Setting the framework
relevant scales for nuclei

<table>
<thead>
<tr>
<th>Degrees of Freedom</th>
<th>Energy (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>quarks, gluons</td>
<td>940 neutron mass</td>
</tr>
<tr>
<td>constituent quarks</td>
<td>140 pion mass</td>
</tr>
<tr>
<td>baryons, mesons</td>
<td>8 proton separation energy in lead</td>
</tr>
<tr>
<td>nucleonic densities and currents</td>
<td>1.32 vibrational state in tin</td>
</tr>
<tr>
<td>collective coordinates</td>
<td>0.043 rotational state in uranium</td>
</tr>
</tbody>
</table>
understanding nuclei

nuclear shell model

electronic shells
understanding nuclei

nuclear shell model

magic numbers

$^{208}_{\text{Pb}}$

$^{100}_{\text{Sn}}, ^{132}_{\text{Sn}}$

$^{56}_{\text{Ni}}, ^{78}_{\text{Ni}}$

$^{40}_{\text{Ca}}, ^{48}_{\text{Ca}}$

$^{16}_{\text{O}}$
nuclear states

![Diagram of nuclear states]

- Excitation energy (MeV)
- States:
  - $^{13}$C
  - $1/2^-$
  - $3/2^-$
  - $5/2^-$
  - $7/2^+$
  - $3/2^+$
  - $9/2^+$
  - $3/2^+$
  - $5/2^+$
nuclear stability

Fig. 1.1. Binding energies per nucleon, $B(A, Z)/A$, for all naturally occurring long-lived isotopes of $A$ nucleons.
nuclear chart
Important concepts:

- projectile (A)
- target (B)
- residual nuclei (C+D)
- q-value of a reaction

\[ Q = (m_A + m_B - m_C - m_D)c^2 \]

Notations for the reaction

\[ B(A,C)D \]

\[ A+B \Leftrightarrow C+D \]
what sort of reactions?

- elastic scattering
  - optical potential, densities
- inelastic scattering
  - nuclear shapes or E.M. transition probabilities
- breakup, transfer, knockout
  - bound/continuum properties, correlations, pairing, astro
- fusion and compound
  - production yields, astrophysical rates, applications (energy)

need accurate reaction models!
why reactions?

- shell structure
- correlations
- pairing
- weakly bound systems
- role of continuum
- ...

need accurate reaction models!

transfer versus knockout

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[Jenny Lee et al, PRL 2009]

[Gade et al, Phys. Rev. Lett. 93, 042501]
why reactions?

astrophysical reaction rates through indirect methods

- ANC method (using peripheral transfer or breakup)
- Coulomb dissociation method
- Trojan horse method

\[ ^{14}\text{C}(n,\gamma)^{15}\text{C} \]

Summers and Nunes, PRC78(2009)069908
many body bound problem

\[ H_A = - \sum_{i=1}^{A} \frac{\hbar^2}{2m_i} \nabla_{r_i}^2 + \frac{\hbar^2}{2M} \nabla_{S}^2 + \sum_{i>j} A V^{(2)}(r_i-r_j) \]

\[ H_A \Phi_{I\mu}(\rho_1, \ldots, \rho_{A-1}) = E_I \Phi_{I\mu} \]

\[ \lim_{\rho_i \to \infty} \Phi_{I\mu}(\ldots, \rho_i, \ldots) = 0 \]

\[ \int d\rho_1 \ldots \int d\rho_{A-1} |\Phi_{I\mu}(\rho_1, \ldots, \rho_{A-1})|^2 = 1 \]
Ab-initio methods for reactions

- area with huge advances in the last decade
- need NNN forces (also important for structure)
- need three-body dynamics
Ab-initio reactions with 3N forces

- **he4+n elastic scattering**

**FIG. 5.** (Color online) Dependence of the $n^{-4}$He phase shifts on the considered target eigenstates. Results with only the g.s. of $^{4}$He (thin gray long-dashed lines) are compared to those obtained by including in addition up to the $0^{+0}$ (thin black dashed lines), $0^{-0}$ (thin violet lines), $2^{-0}$ (thick brown dotted lines), $2^{-1}$ (thick green long-dashed lines), $1^{-1}$ (thick blue dashed lines), and $1^{-0}$ (thick red lines) excited states of $^{4}$He, respectively. The model space is truncated at $N_{\text{max}} = 13$. Other parameters are identical to those of Fig. 2.

**FIG. 10.** (Color online) Comparison of the $n^{-4}$He (a) and $p^{-4}$He
Ab-initio states with 3body dynamics

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### TABLE I. Computed NCSM $^4$He, NCSM/RGM $^6$He [as $^4$He(g.s.) + $n + n$] and NCSM $^6$He g.s. energies (in MeV) as a function of the absolute HO model space size $N_{tot} = N_0 + N_{max}$, where $N_0$ is the number of oscillator quanta shared by the nucleons in their lowest configuration. For the $^4$He nucleus and for the NCSM/RGM $^4$He + $n + n$ system, $N_0 = 0$ and $N_{max} = N_{tot}$. However, for the $p$-shell $^6$He nucleus within the NCSM, $N_0 = 2$ and $N_{max} = N_{tot} - 2$. The last two rows show the extrapolated values for the calculations with their uncertainties and the experimental values.

<table>
<thead>
<tr>
<th>$N_{tot}$</th>
<th>$^4$He NCSM</th>
<th>$^6$He NCSM/RGM</th>
<th>$^6$He NCSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>-27.984</td>
<td>-28.907</td>
<td>-27.705</td>
</tr>
<tr>
<td>Extrapolation</td>
<td>-28.230(5)</td>
<td>-28.70(3)</td>
<td>-29.84(4)</td>
</tr>
<tr>
<td>Experimental</td>
<td>-28.296</td>
<td>-29.268</td>
<td></td>
</tr>
</tbody>
</table>

- still as the bound state level
- extremely demanding computaitonally
Ab-initio states with 3-body dynamics

PRC 86, 021602R (2012)

FIG. 5. (Color online) Differential cross section from coupled-cluster calculations divided by Rutherford cross section for elastic proton scattering on $^{40}$Ca at $E_{\text{cm}} = 9.6$ MeV (solid line), experimental data (dots), and optical model potential fits (dashed line), taken from Ref. [31].

FIG. 6. (Color online) Same caption as in Fig. 5 except that the energy is $E_{\text{cm}} = 12.44$ MeV.

- ab-initio coupled cluster calculations
- single nucleon correlations insufficient
reducing the many body to a few body problem

- isolating the important degrees of freedom in a reaction
- keeping track of all relevant channels
- connecting back to the many-body problem

- effective nucleon-nucleus interactions (or nucleus-nucleus)
  (energy dependence/non-local?)
- many body input (often not available)
- reliable solution of the few-body problem
forces present in reactions

- nuclear (strong)
- weak
- electromagnetic
- gravitational
Connection to astrophysics
Coulomb effects in reactions

Astrophysical S-factor

\[ \sigma(E) = \frac{1}{E} e^{-2\pi \eta S(E)} \]

Sommerfeld parameter

\[ \eta = Z_1 Z_2 e^2 / (\hbar v) \]

Fig. 1.4. Dependence of cross section and \( S(E) \) on energy, for the reaction \(^3\text{He}(\alpha, \gamma)^7\text{Be}\). The solid curve is a calculation to be discussed in Appendix B.
reaction rates and the Gamow peak

\[ \langle \sigma v \rangle = \frac{8}{\pi \mu_{12}(k_B T)^3} \int_0^\infty S(E) \exp\left( -\frac{E}{k_B T} - \sqrt{\frac{E_G}{E}} \right) \, dE. \]

\[ E_G = 4\pi^2 \eta^2 E = 2\mu_{12}(\pi Z_1 Z_2 e^2)^2 / \hbar^2 \]

\[ E_0 = \left( \frac{E_G k_B^2 T^2}{4} \right)^{1/3} \]
history of the universe
primordial nucleosynthesis

Q-value for $p(n, g)d$ 2.26 MeV

$T(\text{universe to cool down to } E=2.26)=7$ min

slightly less than $T(\text{free neutron decay})$
reactions in light stars

\[ p + p \rightarrow d + e^+ + \nu \]
\[ p + d \rightarrow ^3\text{He} + \gamma \]

86% \[ ^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + 2p \]

Chain I
\[ Q_{ret} = 26.20 \text{ MeV} \]

14% \[ ^3\text{He} + ^4\text{He} \rightarrow ^7\text{Be} + \gamma \]

99.7% \[ ^7\text{Be} + e^- \rightarrow ^7\text{Li} + \nu \]
\[ ^7\text{Li} + p \rightarrow 2^4\text{He} \]

Chain II
\[ Q_{ret} = 25.66 \text{ MeV} \]

0.3% \[ ^7\text{Be} + p \rightarrow ^8\text{B} + \gamma \]
\[ ^8\text{B} \rightarrow ^8\text{Be} + e^+ + \nu \]

Chain III

Overall result: \[ 4p \rightarrow ^4\text{He} + 2e^+ + 2\nu + Q_{ret} \]
solar neutrino puzzle and nuclear reactions

status in the nineties: measured solar neutrino flux $\ll$ predicted flux

**FIG. 8 (color online).** $S_{17}$ values from CD experiments. Full circles: latest analysis of the GSI CD experiment (Schümann et al., 2006); open stars: Kikuchi et al. (1998) analyzed in first-order perturbation theory; open squares: Davids and Typel (2003). The error bars include statistical and estimated systematic errors. The curve is taken from the cluster-model theory of Descouvemont et al. (2004), normalized to $S_{17}(0) = 20.8$ eV b.

**FIG. 9 (color online).** $S_{17}(E)$ vs center-of-mass energy $E$, for $E \leq 1250$ keV. Data points are shown with total errors, including systematic errors. Dashed line: scaled Descouvemont (2004) curve with $S_{17}(0) = 20.8$ eV b; solid line: including a fitted $1^+$ resonance shape.
triple-alpha reaction

\[
\begin{array}{c}
\alpha \\
\alpha \\
\alpha \\
\downarrow \\
\gamma \\
\gamma \\
\gamma \\
\gamma
\end{array}
\]

\[
\begin{array}{c}
\frac{7.2747}{3\alpha} \\
7.6542 \quad 0^+ \\
4.4389 \quad 2^+ \\
J^\pi = 0, \text{ T = 0}
\end{array}
\]

\[
\begin{array}{c}
7.3666 \\
\alpha + ^8\text{Be}
\end{array}
\]

\[
^12\text{C}
\]
CNO cycles

\[
\begin{align*}
^{12}\text{C} & \xrightarrow{(p,\gamma)} ^{13}\text{N} \xrightarrow{(e^++\nu)} ^{13}\text{N} & & ^{14}\text{N} & \xrightarrow{(p,\gamma)} ^{15}\text{O} \xrightarrow{(e^++\nu)} ^{15}\text{O} \xrightarrow{(p,\gamma)} ^{13}\text{C} \\
^{12}\text{C} & \xrightarrow{(p,\alpha)} ^{15}\text{N} \xrightarrow{(e^++\nu)} ^{13}\text{N} & & ^{16}\text{O} & \xrightarrow{(p,\gamma)} ^{15}\text{N} \xrightarrow{(e^++\nu)} ^{17}\text{O} \xrightarrow{(p,\alpha)} ^{13}\text{C}
\end{align*}
\]
heavier elements
medium mass elements and red giants

- Hydrogen, Helium
- Helium, Nitrogen
- Carbon, Oxygen, Neon
- Iron
- Silicon, Sulfur
- Oxygen, Neon, Sodium
heavy elements and the s-process

- $^{64}\text{Cu}, \text{t}_{1/2}=12 \text{ h}, 40\% (\beta^-), 60\% (\beta^+)$
- $^{80}\text{Br}, \text{t}_{1/2}=17 \text{ min}, 92\% (\beta^-), 8\% (\beta^+)$
- $^{85}\text{Kr}, \text{t}_{1/2}=11 \text{ yr}$
- $^{79}\text{Se}, \text{t}_{1/2}=295 \text{ k yr}$
- $^{63}\text{Ni}, \text{t}_{1/2}=101 \text{ yr}$
heavy elements: elemental abundances
Question 3
How were the heavy elements made?
Where did they come from?
heavy element: r-process in the chart
how do we measure reactions rates?

$^{14}\text{C}(n,\gamma)^{15}\text{C}$ has impact on:

- neutron-induced CNO cycle
- heaviest element in non-homogenous bigbang
- abundancies from the r-process in supernovae
indirect methods: nuclear reactions

- direct measurement \( ^{14}\text{C}(n,\gamma)^{15}\text{C} \)

- transfer reaction \( ^{14}\text{C}(d,\text{p})^{15}\text{C} \)

- Coulomb dissociation

\[ ^{208}\text{Pb} \rightarrow ^{15}\text{C} + ^{14}\text{C} + \gamma \text{ (low relative energy)} \]
breakup reactions and \((n,\gamma)\)

\[ ^{208}\text{Pb}\left(^{15}\text{C},^{14}\text{C}+n\right)^{208}\text{Pb}@68\text{ MeV/u} \]

Nakamura et al, NPA722(2003)301c
Reifarth et al, PRC77,015804 (2008)