Chapter 3: vectors and 2d motion

Summing vectors graphically

**Triangular method**

Resultant vector
Summing vectors graphically

Parallelogram method

Resultant vector

Subtracting vectors graphically
Components of a vector

\[ A = \sqrt{A_x^2 + A_y^2} \]

\[ \tan \theta = \frac{A_y}{A_x} \]

\[ A_x = A \cos \theta \]
\[ A_y = A \sin \theta \]

Components of vectors

What are the components of the displacement vector of superman?
Properties of vectors

Equality: two vectors are equal if they have the same magnitude and direction

Addition: triangular method; parallelogram method or adding components (note that \(A+B=B+A\))

\[ A + B = (A_x + B_x, A_y + B_y) \]

Subtraction: \(A-B=A+(-B)\)

Multiplying or dividing by a scalar: \(3A\) is a vector with the same direction as \(A\) and three times the magnitude.

\[ 3A = (3A_x, 3A_y) \]

Strategy with vectors

- Select a coordinate system
- Sketch the vector to be added or subtract and label them
- Find the \(x\) and \(y\) component of each vector
- Find the resultant components
- Use the Pythagorean theorem to find the magnitude of the resultant vector
- Use a suitable trigonometric function to find the angle the resultant vector makes with the positive \(x\) axis
A hiker begins a trip by first walking 25.0 km southeast from her base camp. On the second day she walks 40.0 km in the direction 60.0 degrees northeast, at which point she discovers a forest ranger’s tower.

a) Determine the components of the hiker’s displacement on the 1st and 2nd day
b) Determine the total displacement

Example of summing vectors

What is the magnitude and direction of the resultant force applied to the donkey?
2-D motion

displacement

\[ \Delta \vec{r} = \vec{r}_f - \vec{r}_i \]

Average velocity

\[ \vec{v}_a = \frac{\Delta \vec{r}}{\Delta t} \]

Instantaneous velocity

\[ \vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} \]

Average acceleration

\[ \vec{a}_a = \frac{\Delta \vec{v}}{\Delta t} \]

Instantaneous acceleration

\[ \vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} \]
2-D motion: questions

A body cannot have acceleration if its speed is constant
a) True   b) False

A particle can have constant velocity and varying speed
a) True   b) False

There may be acceleration in a car when your car is on cruise control
a) True   b) False

There is acceleration in your car when you hit the brakes
a) True   b) False

Projectile motion
Projectile motion

- It can be described as a superposition of two independent motions in the x and y directions
- Provided air resistance is negligible, the horizontal component of velocity $v_x$ remains constant
- The vertical component of the acceleration is equal to the free fall acceleration $g$
- The vertical component of the velocity $v_y$ and the displacement in the y direction are identical to those of a freely falling body

Projectile motion: strategy

- Select a coordinate system and sketch the path of the particle
- Resolve the initial velocity in its x and y components
- Treat the horizontal and vertical motion independently
- Follow method for constant velocity to analyse x motion
- Follow method for constant acceleration to analyse the y motion
Projectile motion: example

An Alaskan rescue plane drops a package of emergency rations to stranded hikers. The plane is traveling horizontally at 40.0 m/s at height of 100 m above the ground.

a) Where does the package strike the ground relative to the point at which it was released?

b) What are the horizontal and vertical components of the velocity of the package just before it hits the ground?

Projectile motion

As a projectile moves in its parabolic path, the velocity and acceleration vectors are perpendicular to each other:

a) Everywhere along its path
b) At the peak of its path
c) Nowhere along its path
d) Not enough information

If you are carrying a ball and running at constant speed and wish to throw the ball so that you can catch it when it comes back down, should you:

a) Throw the ball at an angle of 45deg and maintain the same speed?

b) Throw the ball straight in the air, and slow down to catch it?

c) Throw the ball straight in the air and maintain the same speed?

d) Not enough information
**Projectile motion: example**

A basketball player 2.0 m tall, wants to make a basket from a distance of 10.0 m. If he shoots the ball at 45.0 deg, at what initial speed must he throw the ball so that it goes through the hoop without striking the backboard?

![Diagram of a basketball player shooting a ball into a hoop](image)

**Relative velocity**

A passenger at the rear of a train traveling at 15 m/s relative to the Earth throws a baseball with a speed 15 m/s in the direction opposite to the motion of the train. What is the velocity of the baseball relative to the Earth?

\[ \vec{v}_{\text{ball-earth}} = \vec{v}_{\text{ball-train}} + \vec{v}_{\text{train-earth}} \]
Relative velocity

The boat is heading due north as it crosses a wide river with velocity of 10.0 km/h relative to the water. The river has a uniform velocity of 5.00 km/h due east. Determine the velocity of the boat with respect to an observer on the riverbank?

\[ \vec{v}_{\text{boat-earth}} = \vec{v}_{\text{boat-river}} + \vec{v}_{\text{river-earth}} \]

Relative velocity

If now the boat moves at the same speed 10 km/h relative to the water but wants to travel due north, in what direction should it move relative to the water?

\[ \vec{v}_{\text{boat-earth}} = \vec{v}_{\text{boat-river}} + \vec{v}_{\text{river-earth}} \]
Motion on an incline

A crate moves down along an incline, starting from rest at a height of 50 cm. The incline makes 30 deg with the horizontal axis. Suppose there is no friction between the crate and the incline.

a) Determine the acceleration along the direction of the motion
b) How does the acceleration vary with time?
c) How does the velocity vary with time?
d) How does the position vary with time?
(provide equation and graph)

Circular motion

Angular displacement

\[ \Delta \theta = \frac{\Delta s}{r} \]

Period (T) time for one rotation

Acceleration in circular motion

\[ a = \frac{v^2}{r} \]