Setting the framework
relevant scales for nuclei

**Physics of Hadrons**
- Degrees of Freedom: quarks, gluons
  - 940 MeV
    - neutron mass
- Degrees of Freedom: constituent quarks
  - 140 MeV
    - pion mass
- Degrees of Freedom: baryons, mesons

**Physics of Nuclei**
- Degrees of Freedom: protons, neutrons
  - 8 MeV
    - proton separation energy in lead
- Degrees of Freedom: nucleonic densities and currents
  - 1.32 MeV
    - vibrational state in tin
- Degrees of Freedom: collective coordinates
  - 0.043 MeV
    - rotational state in uranium
understanding nuclei

nuclear shell model

electronic shells
understanding nuclei

nuclear shell model

magic numbers

\[ ^{208}\text{Pb} \]
\[ ^{100}\text{Sn}, \ ^{132}\text{Sn} \]
\[ ^{56}\text{Ni}, \ ^{78}\text{Ni} \]
\[ ^{40}\text{Ca}, \ ^{48}\text{Ca} \]
\[ ^{16}\text{O} \]
nuclear states
Fig. 1.1. Binding energies per nucleon, $B(A, Z)/A$, for all naturally occurring long-lived isotopes of $A$ nucleons.
nuclear chart
nuclear reactions

Important concepts:

- projectile (A)
- target (B)
- residual nuclei (C+D)
- q-value of a reaction

\[ Q = (m_A + m_B - m_C - m_D)c^2 \]

Notations for the reaction:

- B(A,C)D
- A+B ⇔ C+D
what sort of reactions?

- elastic scattering
  - optical potential, densities
- inelastic scattering
  - nuclear shapes or E.M. transition probabilities
- breakup, transfer, knockout
  - bound/continuum properties, correlations, pairing, astro
- fusion and compound
  - production yields, astrophysical rates, applications (energy)

need accurate reaction models!
why reactions?

- shell structure
- correlations
- pairing
- weakly bound systems
- role of continuum
- ...

transfer versus knockout

need accurate reaction models!

[Jenny Lee et al, PRL 2009]

[Gade et al, Phys. Rev. Lett. 93, 042501]
why reactions?

astrophysical reaction rates through indirect methods

- ANC method (using peripheral transfer or breakup)
- Coulomb dissociation method
- Trojan horse method

Summers and Nunes, PRC78(2009)069908
many body bound problem

\[ H_A = -\sum_{i=1}^{A} \frac{\hbar^2}{2m_i} \nabla_{r_i}^2 + \frac{\hbar^2}{2M} \nabla_S^2 + \sum_{i>j}^{A} V^{(2)}(r_i-r_j) \]

\[ H_A \Phi_{I\mu}(\rho_1, \ldots, \rho_{A-1}) = E_I \Phi_{I\mu} \]

\[ \lim_{\rho_i \to \infty} \Phi_{I\mu}(\ldots, \rho_i, \ldots) = 0 \]

\[ \int d\rho_1 \ldots \int d\rho_{A-1} |\Phi_{I\mu}(\rho_1, \ldots, \rho_{A-1})|^2 = 1 \]
Ab-initio methods for reactions

- area with huge advances in the last decade
- need NNN forces (also important for structure)
- need three-body dynamics
Ab-initio reactions with 3N forces

FIG. 5. (Color online) Dependence of the $n^{-4}$He phase shifts on the considered target eigenstates. Results with only the g.s. of $^4$He (thin gray long-dashed lines) are compared to those obtained by including in addition up to the $0^+0^+$ (thin black dashed lines), $0^{-0}$ (thin violet lines), $2^+0^+$ (thick brown dotted lines), $2^+1^+$ (thick green long-dashed lines), $1^{-1}$ (thick blue dashed lines), and $1^+0^+$ (thick red lines) excited states of $^4$He, respectively. The model space is truncated at $N_{\text{max}} = 13$. Other parameters are identical to those of Fig. 2.

FIG. 10. (Color online) Comparison of the $n^{-4}$He (a) and $p^{-4}$He (b) phase shifts as a function of the kinetic energy $E_{\text{kin}}$ for different target eigenstates. The solid lines represent the full $NN+3N$ interaction, while the dashed lines correspond to the induced interaction. The symbols represent experimental data. The model space is truncated at $N_{\text{max}} = 13$. Other parameters are identical to those of Fig. 2.

- $he4+n$ elastic scattering
Ab-initio states with 3body dynamics

• still as the bound state level
• extremely demanding computaitonally
Ab-initio states with 3-body dynamics

PRC 86, 021602R (2012)

FIG. 5. (Color online) Differential cross section from coupled-cluster calculations divided by Rutherford cross section for elastic proton scattering on \(^{40}\)Ca at \(E_{\text{c.m.}} = 9.6\) MeV (solid line), experimental data (dots), and optical model potential fits (dashed line), taken from Ref. [31].

FIG. 6. (Color online) Same caption as in Fig. 5 except that the energy is \(E_{\text{c.m.}} = 12.44\) MeV.

• ab-initio coupled cluster calculations
• single nucleon correlations insufficient
reducing the many body to a few body problem

- isolating the important degrees of freedom in a reaction
- keeping track of all relevant channels
- connecting back to the many-body problem

- effective nucleon-nucleus interactions (or nucleus-nucleus)
  (energy dependence/non-local?)
- many body input (often not available)
- reliable solution of the few-body problem
forces present in reactions

- nuclear (strong)
- weak
- electromagnetic
- gravitational
Connection to astrophysics
Coulomb effects in reactions

\[ \sigma(E) = \frac{1}{E} e^{-2\pi \eta S(E)} \]

Astrophysical S-factor

\[ \eta = \frac{Z_1 Z_2 e^2}{(\hbar v)} \]

Sommerfeld parameter

Fig. 1.4. Dependence of cross section and \( S(E) \) on energy, for the reaction \(^3\text{He}(\alpha, \gamma)^7\text{Be}\). The solid curve is a calculation to be discussed in Appendix B.
reaction rates and the Gamow peak

\[
\langle \sigma v \rangle = \sqrt{\frac{8}{\pi \mu_{12}(k_B T)^3}} \int_0^\infty S(E) \exp\left(-\frac{E}{k_B T} - \sqrt{\frac{E_G}{E}}\right) dE.
\]

\[E_G = 4\pi^2 \eta^2 E = 2\mu_{12}(\pi Z_1 Z_2 e^2)^2 / \hbar^2\]

\[E_0 = \left(\frac{E_G k_B T^2}{4}\right)^{\frac{1}{3}}\]
history of the universe
primordial nucleosynthesis

1: \( n \leftrightarrow p \)
2: \( p(n, \gamma)d \)
3: \( d(p, \gamma)^3\text{He} \)
4: \( d(d, n)^3\text{He} \)
5: \( d(d, p)t \)
6: \( t(d, n)\alpha \)
7: \( t(\alpha, \gamma)^7\text{Li} \)
8: \( ^3\text{He}(n, p)t \)
9: \( ^3\text{He}(d, p)^4\text{He} \)
10: \( ^3\text{He}(\alpha, \gamma)^7\text{Be} \)
11: \( ^7\text{Li}(p, \alpha)^4\text{He} \)
12: \( ^7\text{Be}(n, p)^7\text{Li} \)

Q-value for \( p(n, g)d \) 2.26 MeV

\( T(\text{universe to cool down to } E=2.26)=7 \text{ min} \)

slightly less than \( T(\text{free neutron decay}) \)
reactions in light stars

\[ p + p \rightarrow d + e^+ + \nu \]
\[ p + d \rightarrow ^3\text{He} + \gamma \]
\[ ^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + 2p \]

Chain I
\[ Q_{\text{ret}} = 26.20 \text{ MeV} \]

\[ ^3\text{He} + ^4\text{He} \rightarrow ^7\text{Be} + \gamma \]
\[ ^7\text{Be} + e^- \rightarrow ^7\text{Li} + \nu \]
\[ ^7\text{Li} + p \rightarrow 2^4\text{He} \]
\[ ^7\text{Be} + p \rightarrow ^8\text{B} + \gamma \]
\[ ^8\text{B} \rightarrow ^8\text{Be} + e^+ + \nu \]

Overall result: \[ 4p \rightarrow ^4\text{He} + 2e^+ + 2\nu + Q_{\text{ret}} \]

Chain III
\[ 2^4\text{He} \quad Q_{\text{ret}} = 19.17 \text{ MeV} \]
solar neutrino puzzle and nuclear reactions

status in the nineties: measured solar neutrino flux $<<$ predicted flux

FIG. 8 (color online). $S_{17}$ values from CD experiments. Full circles: latest analysis of the GSI CD experiment (Schümann et al., 2006); open stars: Kikuchi et al. (1998) analyzed in first-order perturbation theory; open squares: Davids and Typel (2003). The error bars include statistical and estimated systematic errors. The curve is taken from the cluster-model theory of Descouvemont et al. (2004), normalized to $S_{17}(0) = 20.8$ eV b.

FIG. 9 (color online). $S_{17}(E)$ vs center-of-mass energy $E$, for $E \leq 1250$ keV. Data points are shown with total errors, including systematic errors. Dashed line: scaled Descouvemont (2004) curve with $S_{17}(0) = 20.8$ eV b; solid line: including a fitted $1^+$ resonance shape.
triple-alpha reaction

\[ \frac{7.2747}{3\alpha} \rightarrow 1^2\text{C} \]

\[ 7.6542 \quad 0^+ \quad \gamma \quad e^+ - e^- \]
\[ 4.4389 \quad 2^+ \]
\[ J^\pi = 0, T = 0 \]

\[ \frac{7.3666}{\alpha + ^8\text{Be}} \]
CNO cycles

Diagram showing the cycles involving carbon (13C), nitrogen (15N), and oxygen isotopes (14N and 17O or 17F). The reactions include:

- $^{13}\text{C} \rightarrow ^{14}\text{N}$ via $(p,\gamma)$
- $(e^+ + \nu)$
- $(p,\gamma)$
- $^{13}\text{N} \rightarrow ^{15}\text{O}$
- $(p,\gamma)$
- $(e^+ + \nu)$
- $(p,\alpha)$
- $^{12}\text{C} \rightarrow ^{15}\text{N}$
- $(p,\alpha)$
- $(e^+ + \nu)$

Diagram on the right shows the extension to $^{17}\text{O}$ or $^{17}\text{F}$.

Diagram on the left shows $^{16}\text{O}$.

Additional reactions include:

- $(p,\gamma)$
- $(p,\gamma)$
- $(e^+ + \nu)$
- $(p,\gamma)$
heavier elements
medium mass elements and red giants

- Hydrogen, Helium
- Helium, Nitrogen
- Carbon, Oxygen, Neon
- Iron
- Silicon, Sulfur
- Oxygen, Neon, Sodium
heavy elements and the s-process
heavy elements: elemental abundancies
Question 3
How were the heavy elements made?
Where did they come from?
heavy element: r-process in the chart

- r-process in the chart
how do we measure reactions rates?

$^{14}\text{C}(n,\gamma)^{15}\text{C}$ has impact on:
- neutron-induced CNO cycle
- heaviest element in non-homogenous bigbang
- abundancies from the r-process in supernovae
indirect methods: nuclear reactions

- direct measurement $^{14}\text{C}(n,\gamma)^{15}\text{C}$
- transfer reaction $^{14}\text{C}(d,p)^{15}\text{C}$
- Coulomb dissociation

\[ \text{low relative energy} \]
breakup reactions and \((n,\gamma)\)

\[
\begin{align*}
\text{Nakamura} &\quad \text{Reifarth} \\
^{208}\text{Pb}^{(15}C,^{14}C+n)^{208}\text{Pb@68 MeV/u} &\quad {^{14}C(n,\gamma)^{15}C}
\end{align*}
\]
Some basics
classification of reactions

Direct reactions
transfer momentum is small compared to initial momentum
typically peripheral
short timescale ($10^{-22}$ s)
E>$10$ MeV
mostly one step
final states keep memory of initial states

Resonance reactions
reactions that go through a resonance (peak in the cross section)
intermediate step in the reaction
longer time scale

Compound reactions
longer timescale
many steps in the reaction
all nucleons share the beam energy
loss of memory from the initial state
low energy reactions
direct reactions

Capture

\[ \phi_{k\ell j}^m(\mathbf{r}) \]

Inelastic excitations (bound to bound states) DWBA

\[ \phi_{n\ell j}^m(\mathbf{r}) \]
\[ \chi_{k,\sigma}(\mathbf{R}) \]
\[ \phi_{n'\ell'j'}^{m'}(\mathbf{r}) \]

target
direct reactions

Inelastic excitations (breakup)

\[ \phi_{n\ell j}^m (\vec{r}) \rightarrow \chi_{\vec{k},m} \rightarrow \chi_{\vec{k}_{v},\sigma_v} \rightarrow \phi_{k'\ell' j'}^{m'} (\vec{r'}) \rightarrow \vec{k}_{v}, \sigma_v \rightarrow \vec{k}_c, \sigma_c \]

target

Transfer reactions

\[ \phi_{n_i \ell_i j_i}^{m_i} (\vec{r'}) \rightarrow \chi_{\vec{k}_i, m_i} \rightarrow \chi_{\vec{k}_f, m_f} \rightarrow \phi_{n_f \ell_f j_f}^{m_f} (\vec{r'}) \rightarrow \vec{k}_f \rightarrow \vec{j}_f \]

target
FIG. 10. Elastic scattering for $^6$He+$^{12}$C at 38.3 MeV/nucleon in comparison with the OM results given by the real folded potential (obtained with the CDM3Y6 interaction and the Gaussian $g\alpha$ density for $^6$He). The dashed curve is obtained with the unrenormalized folded potential only. The solid curve is obtained by adding a complex surface polarization potential to the real folded potential. Its parameters, and those of the imaginary part, are explained in the text. The dotted line is obtained by folding the CDM3Y6 interaction with the compact Gaussian density $ro$.

[Lapoux et al, PRC 66 (02) 034608]
why do reactions? inelastic

traditionally used to extract electromagnetic transitions or nuclear deformations

Fig. 2. Comparison of $B(E1)$ values obtained from lifetime and Coulomb excitation measurements. The weighted average of lifetime measurements [3] (open circle) is plotted on the left along with the weighted average (solid circle) of three Coulomb excitation measurements (solid symbols). The individual Coulomb excitation measurements, GANIL (this work, square), MSU (up triangle) [6], RIKEN (down triangle) [7], and a previous GANIL experiment (diamond) [4], are plotted versus the beam energy.

[Summers et al, PLB 650 (2007) 124]
why do reactions? transfer

traditionally used to extract spin, parity and spectroscopic factors

\[ d^{(132}\text{Sn},^{133}\text{Sn})p@5 \text{ MeV/u} \]

why do reactions? transfer

\[ ^{11}\text{Li}(p,t)^{9}\text{Li} @ 3 \text{ A MeV} \]
measured both ground state and excited state \(^9\text{Li}\)

\[ [\text{Tanihata et al, PRL 100, 192502 (2008)] \]

\textit{traditionally used to study two nucleon correlations and pairing}
why do reactions? breakup

\[ ^{14}\text{Be} \rightarrow n+n+^{12}\text{Be} \]

two nucleon correlation function

\[ ^{23}\text{O}(^{208}\text{Pb, ^{208}\text{Pb}})^{22}\text{O}+n+\gamma \]

[Nociforo et al, PLB 605 (2005) 79]

[Marques et al, PRC 64 (2001) 061301]

Fig. 1. Doppler corrected \( \gamma \)-ray spectra measured in coincidence with an \(^{22}\text{O} \) fragment and one neutron for Pb (symbols) and C (shaded area) targets. Arrows indicate the strongest \( \gamma \) transitions as expected from the \(^{22}\text{O} \) level scheme of Ref. [10] (partial level scheme shown as inset; level energies are in keV).
why do reactions? fusion

**Fusion of Stable vs Unstable Nuclei**

![Graph showing reduced cross sections for fusion of halo, normal/weakly bound, and strongly bound nuclei.](image)

- **Halo**
- **Strongly bound**
- **Normal, weakly bound**

Fig. 8. Reduced cross sections for the fusion of halo, normal/weakly bound, and strongly bound nuclei. (Courtesy of Kolata).

After geometric effects are scaled out, fusion enhanced for halo nuclei!

*Superheavies Halos Applications: energy*
The overlap function for $^{19}\text{C} \rightarrow n^{+^{18}\text{C}}$ in arbitrary units. The radial sensitivity of the $^{18}\text{C}(d,p)^{19}\text{C}$ cross section is represented by the colored bars for different beam energies.
resonant reaction

\[ a + A \rightarrow C^{*} \]

Breit-Wigner shape

\[ \sigma_{\text{scatt}} = \frac{\pi}{k^2} \left(2\ell + 1\right) \frac{\Gamma^2}{(E - E_R)^2 + \Gamma^2 / 4} \]

\[ \Gamma = \text{FWHM} \]

\[ E_R = \text{resonant energy} \]
compound reactions

the decay of the compound state does not depend on the initial state.
classification of reactions
classification of reactions
selectivity of the reaction to resonances
Some nucleons end up having enough energy to evaporate
energy distribution: compound vs direct

pure compound nucleus formation can be seen at backward angles

The more forward you go, the more pronounced direct components will appear
angular distribution: compound vs direct

Direct reactions (ID):
Forward peaked (large b)

Compound reactions (NC):
Distribution is generally isotropic (except for heavy ion collision where $\Delta L$ large)
kinematic of reactions

\[ E_{\text{tot}} = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 \]

\[ E_{\text{tot}} = \frac{1}{2} m_{AB} \dot{S}^2 + \frac{1}{2} \mu \dot{R}^2 \]

energy in the relative motion

\[ E = \frac{m_B}{m_{AB}} E_A = \frac{1}{2} \mu v_A^2. \]
apply laws of conservation  
conservation of mass  
conservation of energy  
conservation of momentum

\[ m_A + m_B = m_C + m_D, \]
\[ Q + E_A + E_B = E_C + E_D, \]
\[ p_A + p_B = p_C + p_D, \]
Figure 1.7. Energy $E_b$ of boron nuclei in the reaction $^{12}\text{C} + ^{14}\text{N} \rightarrow ^{10}\text{I}$ as function of the energy $E_a$ of the incident $^{12}\text{C}$ nuclei, for several scattering angles.
equations of motion

\[
\left[ -\frac{\hbar^2}{2m_A} \nabla^2_{\mathbf{r}_A} - \frac{\hbar^2}{2m_B} \nabla^2_{\mathbf{r}_B} + V(\mathbf{r}_A - \mathbf{r}_B) - E_{\text{tot}} \right] \Psi(\mathbf{r}_A, \mathbf{r}_B) = 0. \]

laboratory

Center of mass

\[
\left[ -\frac{\hbar^2}{2m_{AB}} \nabla^2_{\mathbf{S}} - \frac{\hbar^2}{2\mu} \nabla^2_{\mathbf{R}} + V(\mathbf{R}) - E_{\text{tot}} \right] \Psi(\mathbf{S}, \mathbf{R}) = 0. \]

\[ \Psi(\mathbf{S}, \mathbf{R}) = \Phi(\mathbf{S})\psi(\mathbf{R}) \]

\[ \Phi(\mathbf{S}) = A \exp(i\mathbf{K} \cdot \mathbf{S}) \]

\[ -\frac{\hbar^2}{2m_{AB}} \nabla^2_{\mathbf{S}} \Phi(\mathbf{S}) = (E_{\text{tot}} - E) \Phi(\mathbf{S}) \]

and

\[ \left[ -\frac{\hbar^2}{2\mu} \nabla^2_{\mathbf{R}} + V(\mathbf{R}) \right] \psi(\mathbf{R}) = E \psi(\mathbf{R}) \]
The number of particles entering a detector depends on:
- solid angular size of detector
- number of scattering centers in the target
- flux of the incident beam
- the cross sectional area for the reaction to occur

\[
\frac{dN}{dt} = j_i \ n \ \Delta \Omega \ \sigma
\]

\[j = v |\psi|^2\]
**cross section**

**Definition of cross section:**  
the area within which a projectile and a target will interact and give rise to a specific product.

**Units**  
1b (barn) = 10 fm x 10 fm

If we consider just one scattering center \( n = 1 \), and measure the scattered angular flux in the final state as \( \hat{j}_f (\theta, \phi) \) particles/second/steradian, then

\[
\sigma (\theta, \phi) = \frac{\hat{j}_f (\theta, \phi)}{j_i}
\]
cross section in c.m. and lab

Total cross section:
the same in center of mass and laboratory

Angular distribution of the cross section:

\[ \sigma(\theta, \phi) \ d\phi \ \sin \theta \, d\theta = \sigma_{\text{lab}}(\theta_{\text{lab}}, \phi_{\text{lab}}) \ d\phi_{\text{lab}} \ \sin \theta_{\text{lab}} \, d\theta_{\text{lab}} \]