k = \mu_k N \)
  – static friction \( f_s < \mu_s N \)
• Examples.
Example

- Two forces, $\vec{F}_1$ and $\vec{F}_2$ act on the 5.00 kg block shown in the drawing. The magnitudes of the forces are $F_1=60$ N and $F_2 = 25$ N. What is the horizontal acceleration (magnitude and direction) of the block?

Normal force cancels the y components of $\vec{F}_1$ and $\vec{W}$

$F_{1,y} + \text{Normal}_y - W = 0$

$F_{1,x} = 60 \cos (60^\circ) \text{N} = 30 \text{N}$

$F_{\text{net}} = F_{1,x} - F_2 = 30 \text{N} - 25 \text{N} = 5 \text{N}$

$a = \frac{F_{\text{net}}}{m} = \frac{5 \text{N}}{5 \text{kg}} = 1 \text{m} / \text{s}^2$

Block accelerates to the right.

<table>
<thead>
<tr>
<th>$F_1$</th>
<th>60 N</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_2$</td>
<td>25 N</td>
</tr>
<tr>
<td>$m$</td>
<td>5 kg</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>60°</td>
</tr>
<tr>
<td>$a$</td>
<td>?</td>
</tr>
</tbody>
</table>
conceptual question

• Consider a person standing in an elevator that is accelerating upward. The upward normal force $N$ exerted by the elevator floor on the person is
  – a) larger than
  – b) identical to
  – c) smaller than
the downward weight $W$ of the person.
Example: with balance scale marked in Newtons, not the usual scale, but a quite reasonable one.

- A 100 kg man stands on a bathroom scale in an elevator. Starting from rest, the elevator ascends, attaining its maximum speed of 1.2 m/s in 0.80 s. It travels with this constant speed for 5.0 s, undergoes a uniform negative acceleration for 1.5 s and comes to rest. What does the scale register (a) before the elevator starts to move? (b) during the first 0.8 s? (c) while the elevator is traveling at constant speed? (d) during the negative acceleration?

<table>
<thead>
<tr>
<th>Scale reading =Normal force</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N - mg = ma : scale = N$ (in Newtons)</td>
</tr>
</tbody>
</table>

**a)**: $a = 0$; $N = mg : scale = N = 980$ Newtons

<table>
<thead>
<tr>
<th>$m$</th>
<th>100 kg</th>
</tr>
</thead>
</table>

**b)**: $N - mg = ma : scale = N$

$a = \frac{\Delta v}{\Delta t} = \frac{(1.2 \text{ m/s})}{0.8 \text{ s}} = 1.5 \text{ m/s}^2$

$N = m(g + a); scale = m(g + a)$

scale $= (100 \text{ kg})(9.8 \text{ m/s}^2 + 1.5 \text{ m/s}^2) = 1130$ Newtons

**c)**: $N = mg : scale = N = 980$ Newtons

<table>
<thead>
<tr>
<th>$v_f$</th>
<th>1.2 m/s</th>
</tr>
</thead>
</table>

$d)$: $a = \frac{(v_f - v_i)}{\Delta t} = -1.2 \text{ m/s} / (1.5 \text{s}) = -0.8 \text{ m/s}^2$

$N = m(g + a); scale = m(g + a) = 900$ Newtons
No Acceleration: Static Equilibrium

- All objects are at rest and remain so. The net force on any object must vanish. I.e. on an object:
  \[ \vec{F}_i = 0 \]

- Example: Three ropes are arranged so as to support a 4 kg mass as shown below. Determine the tension in each rope.

\[
\begin{align*}
F_{i,x} &= 0 = -T_1 + T_2 \cos(60^0) \\
T_2 &= T_3 / \sin(60^0) = 1.16T_3 \\
T_3 &= mg = 39.1N; T_2 = 45.3N; T_1 = 22.6N
\end{align*}
\]
Atwood’s machine

• Consider the Atwood machine to the right. The massless string passes over a massless and frictionless pulley. It is under tension $T$, which we define to be the magnitude of the tension force. By this definition it is a positive number.

• Choosing up to be positive, what is the net force on mass 1?

- a) $T - m_1g$
- b) $T + m_1g$
- c) $m_1g - T$
- d) none of the above
Atwood’s machine

- Choosing up to be positive, what is the net force on mass 2?
  - a) $m_2g-T$
  - b) $T+m_2g$
  - c) $T-m_2g$
  - d) none of the above

$$F_{net} = T - m_2g$$
Atwood’s machine

- From Newton’s 2nd law:
  \[ T - m_1 g = m_1 a_1 \]
  \[ \Rightarrow T = m_1 a_1 + m_1 g \]

- Also
  \[ T - m_2 g = m_2 a_2 \]
  \[ \Rightarrow T = m_2 a_2 + m_2 g \]

- If \( m_2 \) exceeds \( m_1 \), \( m_2 \) goes down and \( m_1 \) goes up. If \( m_1 \) exceeds \( m_2 \), \( m_1 \) goes down and \( m_2 \) goes up. In either case
  \[ \Delta y_2 = -\Delta y_1 \]
  \[ v_2 = -v_1 \]
  \[ a_2 = -a_1 \]

- Putting it together:
  \[ m_1 a_1 + m_1 g = m_2 a_2 + m_2 g \]
  \[ = -m_2 a_1 + m_2 g \]
  \[ m_1 a_1 + m_2 a_1 = m_2 g - m_1 g \]
  \[ a_1 = \frac{(m_2 - m_1)g}{m_1 + m_2} \] pulling \( m_1 \) up and \( m_2 \) down.
  \[ \text{total mass } \Rightarrow \frac{m_1 + m_2}{m_1 + m_2} \]
Reading Quiz

4. An action/reaction pair of forces

A. point in the same direction.
B. act on the same object.
C. are always long-range forces.
D. act on two different objects.
**Newton’s Third Law**

- When a body exerts a force on another, the second body exerts an equal oppositely directed force on the first body.
  - Note: the two forces act on different bodies

\[ \vec{F}_{21} = -\vec{F}_{12} \]

- Force on body 2 due to body 1:
  \[ \vec{F}_{21} \]

- Force on body 1 due to body 2:
  \[ \vec{F}_{12} \]

- 3\textsuperscript{rd} law:
  \[ \vec{F}_{21} = -\vec{F}_{12} \]
Example

Two skaters, an 82 kg man and a 48 kg woman, are standing on ice. Neglect any friction between the skate blades and the ice. The woman pushes the man with a force of 45 N due east. Determine the accelerations (magnitude and direction) of the man and the woman.

\[ a_{\text{man}} = \frac{-45\text{N}}{82\text{kg}} = 0.55\text{m/s}^2 \text{ east} \]

\[ a_{\text{woman}} = \frac{45\text{N}}{48\text{kg}} = 0.94\text{m/s}^2 \text{ west} \]
Two skaters, an 100 kg man and a 50 kg woman, are standing on ice. Neglect any friction between the skate blades and the ice. By pushing the man, the woman is accelerated at 2 m/s$^2$ in the direction of due west. What is the corresponding acceleration of the man?

- a) 4 m/s$^2$ due east
- b) 1 m/s$^2$ due east
- c) 2 m/s$^2$ due east
- d) 1.5 m/s$^2$ due east

\[
\vec{F}_{\text{woman on man}} = -\vec{F}_{\text{man on woman}}
\]

\[
m_{\text{man}} \ddot{a}_{\text{man}} = -m_{\text{woman}} \ddot{a}_{\text{woman}}
\]

choosing west to be negative (just to be contrary)

\[
a_{\text{man,x}} = - \frac{m_{\text{woman}}}{m_{\text{man}}} a_{\text{woman,x}} = - \frac{50}{100} (-2 \text{ m/s}^2) = 1 \text{ m/s}^2 \text{ east}
\]

Note: the direction would still be east even if you chose west to be positive
Example

A block with mass 5 kg and a second block with mass 10 kg are supported by a frictionless surface. A force of 60 N is applied to the 10 kg mass. What is the force of the 5 kg block on the 10 kg block?

x components:

body 2: \( N_{21,x} = m_2 a_x \); \( N_{21,x} = -N_{12,x} \) from Newtons 3\(^{rd}\) law

body 1: \( F_x + N_{12,x} = m_1 a_x \); \( F_x = -N_{12,x} + m_1 a_x \)

\( F_x = N_{21,x} + m_1 a_x = m_2 a_x + m_1 a_x = (m_2 + m_1) a_x \)

\( a_x = F_x / (m_2 + m_1) \)

\( N_{12,x} = -N_{21,x} = -m_2 a_x = -m_2 F_x / (m_2 + m_1) \)

\( N_{12,x} = -5\text{kg}\cdot60\text{N} / (15\text{kg}) = -20\text{N} \)

\( N_{12,x} = 20\text{N} \) to the left

If objects move together, the acceleration is governed by the total mass
Friction

- Friction impedes the motion of one object along the surfaces of another. It occurs because the surfaces of the two objects temporarily stick together via “microwelds”. The frictional force can be larger if the two surfaces are at rest with respect to each other.

- Experimentally we have two cases:
  - kinetic friction:
    \[ f_k = \mu_k N \]
  - static friction
    \[ f_s < \mu_s N \]

- The coefficient of static friction generally exceeds that for kinetic friction: \( \mu_s > \mu_k \)

- Frictional forces always oppose the motion of one surface with respect to the other.
Example with static friction

- Consider the figure below, with $M_1=105$ kg and $M_2=44.1$ kg. What is the minimum static coefficient of friction necessary to keep the block from slipping.

\[ T = M_2g \]

If $M_1$ doesn’t move

\[ T = f_s \leq \mu_s N = \mu_s M_1 g \]

Putting it together

\[ M_2 g \leq \mu_s M_1 g \]

\[ \frac{M_2}{M_1} \leq \mu_s \]

If this isn’t true, $M_1$ will slip
Example kinetic friction with ramp

- The block shown below starts sliding down the ramp. Assuming the coefficient of kinetic friction $\mu_k = 0.3$, how long does it take for the block to travel 2m to the bottom of the ramp?

1. Draw the forces.
2. Choose an appropriate coordinate system.
3. Calculate the components.
4. Use Newton’s $2^{nd}$ to get $t$

$W, f_k, N$ are magnitude of forces

$x : \sum_i F_{i,x} = f_k - W \sin (\theta) = m a_x$

$\Rightarrow \mu_k N - mg \sin (\theta) = m a_x$

$y : \sum_i F_{i,y} = N - W \cos (\theta) = 0$

$\Rightarrow N = mg \cos (\theta)$

$\Rightarrow \mu_k \frac{mg \cos (40^\circ)}{N} - mg \sin (40^\circ) = m a_x$

$\Rightarrow a_x = -4.05 \text{m/s}^2$

$\Delta x = \frac{1}{2} a_x t^2 \Rightarrow t = \sqrt{\frac{2\Delta x}{a_x}} = \sqrt{\frac{-4m}{-4.05 \text{m/s}^2}} = 1s$
Example of static friction with ramp

- Chucky puts a block on an incline plane as shown below. He increases the angle to 40°, at which point the block begins to slide. What is the static coefficient of friction?

1. Draw the forces.
2. Choose an appropriate coordinate system.
3. Calculate the components.
4. Use Newton’s 2nd law to get \( \mu_s \)

\( W, N, f_s \) are magnitudes of forces

\[ x : \sum_i F_{i,x} = 0 = f_s - W \sin(\theta) \]
\[ \Rightarrow \mu_s N > f_s = W \sin(\theta) \]

\[ y : \sum_i F_{i,y} = 0 = N - W \cos(\theta) \]
\[ \Rightarrow N = W \cos(\theta) \]
\[ \Rightarrow \mu_s W \cos(\theta) > W \sin(\theta) \]
\[ \Rightarrow \mu_s > \tan(\theta) \]

at max angle, \( \mu_s = \tan(40^\circ) = .84 \)

\[ \theta = 40^\circ \]