Basic ideas on compound reactions

Compound versus direct

- fast – direct reactions (forward-peaked)
- slow – compound nucleus (symmetric contributions around 90 deg)
- usually possible to separate these contributions
**Compound nucleus - definitions**

- Compound nucleus cross sections involve averaging over many compound nucleus levels – statistical features.

\[ \hat{\Gamma}_{pq} = 2 \sum_\alpha \gamma_{p\alpha} P_{\alpha} \gamma_{q\alpha} \]

- R-matrix phenomenology: width level matrix for state \( q \) to decay to state \( p \).

- \( P \) (penetrabilities depend on energy)

- \( \gamma \) (depend on wavefunction at the boundary of the box)

- Mean square value over energy interval \( I \)

- Average width \( <\Gamma> \)

- Energy interval \( I \)

**Porter Thomas distribution**

- Resonant states are complicated configurations

- Reduced width amplitudes arise from random effects of the Hamiltonian

- Should be governed by normal distribution centered about zero

- Probability density function for an individual reduced width amplitude \( \gamma \)

\[ P_{\alpha}^{PT}(\gamma) = \frac{1}{\sqrt{2\pi}(\gamma^2)_\alpha} \exp \left( -\frac{\gamma^2}{(\gamma^2)_\alpha} \right) \]

- Mean square value over energy interval

- Mean partial width

\[ <\Gamma> = 2(\gamma_{p\alpha}^2)P_{\alpha} \]

- Probability density for partial widths

\[ x = \Gamma/<\Gamma>_{\alpha} \]

\[ P_1(x) = \frac{1}{\sqrt{2\pi}} x^{-\frac{1}{2}} e^{-x/2} \]
Porter Thomas distribution

- mean partial width
  \( \langle \Gamma \rangle_{\alpha} = 2 \langle \gamma_{\alpha}^2 \rangle P_{\alpha} \)

- probability density for partial widths
  \( x = \Gamma / \langle \Gamma \rangle_{\alpha} \)
  \( P_1(x) = \frac{1}{\sqrt{2\pi}} x^{-\frac{1}{2}} e^{-x/2} \)

- This assumes that the penetrability will vary only slowly over many narrow resonances to be considered in medium and heavy nuclei

- total widths
  \( \Gamma_p = \sum_{\alpha} \Gamma_{\alpha p} = 2 \sum_{\alpha} \gamma_{\alpha}^2 P_{\alpha} \)

- Chi\(^2\) distribution for \( n \) terms
  \( \gamma_{\alpha}(x) = n f(n x) = \frac{n}{2 \Gamma(\frac{1}{2})} \left( \frac{nx}{2} \right)^{\frac{1}{2} - 1} e^{-\frac{nx}{2}} \)
  \( x = \Gamma / \langle \Gamma \rangle_{\alpha} \)

- mean = 1 and variance \( 2/n \)
- \( n = \) number of deg of freedom

---

Fig. 11.2. An early analysis of distributions of widths, in \(^{233}\text{U}\) from analysis of 12 low-lying resonances in \( n + \text{\(^{233}\text{U}\)}\) scattering, from [2]. The \( N(x) \) is the number of levels having a value of \( \Gamma / \langle \Gamma \rangle \) greater than \( x \). The curves are the Porter-Thomas distributions for the indicated number of degrees of freedom (\( \nu \), our \( n \)). We see (left) that neutron widths show 1 or 2 open channels (degrees of freedom), while the photon widths (right) have \( \sim 20 \). Reprinted with permission from M. S. Moore and C. W. Reich. Phys. Rev. 118 (1960) 718. Copyright (1960) by the American Physical Society.
Neglecting correlations

Reich-Moore approximation
- Many off diagonal terms of the width level matrix should average to zero because the reduced widths have random sign and random variations. Neglect only photon off diagonal

\[
\hat{\Gamma}_{pq} = \begin{cases} 
\Gamma_p & p = q \\
0 & p \text{ or } q \text{ are gamma channels} \\
2 \sum_{\alpha} \gamma_{\rho\alpha} p_{\rho\alpha} & p \text{ and } q \text{ are particle channels} 
\end{cases}
\]

Multi-level Breit-Wigner
- Neglect all off diagonal

\[
S_{\alpha'} = \Omega_{\alpha} \left[ \delta_{\alpha\alpha'} + i \sum_{\alpha} \frac{\Gamma_{\alpha p}^{1/2} \Gamma_{\alpha' p}^{1/2}}{E_p - E - \frac{i}{2} \Gamma_p} \right] \Omega_{\alpha'}
\]

Angle integrated cross section

\[
\sigma_{\alpha'\alpha}(J_{\text{tot}}; E) = \frac{\pi}{k^2} g_{J_{\text{hot}}} \sum_p \frac{\Gamma_{\alpha p} \Gamma_{\alpha' p}}{(E - E_p)^2 + \Gamma_p^2/4}
\]

Multi-level Breit Wigner formula

Angle integrated cross section

\[
\sigma_{\alpha'\alpha}(J_{\text{tot}}; E) = \frac{\pi}{k^2} g_{J_{\text{hot}}} \sum_p \frac{\Gamma_{\alpha p} \Gamma_{\alpha' p}}{(E - E_p)^2 + \Gamma_p^2/4}
\]

Each level \( p \) has an energy integrated cross section

\[
\int_0^\infty dE \sigma_{\alpha'\alpha}^{(p)}(J_{\text{tot}}; E) = \frac{2\pi^2}{k^2} g_{J_{\text{hot}}} \frac{\Gamma_{\alpha p} \Gamma_{\alpha' p}}{\Gamma_p}
\]

Most accurate if all widths are much smaller than spacing \( D \) (non overlapping well-separated resonances)

1) Adding cross sections above over all final states should give reaction cross section (loss of flux from entrance channel)

2) If we know optical potential (imaginary part) we can use to normalize total width for decay of compound states.
Hauser Feshbach theory

- energy interval $I$
- mean level spacing $D$

$I \gg D$

Energy average cross section is

Assume peaks are similar on average

Integral over a single resonance

Energy average cross section:

$$
\langle \sigma(E) \rangle = \frac{1}{I} \int_{E-I/2}^{E+I/2} \sum_p \sigma_p(E) dE = \frac{1}{I} \sum_{p, E_p \in [E \pm I/2]} \int_0^\infty \sigma_p(E) dE = \frac{1}{I} \int_0^\infty \sigma_p(E) dE = \frac{1}{I} \int_0^\infty \sigma_p(E) dE
$$

Width fluctuation corrections

- There are possible correlations appearing in the widths when averaging

$$
\left\langle \frac{\Gamma_{ap} \Gamma_{a'p}}{\Gamma_p} \right\rangle \neq \frac{\langle \Gamma_a \rangle \langle \Gamma_{a'} \rangle}{\langle \Gamma \rangle}
$$

- Width fluctuation factor

$$
\left\langle \frac{\Gamma_{ap} \Gamma_{a'p}}{\Gamma_p} \right\rangle = W_{a' a} \frac{\langle \Gamma_a \rangle \langle \Gamma_{a'} \rangle}{\langle \Gamma \rangle}
$$

- So the energy average cross section becomes

$$
\langle \sigma_{a' a} (J_{wa}, E) \rangle = \frac{\pi}{k^2} \sigma_{J_{wa}} W_{a' a} \frac{2\pi}{D} \frac{\langle \Gamma_a \rangle \langle \Gamma_{a'} \rangle}{\langle \Gamma \rangle}
$$

$$
\langle \Gamma \rangle = \sum_{a} \langle \Gamma_a \rangle
$$
Width fluctuation corrections: estimates

\[ \langle \frac{\Gamma_{\alpha p} \Gamma_{\alpha' p}}{\Gamma_p} \rangle = W_{\alpha \alpha'} \langle \frac{\Gamma_{\alpha}}{\Gamma} \rangle \]

estimates for non-overlapping well separated resonances

- factorize penetrability factors \( \langle \gamma_{p\alpha}^2 \gamma_{p\alpha'}^2 \rangle = W_{\alpha \alpha'} \langle \gamma_{p\alpha}^2 \rangle \langle \gamma_{p\alpha'}^2 \rangle \)

- reduced width for inelastic channels should be statistically independent

\[ \langle \gamma_{p\alpha}^2 \gamma_{p\alpha'}^2 \rangle \sim \langle \gamma_{p\alpha}^2 \rangle \langle \gamma_{p\alpha'}^2 \rangle \]

- for elastic channel it should go with the fourth moment
  (for normal distributions \( W_{\alpha \alpha} = 3 \))

\[ \langle \gamma_{p\alpha}^4 \rangle = W_{\alpha \alpha} \langle \gamma_{p\alpha}^2 \rangle^2 \]

Connecting to optical model

\[ \langle \sigma_{\alpha' \alpha} (J_{\text{tot}}^\pi; E) \rangle = \frac{\pi}{k_i^2} g_{\text{tot}} W_{\alpha \alpha'} \frac{2\pi \langle \Gamma_{\alpha} \rangle \langle \Gamma_{\alpha'} \rangle}{\langle \Gamma \rangle} \]

- If we sum over all outgoing channels (and also include the elastic)

\[ \int_0^\infty \sigma^R_{\alpha} (J_{\text{tot}}^\pi; E) \, dE = \frac{\pi}{k^2} g_{\text{tot}} 2\pi \Gamma_{\alpha} \rightarrow \langle \sigma^R_{\alpha} (J_{\text{tot}}^\pi; E) \rangle = \frac{\pi}{k^2} g_{\text{tot}} \frac{2\pi \langle \Gamma_{\alpha} \rangle}{D} \]

- in the optical model the reaction cross sections related to S matrix

\[ \sigma^R_{\alpha} (J_{\text{tot}}^\pi; E) = \frac{\pi}{k^2} g_{\text{tot}} (1 - |S_{\alpha' \alpha}^{\pi \text{opt}}|^2) \]

- this implies a normalized for the widths

\[ 1 - \left| S_{\alpha' \alpha}^{\pi \text{opt}} \right|^2 = \frac{2\pi \langle \Gamma_{\alpha} \rangle}{D} \]
Transmission coefficients

- We now introduce the transmission coefficients $T_\alpha = 1 - |S_{\alpha\alpha}^{\text{opt}}|^2$.

These describe the coupling between the external scattering and internal compound nucleus production due to the imaginary potential.

What is the value of $T_\alpha$ for real potentials?
What is the limiting value of $T_\alpha$ for strongly absorbed channels?

Not barrier tunneling coefficients!!!

Cross sections in terms of transmission coefficients

$$\langle \sigma_{\alpha'\alpha} (J^\pi_{\text{ini}}; L) \rangle = \frac{\pi}{k^2} g J_{\text{ini}}^* W_{\text{ini}} T_\alpha T_{\alpha'} \sum_{\alpha''} \frac{T_{\alpha''}}{T_{\alpha''}}$$

Branching ratios

- Hauser-Feshbach branching ratios
  $$B_{\alpha'} = \frac{T_{\alpha'}}{\sum_{\alpha''} T_{\alpha''}}$$

  The branching ratio of a given compound nuclear state to final channel $\alpha'$ is proportional to the transmission, normalized by the summed coefficients.

- Then the total probability is 1.
Weiskopf-Ewing approximation

- Considering all partial wave contributions
  \[ \langle \sigma_{\alpha'\alpha} (E) \rangle = \frac{\pi}{k^2} \sum_{J_{tot}} g_{J_{tot}} W_{\alpha\alpha'} \frac{T_{\alpha} T_{\alpha'}}{\sum_{\alpha''} T_{\alpha''}} \]

- \( \alpha \) does not factorize into product of production and decay probabilities
- This will only factorize if either:
  - Sum of \( J \) can be ignored
  - All transmissions have the same \( J \) dependence

\[ \langle \sigma_{\alpha'\alpha} (E) \rangle = \sigma_{\alpha}^R (E) \frac{T_{\alpha'}}{\sum_{\alpha''} T_{\alpha''}} \equiv \sigma_{\alpha}^R (E) B_{\alpha'} \]

Strong coupling limit

- Isolated resonances assumption hold for neutron scattering on heavy nuclei up to 100-200 keV.
- Above this energy, resonances are too close and too wide
  
  Hauser Feshbach still holds but needs fixes!

Corrected transmissions \( T \leq 1 \)
\[ T_{\alpha} = \frac{2\pi \langle T_{\alpha} \rangle}{D} \]
\[ T_{\alpha} = 1 - \exp(-2\pi \langle T_{\alpha} \rangle / D) \]

Transmission matrix based on S matrix
\[ T_{\alpha'\alpha} = \delta_{\alpha\alpha'} - \sum_{\alpha''} (S_{\alpha'\alpha''}^{opt}) \ast S_{\alpha''\alpha}^{opt} \]

Corrected width-fluctuation: Monte-Carlo simulations suggest
\[ W_{\alpha\alpha} = 1 + \frac{2}{1 + T_{\alpha}^{1/2}} \]
Decay models

- simplest: irreversible chain of an excited compound nucleus
- successive particle emission results in 3- and 4- body final states...
- cannot be coherently described in our two-body formulation
- if each decay is statistically independent (decoherent) then ok.

- projectile pi fuses with target ti to form compound nucleus
  \[ \beta_i = \{x_i, 1\} \quad \beta = \{x, t\} \quad x = x_i + p_i \]

- cross section for productions
  \[ \sigma^{(0)}_\beta = \sum_{I_i, I_p} \frac{2I_i + 1}{2I_p + 1} \frac{T_{L_i L_p}}{L_i L_p} \]

- transmission coefficients
  \[ T^{L_p}_{\beta; \beta'} = 1 - |S_{\alpha \beta}^{L_f, \text{opt}} (\epsilon_i - \epsilon_{i'} - Q_{\alpha}^{Y})|^2 \]

Decay models

- generalize the one step relations to n-steps
  \[ \sigma^{(n)}_{\beta'} = \sum_{\beta} \mathcal{B}_{\beta'; \beta} \mathcal{G}_{\beta}^{(n-1)} \]
  \[ \mathcal{B}_{\beta'; \beta} = \sum_{L_p} \frac{T_{L_i L_p}}{L_i L_p} \sum_{\beta' L' \beta'} T^{L' L_i}_{\beta'} \]

ignoring width fluctuation corrections

States are not all known
Level density from theory!

\[ D^{-1} = \rho_0 (\epsilon, I, \pi_t) \]

Fig. 11.4. Multi-stage decay processes may be modeled with the Hauser-Feshbach theory, following an initial reaction of neutrons incident on \( ^AX \). The initial compound nucleus is \( ^{A+1}X \) on the right, which may decay by emission of 1, 2 or 3 neutrons to the nuclei to the left. Figure courtesy of Erich Ormand.
Hauser Feshbach method

- implemented systematically (publically available)

**Statistical and pre equilibrium code**: Calculation of energy-averaged cross sections of particle induced nuclear reactions with emission of particles and gamma rays, and fission. The models employed are the Hauser-Feshbach-Moldauer statistical model including a gamma-ray cascade model, and the pre-equilibrium (PE) emission exciton and Geometry-Dependent Hybrid (GDH) models. The optical-potential transmission coefficients are calculated internally by using the optical model code SCAT-2.

**EMPIRE** is a flexible code for calculation of nuclear reactions in the frame of combined optical, Multistep Direct (TUL), Multistep Compound (NVWY) and statistical (Hauser-Feshbach) models. Incident particle can be a nucleon or any nucleus (Heavy Ion). Isomer ratios, residue production cross sections and emission spectra for neutrons, protons, alpha-particles, gamma-rays, and one type of Light Ion can be calculated. The energy range starts just above the resonance region for neutron induced reactions and extends up to several hundreds of MeV for the Heavy Ion induced reactions.

**TALYS** is software for the simulation of nuclear reactions. Many state-of-the-art nuclear models are included to cover all main reaction mechanisms encountered in light particle-induced nuclear reactions. TALYS provides a complete description of all reaction channels and observables, and is user-friendly.

Sources for optical potential

- most of real part – folding NN with projectile and target densities
- imaginary (dynamic polarization potential DPP) – all other effects...
- effect of neglecting direct channels

\[
\begin{align*}
[T_1 \mid U_1 & : E_1 \psi_1 (R) \mid V_{12} \psi_2 (R) = 0 \\
[T_2 \mid U_2 & : E_2 \psi_2 (R) \mid V_{21} \psi_1 (R) = 0
\end{align*}
\]

\[
[T_1 + U_1 + V_{12} \hat{G}_2 V_{21} - E_1] \psi_1 (R) = 0
\]

- the bare interaction $U_1$ is modified by a dynamic polarization

\[
V_{DPP} \psi_1 = V_{12} \hat{G}_2 V_{21} \psi_1 = V_{12} \int_0^\infty G_2 (R, R'; E_2) V_{21} (R') \psi (R') dR'
\]
Sources for optical potential: direct

- the bare interaction $U_1$ is modified by a dynamic polarization (non-local and $E,L$ dependent)
  \[ V_{DPP}(R) = V_{12} \hat{G}_2 V_{21} \psi_1 \]
  \[ = V_{12}(R) \int_0^\infty G_2(R, R'; E_2) V_{21}(R') \psi(R') dR' \]

- trivially equivalent local potential
  \[ [T_1 + U_1 - E_1] \psi_1(R) + V_{12} \psi_2(R) = 0 \quad \Rightarrow \quad V_{\text{celp}}(R) = \frac{V_{12} \psi_2(R)}{\psi_1(R)} \]

- weighted equivalent local potential
  \[ V_{\text{welp}}(R) = \frac{\sum_{J_{\text{tot}},\alpha' \neq \alpha} |\psi_{\alpha'}^{J_{\text{tot}}}(R)|^2 V_{\alpha\alpha'} \psi_{\alpha}^{J_{\text{tot}}}(R)}{\sum_{J_{\text{tot}}} |\psi_{\alpha}^{J_{\text{tot}}}(R)|^2} \]

Sources for optical potential: direct

- fitting S-matrix (angular distributions) – inversion techniques

Fig. 11.6. Left: Data for the $^{40}$Ca(d,d) elastic scattering compared with calculations including no coupling (dashed curve), deuteron breakup only (dotted curve), and deuteron breakup plus (d) and (d,$^3$He) pickup (solid curve) [28]. The dot-dashed curve gives the result for the $J$-weighte5d S matrix from the full CRC calculation. Right: Comparing the bare potential (solid line) and the two inverted potentials, with the real part in the upper panel and the imaginary part below. Reprinted with permission from N. Keeley and R. S. Mackintosh, Phys. Rev. C 77 (2008) 054603. Copyright (2008) by the American Physical Society.
Sources for optical potential

- Effect of neglecting compound-nucleus resonances
  - Assume we can average S matrix amplitudes over energy range with many resonances – **optical average S matrix elements**
  \[ S_{\alpha'\alpha}^{\text{opt}} = \bar{S}_{\alpha'\alpha} \]
  - Energy interval >> D spacing >> \( \Gamma \) average width
  - Assume shift function and penetrability do not vary much
  \[ \bar{S}(E) = \int dE' f(E-E') S(E') \]
  - Empirically introduce Lorentzian smoothing function
  \[ f(\Delta E) = \frac{\Gamma}{\pi} \frac{1}{\Delta E^2 + \Gamma^2} \]

Average S-matrix elements

The average S matrix does not give an average elastic or reaction cross section!

- Elastic cross section
  \[ \sigma_R = \frac{\pi}{k_i^2} \sum_{J_{\text{tot}}} g_{J_{\text{tot}}} (1 - |S|^2) \text{ and } \sigma_{\text{el}} = \frac{2\pi}{k_i^2} \sum_{J_{\text{tot}}} g_{J_{\text{tot}}} |1 - S|^2 \]

- Reaction cross section
  \[ \sigma_{R} = \sigma_R(\text{opt}) - \sigma_{\text{fl}} \]
  \[ \sigma_{\text{el}} = \sigma_{\text{el}}(\text{opt}) + \sigma_{\text{fl}} \]

There is a fluctuation cross section that depends on the variance of S

- Elastic cross section
  \[ \sigma_{R} = \frac{\pi}{k_i^2} \sum_{J_{\text{tot}}} g_{J_{\text{tot}}} |S - \bar{S}|^2 \]

The average S matrix does give an average total cross section

- Elastic cross section
  \[ \sigma_{T} = \frac{2\pi}{k_i^2} \sum_{J_{\text{tot}}} g_{J_{\text{tot}}} (1 - \text{Re}S) \]

- Reaction cross section
  \[ \sigma_{T} = \frac{2\pi}{k_i^2} \sum_{J_{\text{tot}}} g_{J_{\text{tot}}} (1 - \text{Re}\bar{S}) = \sigma_T(\text{opt}) \]
S-matrix average and Hauser Feshbach

Obvious approximation to the Hauser Feshbach based on $\chi$ fluctuation

$$\sigma_\text{eff} = \frac{\pi}{k_f^2} \sum_{\lambda \alpha \beta} s_{\lambda \alpha \beta} - \overline{s}^2$$

$$|s_{\chi \alpha \alpha} - \overline{s}_{\chi \alpha \alpha}|^2 - W_{\chi \alpha \alpha} \sum_{\alpha'} \frac{T_{\alpha \alpha'}}{T_{\alpha'}}$$

Porter Thomas and Hauser Feshbach

Anomalous Fluctuations of $s$-Wave Reduced Neutron Widths of $^{192,194}$Pt Resonances

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We obtained an unprecedentedly large number of $s$-wave neutron widths through $R$-matrix analysis of neutron cross-section measurements on enriched Pt samples. Careful analysis of these data rejects the validity of the Porter-Thomas distribution with a statistical significance of at least 99.997%.
Distribution of Resonance Widths and Dynamics of Continuum Coupling

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We analyze the statistics of resonance widths in a many-body Fermi system with open decay channels. Depending on the strength of continuum coupling, such a system reveals growing deviations from the standard chi-square (Porter-Thomas) width distribution. The deviations emerge from the process of increasing interaction of intrinsic states through common decay channels; in the limit of perfect coupling this process leads to the superradiance phase transition. The width distribution depends also on the intrinsic dynamics (chaotic versus regular). The results presented here are important for understanding the recent experimental data concerning the width distribution for neutron resonances in nuclei.