Solids and Fluids

Crystalline solid
Amorphous solid
Liquids
Gases
Plasmas

Deformation in solids

**Stress** is related to the force causing a deformation

**Strain** is a measure of the degree of deformation

Elastic modulus = \( \frac{\text{stress}}{\text{strain}} \)
Elasticity in length

**Tensile stress** is the ratio of the magnitude of the external force per sectional area $A$:

\[
\text{Tensile stress} = \frac{\text{force}}{\text{area}} = \frac{F}{A}
\]

Units: $1 \text{ Pa} \equiv 1 \text{ Nm}^{-2}$

**Tensile strain** is the ratio of the change of length to the original length:

\[
\text{Tensile strain} = \frac{\Delta L}{L_0}
\]

**Young’s modulus** is the ratio of the tensile stress over the tensile strain:

\[
Y \equiv \frac{F/A}{\Delta L/L_0} = \frac{F/L_0}{\Delta L/A}
\]

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Elastic limit

![Stress-Strain Curve](image-url)

- **Elastic limit**
- **Breaking point**
- **Elastic behavior**

Stress (MPa) vs. Strain
Shear deformation

Shear stress is the ratio of the parallel force to the area $A$ of the face being sheared:

$$\text{Shear stress} = \frac{F_{parallel}}{area}$$

Units: $1 \text{ Pa} \equiv 1 \text{ Nm}^{-2}$

Shear strain is the ratio of the distance sheared to the height:

$$\text{Shear strain} = \frac{\Delta x}{h}$$

Shear modulus is the ratio of the shear stress over the shear strain:

$$S = \frac{F_{parallel} / A}{\Delta x / h} = \frac{F_{parallel} h}{\Delta x A}$$
Volume elasticity

Volume elasticity

Volume stress is the change in the applied force per surface area

\[ \Delta P \equiv \frac{\Delta F}{A_{\text{surface}}} \]

Volume strain is the ratio of the change of volume to the original volume

Volume strain = \[ \frac{\Delta V}{V} \]

Bulk modulus is the ratio of the volume stress over the volume strain:

\[ B \equiv -\frac{\Delta F / A}{\Delta V / V} = -\frac{\Delta P}{\Delta V / V} \]
Elasticity in volume

For the Bulk modulus to be positive, as an increase of pressure always produces a decrease of volume, one introduces a negative sign in its definition!

**Bulk modulus** is the ratio of the volume stress over the volume strain:

\[
B \equiv -\frac{\Delta F / A}{\Delta V / V} = -\frac{\Delta P}{\Delta V / V}
\]

Elastic modulus: question time

Which material do you expect will have a larger Young’s modulus, a) copper; b) rubber; c) both the same?

Which of the following don’t make sense:
- a) The Young’s modulus of water?
- b) The Shear modulus of glass?
- c) The Bulk modulus of liquid helium?

Do you expect the Young’s modulus of the bone to be:
- a) larger, b) smaller, c) the same;
  as the Shear modulus?
Elastic modulus

**TABLE 9.1** Typical Values for Elastic Moduli

<table>
<thead>
<tr>
<th>Substance</th>
<th>Young’s Modulus (Pa)</th>
<th>Shear Modulus (Pa)</th>
<th>Bulk Modulus (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>$7.0 \times 10^{10}$</td>
<td>$2.5 \times 10^{10}$</td>
<td>$7.0 \times 10^{10}$</td>
</tr>
<tr>
<td>Bone</td>
<td>$1.8 \times 10^{10}$</td>
<td>$8.0 \times 10^{10}$</td>
<td>—</td>
</tr>
<tr>
<td>Brass</td>
<td>$9.1 \times 10^{10}$</td>
<td>$3.5 \times 10^{10}$</td>
<td>$6.1 \times 10^{10}$</td>
</tr>
<tr>
<td>Copper</td>
<td>$11 \times 10^{10}$</td>
<td>$4.2 \times 10^{10}$</td>
<td>$14 \times 10^{10}$</td>
</tr>
<tr>
<td>Steel</td>
<td>$20 \times 10^{10}$</td>
<td>$8.4 \times 10^{10}$</td>
<td>$16 \times 10^{10}$</td>
</tr>
<tr>
<td>Tungsten</td>
<td>$35 \times 10^{10}$</td>
<td>$14 \times 10^{10}$</td>
<td>$20 \times 10^{10}$</td>
</tr>
<tr>
<td>Glass</td>
<td>$6.5-7.8 \times 10^{10}$</td>
<td>$2.6-3.2 \times 10^{10}$</td>
<td>$5.0-5.5 \times 10^{10}$</td>
</tr>
<tr>
<td>Quartz</td>
<td>$5.6 \times 10^{10}$</td>
<td>$2.6 \times 10^{10}$</td>
<td>$2.7 \times 10^{10}$</td>
</tr>
<tr>
<td>Rib Cartilage</td>
<td>$1.2 \times 10^{7}$</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Rubber</td>
<td>$0.1 \times 10^{7}$</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Tendon</td>
<td>$2 \times 10^{7}$</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Water</td>
<td>—</td>
<td>—</td>
<td>$0.21 \times 10^{10}$</td>
</tr>
<tr>
<td>Mercury</td>
<td>—</td>
<td>—</td>
<td>$2.8 \times 10^{10}$</td>
</tr>
</tbody>
</table>

Ultimate strength of materials

The maximum force per unit area the material can withstand before breaking or fracturing.

**TABLE 9.2** Ultimate Strength of Materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Tensile Strength (N/m²)</th>
<th>Compressive Strength (N/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iron</td>
<td>$1.7 \times 10^{8}$</td>
<td>$5.5 \times 10^{8}$</td>
</tr>
<tr>
<td>Steel</td>
<td>$5.0 \times 10^{8}$</td>
<td>$5.0 \times 10^{8}$</td>
</tr>
<tr>
<td>Aluminum</td>
<td>$2.0 \times 10^{8}$</td>
<td>$2.0 \times 10^{8}$</td>
</tr>
<tr>
<td>Bone</td>
<td>$1.2 \times 10^{8}$</td>
<td>$1.5 \times 10^{8}$</td>
</tr>
<tr>
<td>Marble</td>
<td>—</td>
<td>$8.0 \times 10^{7}$</td>
</tr>
<tr>
<td>Brick</td>
<td>$1 \times 10^{6}$</td>
<td>$3.5 \times 10^{7}$</td>
</tr>
<tr>
<td>Concrete</td>
<td>$2 \times 10^{6}$</td>
<td>$2 \times 10^{7}$</td>
</tr>
</tbody>
</table>
Density

\[ \rho \equiv \frac{M}{V} \]

Which do you expect will have a larger density
a) lead;  b) glycerin; c) air?

TABLE 9.3 Density of Some Common Substances

<table>
<thead>
<tr>
<th>Substance</th>
<th>( \rho (\text{kg/m}^3) )</th>
<th>Substance</th>
<th>( \rho (\text{kg/m}^3) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ice</td>
<td>( 0.917 \times 10^3 )</td>
<td>Water</td>
<td>( 1.00 \times 10^3 )</td>
</tr>
<tr>
<td>Aluminum</td>
<td>( 2.70 \times 10^3 )</td>
<td>Glycerin</td>
<td>( 1.26 \times 10^3 )</td>
</tr>
<tr>
<td>Iron</td>
<td>( 7.86 \times 10^3 )</td>
<td>Ethyl alcohol</td>
<td>( 0.806 \times 10^3 )</td>
</tr>
<tr>
<td>Copper</td>
<td>( 8.92 \times 10^3 )</td>
<td>Benzene</td>
<td>( 0.879 \times 10^3 )</td>
</tr>
<tr>
<td>Silver</td>
<td>( 10.5 \times 10^3 )</td>
<td>Mercury</td>
<td>( 13.6 \times 10^3 )</td>
</tr>
<tr>
<td>Lead</td>
<td>( 11.3 \times 10^3 )</td>
<td>Air</td>
<td>1.29</td>
</tr>
<tr>
<td>Gold</td>
<td>( 19.3 \times 10^3 )</td>
<td>Oxygen</td>
<td>1.43</td>
</tr>
<tr>
<td>Platinum</td>
<td>( 21.4 \times 10^3 )</td>
<td>Hydrogen</td>
<td>( 8.99 \times 10^{-2} )</td>
</tr>
<tr>
<td>Uranium</td>
<td>( 18.7 \times 10^3 )</td>
<td>Helium</td>
<td>( 1.79 \times 10^{-1} )</td>
</tr>
</tbody>
</table>

\[ ^a \text{ All values are at standard atmospheric temperature and pressure (STP), defined as 0°C (273 K) and 1 atm (1.013 \times 10^5 \text{ Pa}). To convert to grams per cubic centimeter, multiply by 10^{-3}.} \]
Nail beds: how do they do it?

So relaxing!

Pressure

The average pressure in the fluid at a certain depth is the force per area

\[ P \equiv \frac{F}{A} \]
**Pressure: question time**

Someone stomps on your foot with the heel of a shoe. Would it be better if that person were:

a) a basketball player wearing sneakers or
b) a petite woman wearing spike-heeled shoes?

A water bed is 2.00 m on each side and 30 cm deep.

a) Find its weight.
b) Find the pressure that the water bed exerts on the floor.

---

**Variation of pressure with depth**

All portions of the fluid must be in static equilibrium

\[ F_2 = F_1 \]

All points at the same depth must be at the same pressure

\[ PA = P_0A + Mg \]
\[ P = P_0 + \rho gh \]

The pressure at a depth \( h \) below the surface of a liquid, open to the atmosphere, is greater than the atmospheric pressure by the amount \( \rho gh \).
Pressure: question time

Which of the following statements is true:

a) The pressure in A is larger than in B
b) The pressure in D is smaller than in C
c) The pressures in A, B, C and D are the same

Pascal’s principle

\[ \frac{F_1}{A_1} = \frac{F_2}{A_2} \]

A change in pressure applied to an enclosed fluid is transmitted undiminished to every point of the fluid and to the walls of the container.
Measuring pressure

One atmosphere is the pressure equivalent of a column of mercury that is exactly 0.76 m in height at 0 °C with \( g = 9.806 \text{m/s}^2 \).

\[
P_0 = \rho gh
= (13.595 \times 10^3 \text{Kg/m}^3)(9.806 \text{m/s}^2)(0.760 \text{m})
= 1.013 \times 10^5 \text{Pa} \equiv 1 \text{atm}
\]

Measuring pressure: question time

The blood pressure is measured with the manometer around the arm. Suppose that it was measured with the manometer around the calf of the leg. Would the reading be
a) The same
b) Less than
c) Or greater than the reading for the arm?
Transmitting pressure: example

A circular piston of 5cm radius is used to exert pressure on the incompressible liquid that is then transmitted to a second piston of radius 15 cm.

a) What force must be exerted on the first piston to lift a car weighing 13300 N?
b) What air pressure will produce this force?
c) Show that the input energy transfer is equal in magnitude to the output energy transfer.

Buoyant forces

Archimides’ principle
Any object completely or partially submerged in a fluid is buoyed up by a force whose magnitude is equal to the weight of the fluid displaced by the object

\[ B_{\text{fluid}} = W_{\text{obj}} \]
Buoyant forces

If the density of the object is smaller than that of the fluid, the object floats.

If the density of the object is larger than that of the fluid, the object sinks.

If the density of the object is equal to that of the fluid, the object is in equilibrium.

**Buoyant forces**

1. **Totally submerged object**

   \[ F_{net} = B - W_{obj} = (\rho_{fluid} - \rho_{obj})Vg \]

2. **Partially submerged object**

   \[ F_{net} = B - W_{obj} = (\rho_{fluid}V_{fluid} - \rho_{obj}V_{obj})g \]

We have equilibrium when \( F_{net} = 0 \)

\[ \rho_{fluid}V_{fluid} = \rho_{obj}V_{obj} \]
Buoyant force: question time

A lead and an iron cube are submerged in water. Lead is denser than iron, and both are denser than water. Is the buoyant force on a lead object a) greater than b) less than c) equal to the buoyant force on a iron object of the same dimensions.

Buoyant force: example

The weight of a crown on a scale is 7.84 N. The same crown is weighed when immersed in a bucket of water (density 1000 kg/m³) and the scale then reads 6.86 N. Determine whether the crown is made of pure gold.
Buoyant force: example

A raft is constructed of wood (density 600 kg/m\(^3\)). Its surface is 5.7 m\(^2\) and the volume is 0.60 m\(^3\). When the raft is placed on fresh water, to what depth does it sink?

Fluids in motion

**Streamline (laminar) flow:** every particle that passes a particular point moves exactly along a smooth path. The streamline has the same direction as the fluid velocity at any point. This corresponds to the low velocity limit of flow.
Fluids in motion

As the velocity of the fluid increases, there is a point when the flow of the fluid becomes irregular: **Turbulent** motion

**An Ideal Fluid**

1. **non viscous** (no internal friction force)
2. **incompressible** (density constant)
3. the motion is **steady** (velocity, density and pressure at any point are the same over time)
4. it moves **without turbulence** (zero angular velocity about its center)

Equation of continuity

The amount of fluid that enters one end of the tube, in a given time interval, equals the amount of fluid leaving the tube, during the same time interval.

**Flow rate**: amount of fluid passing a cross-sectional area per second

\[ \text{flow rate} = \frac{V}{t} = A \nu \]
Equation of continuity: example

A water hose 2.50 cm in diameter is used by a gardener to fill a 30.0 liter (dm³) bucket. It takes 1.00 min to fill it. A nozzle with 0.5 cm² cross-sectional area is then attached to it. The nozzle is held so that the water is projected horizontally from a point 1.00 m above the ground. Over what horizontal distance can the water be projected?

Bernoulli’s Equation

Energy conservation applied to a fluid: the sum of the pressure, the kinetic energy per unit volume and the potential energy per unit volume has the same value at all points along a streamline.

\[ P_1 + \frac{1}{2} \rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho gh_2 \]
Bernoulli’s equation: question time

Consider a horizontal pipe. Compare the pressure exerted by swiftly moving fluids in that pipe with the pressure exerted by slowly moving fluids:

a) Larger  b) smaller  c) the same

Two helium balloons float next to each other, about 2 cm apart, at the end of strings secured to a table. You blow through the opening between the balloons. What happens?

a) They move toward each other  
b) They move away from each other  
c) They are unaffected

Bernoulli’s equation: example

Consider a water tank with a leak. If the top of the tank is open to atmospheric pressure, determine the speed at which the water leaves the hole when the water level is 0.500 m above the hole.
Bernoulli’s equation: question time

Imagine the dish-washer is draining to the left. Why is there the need for a vent connected to the sink sewer pipe?

a) it helps keeps the trap in place.

b) it allows air to flow out of the system.

c) it helps to free the water in the trap.

Bernoulli’s equation: Applications

- Blood travels faster than normal through constricted regions.
- Lift on aircraft wings.
- Golf ball: Lift experienced by spin.
- Perfume sprayers.
Viscosity

Viscosity relates to the internal friction of the fluid.

In an ideal fluid all particles travel with the same speed.

In a viscous fluid the speed of the particles on the wall is zero, being maximum in the center.

The force needed to move a plate in contact with a fluid is proportional to the area of the plate and the speed of the fluid. It is inversely proportional to the distance between the two plates.

\[ F = \frac{\eta A v}{d} \]

Coefficient of viscosity

Units: 1 N m^-2
Viscosity

**Poiseuille’s Equation**

\[
\text{Rate of flow} = \frac{\Delta V}{\Delta t}
\]

Does the rate of flow of a viscous fluid in a tube increase (a), decrease (b) or remain the same (c) with:

1. the difference in pressure (P1-P2)?
2. The radius?
3. The length?
4. The viscosity?

**TABLE 9.5** The Viscosities of Various Fluids

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Temperature (°C)</th>
<th>Viscosity (N·s/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>20</td>
<td>1.0 × 10⁻³</td>
</tr>
<tr>
<td>Water</td>
<td>100</td>
<td>0.3 × 10⁻³</td>
</tr>
<tr>
<td>Whole blood</td>
<td>37</td>
<td>2.7 × 10⁻³</td>
</tr>
<tr>
<td>Glycerin</td>
<td>20</td>
<td>1.500 × 10⁻³</td>
</tr>
<tr>
<td>10-wt motor oil</td>
<td>30</td>
<td>250 × 10⁻³</td>
</tr>
</tbody>
</table>

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Poiseuille’s Equation

\[
\text{Rate of flow} = \frac{\Delta V}{\Delta t} = \frac{\pi R^4 (P_1 - P_2)}{8 \eta L}
\]

Poiseuille’s Equation: example

A patient receives a blood transfusion through a needle of radius 0.20 mm and length 2.0 cm. The density of blood is 1050 kg/m³. The supply bottle is 0.50 m above the patient’s arm. What is the rate of flow through the needle?