Chap 4: Reaction mechanisms
Collective excitations
Coupling potentials

- Central potentials do not couple together different partial wave channels

- Need to generalize interactions!
  - Transition potentials and multipole expansion

\[ H^\lambda_{\text{intr}}(\xi, R) = \sqrt{4\pi} \, F_\lambda(R) \sum_{m=-\lambda}^{\lambda} T^m_\lambda(\xi)^* Y^m_\lambda(\hat{R}) \]

\[ \equiv \sqrt{4\pi} \, F_\lambda(R) \, T_\lambda(\xi) \cdot Y_\lambda(\hat{R}), \]

- Matrix elements of transition potentials: need angular momentum algebra

Wigner-Eckart Theorem:

\[ \langle j_f m_f | \hat{O}_{jm} | j_i m_i \rangle = \frac{\langle j_i m_i, jm | j_f m_f \rangle}{\hat{j}_f} \langle j_f || \hat{O}_j || j_i \rangle \]

Typical matrix elements:

\[ \langle L_f || Y_\lambda || L_i \rangle = (4\pi)^{-1/2} \hat{x} \hat{L}_i \langle L_i 0, \lambda 0 | L_f 0 \rangle \]

- Rank \( \lambda \):
  - Monopole (0)
  - Dipole (1)
  - Quadrupole (2)
Coupling potentials

\[ \langle (L_f I_f) J_{tot} | H_{\text{intr}}^{\lambda} (\vec{R}, R) | (L_i I_i) J_{tot} \rangle = \sqrt{4\pi} \mathcal{F}_{\lambda} (R) \times (-1)^{\lambda + J_{tot} + L_i + I_f} \left\{ \begin{array}{ccc} L_i & I_i & J_{tot} \\ I_f & L_f & \lambda \end{array} \right\} \langle L_f \parallel Y_{\lambda} \parallel L_i \rangle \langle I_f \parallel T_{\lambda} \parallel I_i \rangle \]

- transition potential

\[ V_{f_i}^{\lambda} (R) = \mathcal{F}_{\lambda} (R) \langle I_f \parallel T_{\lambda} \parallel I_i \rangle \]

- form factor

- reduced matrix element (structure)

- matrix elements of transition potentials:

\[ \langle (L_f I_f) J_{tot} | H_{\text{intr}}^{\lambda} (\vec{R}, R) | (L_i I_i) J_{tot} \rangle \]

\[ \equiv V_{f_i}^{\lambda} (R) (-1)^{\lambda + J_{tot} + L_i + I_f} \hat{\lambda} \hat{L}_i \langle L_i 0, \lambda 0 | L_f 0 \rangle \left\{ \begin{array}{ccc} L_i & I_i & J_{tot} \\ I_f & L_f & \lambda \end{array} \right\} \]

\[ \equiv V_{f_i}^{\lambda} (R) (-1)^{\lambda + J_{tot} - I_i - L_f} \hat{\lambda} \hat{L}_i \langle L_i 0, \lambda 0 | L_f 0 \rangle W (L_i I_i L_f I_f ; J_{tot} \lambda) \]

Transition potential

Angular momentum coupling
Vector forces: Spin-orbit couplings

- $\lambda = 1 \quad V_{\text{so}} = \mathcal{F}_{1}^{\text{so}}(R) \ 2 \mathbf{L} \cdot \mathbf{s}$

- matrix elements diagonal in the J-basis

$$\langle (Ls)J | V_{\text{so}} | (Ls)J \rangle = \mathcal{F}_{1}^{\text{so}}(R)[J(J+1) - L(L+1) - s(s+1)]$$

- form-factor

$$\mathcal{F}_{1}^{\text{so}}(R) = \left(\frac{\hbar}{m_\pi c}\right)^2 \int \frac{1}{R} dR \frac{V_{\text{so}}}{1 + \exp\left(\frac{R - R_{so}}{a_{so}}\right)}$$

$$\frac{\hbar^2}{(m_\pi c)^2} = 2.00 \text{ fm}^2$$

- typical strength in nuclear physics

$$V_{\text{so}} = 5-8 \text{ MeV for nucleons}$$
Vector forces: spin-spin couplings

○ $\lambda = 1 \quad V_{ss} = \mathcal{F}_{1}^{ss}(R) I_p \cdot I_t$

○ matrix elements diagonal in the S-basis

$$\langle L(I_p, I_t)S; J_{tot} | V_{ss} | L(I_p, I_t)S; J_{tot} \rangle = \mathcal{F}_{1}^{ss}(R) \frac{1}{2} [S(S+1) - I_p(I_p+1) - I_t(I_t+1)]$$
Tensor forces:

set of rank-2 operators for two $s=1/2$ system

\[
T_r = (s \cdot \hat{R})^2 - \frac{2}{3}
\]

\[
T_p = (s \cdot p)^2 - \frac{2}{3}p^2
\]

\[
T_L = (s \cdot L)^2 - \frac{1}{2}S \cdot L - \frac{2}{3}L^2
\]

- NN forces often parameterized in terms of these operators
Inelastic couplings

- q-value for inelastic excitation
  \[ Q = \epsilon_i - \epsilon_f \]

- change in angular momentum
  \[ |I_i - I_f| \leq \lambda \leq I_i + I_f \]

- normal parity transitions
  - change nuclear parity for odd multipoles

- transition can be caused by
  - Coulomb
  - nuclear
  - both
collective transitions: rotational model

- nucleus is assumed to behave like a rigid rotor
  bandhead is $I=K$ (K projection of I on intrinsic body-fixed frame)

$$\epsilon_I = \frac{\hbar^2}{2\mathcal{M}} [I(I+1) - K(K+1)]$$

$$\mathcal{M}_{\text{rigid}} = \frac{2}{5} m_u A R_0^2$$

eigenstates of a rotor: rotational matrices

$$\phi_{IM} = \frac{\hat{I}}{\sqrt{8\pi^2}} D_{MK}^I(\omega)^* \phi_K$$

Characterized by deformation:
Typical spectra as well as $B(E\lambda)$
Figure 3. A plot of the $0^+_2$ and $0^-_1$ rotational bands including interband E2 and intraband E1 $\gamma$ transitions for thorium nuclei with $N = 130$ through $N = 144$. The gradual upward shift of the $0^-_1$ band relative to $0^+_2$ as $N$ increases beyond 136 and hence the shift from a reflection-asymmetric, alternating-parity band with interweaving E1 $\gamma$ transitions at lower $N$ to an octupole vibrational excited mode at higher $N$ are illustrated. Parentheses around spins represent uncertainty in $J^*$, and dashed energy levels indicate uncertainty in the existence of that level.
collective transitions: rotational model

- if target is deformed, potential projectile-target is also deformed

\[ V(R, \theta', \phi') = U(R - \tilde{R}(\theta', \phi') + R_0) \]

**Multipole expansion of radius**

\[ \tilde{R}(\theta', \phi') = R_0 + \sum_{q=2}^{q_{\text{max}}} \sum_{\mu=-q}^{+q} d_{q\mu} Y_q^\mu(\theta', \phi'), \]

**Expansion for potential (1\textsuperscript{st} order)**

\[ V(R, \theta', \phi') = U(R) - U'(R) \sum_{\lambda=2}^{q_{\text{max}}} \delta_\lambda Y_\lambda^0(\theta', \phi'). \]

def ormation parameter and deformation length

\[ \beta_q = \delta_q / R_0 \]
collective inelastic: nuclear

- in the simplest model for the nuclear force

\[ V_{fi}^\lambda (R) = \mathcal{F}_\lambda (R) \langle I_f | \mathcal{T}_\lambda | I_i \rangle \]

\[ V_{fi}^\lambda (R) = -\frac{\delta_\lambda}{\sqrt{4\pi}} U'(R) \hat{I}_i \langle I_i K, \lambda 0 | I_f K \rangle \]

\[ \langle I_f M_f | Y_\lambda^m (\xi) | I_i M_i \rangle \]

\[ = \frac{\hat{I}_i \hat{I}_f}{8\pi^2} \int d\omega \langle \phi_K | D_{M_f K}^{I_f} (\omega) Y_{\lambda}^{m*} D_{M_i K}^{I_i} (\omega) \phi_K \rangle \]

\[ = \frac{\hat{I}_i \hat{I}_f}{\sqrt{4\pi}} \langle I_i K, \lambda 0 | I_f K \rangle \hat{I}_f^{-1} \langle I_i M_i, \lambda m | I_f M_f \rangle \]

\[ \langle j_f m_f | \hat{O}_{jm} | j_i m_i \rangle = \frac{\langle j_i m_i, jm | j_f m_f \rangle}{\hat{I}_f} \langle I_f | \hat{O}_j | I_i \rangle \]
what about the Coulomb interaction?

\[ V_C(R) = e \int \frac{d\textbf{r}}{|	extbf{R} - \textbf{r}|} \rho_q(\textbf{r}) \]

\[ \frac{1}{|\textbf{R} - \textbf{r}|} = \sum_{\lambda, \mu} \frac{4\pi}{2\lambda + 1} Y^{\lambda \mu}_\lambda(\textbf{r}) \ast Y^{\lambda \mu}_\lambda(\textbf{R}) f(R, r) \]

‘near field’ and ‘far field’ forms

\[ f(R, r) \equiv \begin{cases} \frac{R^\lambda}{r^{\lambda+1}} & \text{for } R \leq r \\ \frac{r^\lambda}{R^{\lambda+1}} & \text{for } R \geq r \end{cases} \]
collective inelastic: Coulomb

- Interaction Hamiltonian

\[
H_{\text{intr}} = eZ_t V_C(R) = \sum_{\lambda, \mu} \frac{4\pi Z_t e^2}{2\lambda + 1} Y_\lambda^\mu(R) \int Y_\lambda^\mu(r)^* f(R, r) \rho_q(r) \, dr
\]

- Coulomb form factor

\[
\mathcal{F}_\lambda(R) T_\lambda^\mu(\xi) = \frac{\sqrt{4\pi Z_t e^2}}{2\lambda + 1} \int Y_\lambda^\mu(r) f(R, r) \rho_q(r) \, dr
\]

- Matrix elements for transitions

\[
V_{fi}^\lambda(R) = \langle I_f \parallel \mathcal{F}_\lambda T_\lambda \parallel I_i \rangle = \frac{\sqrt{4\pi Z_t e^2}}{2\lambda + 1} \langle I_f \parallel Y_\lambda^\mu(r) f(R, r) \rho_q(r) \parallel I_i \rangle
\]

\[
\Rightarrow \quad \frac{\sqrt{4\pi e^2 Z_t}}{2\lambda + 1} \langle I_f \parallel Y_\lambda^\mu(r) r^\lambda \rho_q(r) \parallel I_i \rangle.
\]

\[
\frac{3Z_t \delta_{\lambda, \lambda_c} R_c^{\lambda - 1}}{4\pi} \hat{I}_i \langle I_i K, \lambda 0 | I_f K \rangle
\]

related to B(E\lambda)
Coulomb inelastic couplings

- Coulomb transition potential:

\[ V_{fi}^\lambda (R) \xrightarrow{R \gg R_n} \frac{\langle I_f \parallel E\lambda \parallel I_i \rangle}{R^{\lambda + 1}} \frac{\sqrt{4\pi} e^2 Z_t}{2\lambda + 1} \]

- Reduced transition probability:

\[ B(E\lambda, I_i \rightarrow I_f) = \frac{1}{2I_i + 1} |\langle I_f \parallel E\lambda \parallel I_i \rangle|^2 \]

- Symmetry property:

\[ \langle I_f \parallel E\lambda \parallel I_i \rangle = \langle I_i \parallel E\lambda \parallel I_f \rangle \]

\[ (2I_i + 1) B(E\lambda, I_i \rightarrow I_f) = (2I_f + 1) B(E\lambda, I_f \rightarrow I_i) \]
Chap 4: Reaction mechanisms
Single particle, transfer, and others
Binding potentials

- introducing single particle degrees of freedom
- quantum numbers $n\ell j$ or $nj^{\pi}$

$$\phi_{\ell sj;b}^m(r) = [Y_{\ell}(\hat{r}) \otimes X_s]_{jm} u_{\ell sj;b}(r)/r$$

- boundary conditions

$$u_{\ell sj;b}(0) = 0$$

$$\left. u_{\ell sj;b}(r) \right|_{r > R_n} = C_{\ell} W_{-\eta, \ell + \frac{1}{2}}(-2k_{1r}) \rightarrow r \gg \rho_p, \quad C_{\ell} e^{-k_{1r}} \rightarrow r \rightarrow \infty 0$$

- normalization of bound states

$$\int |\phi_{\ell sj;b}^m(r)|^2 dr = \int_0^\infty |u_{\ell sj;b}(r)|^2 dr = 1$$
bound states have \( k = ik_l \) with imaginary part \( k_l > 0 \)

\[
k = \sqrt{-2\mu E/\hbar^2}
\]

\[
\eta_l = -Z_1 Z_2 e^2 \mu / (\hbar^2 k_l)
\]

It is therefore common to use a *Whittaker function* that is \( H^+ \) rescaled as

\[
W_{i\eta, \ell+\frac{1}{2}} (2i\rho) = e^{-\pi \eta/2} e^{-i(\pi \ell/2 + \sigma_L(\eta))} H_{\ell}^+ (\eta, \rho).
\]

(4.2.7)

Such a Whittaker function, from Eq. (3.1.69), has asymptotic form

\[
W_{i\eta, \ell+\frac{1}{2}} (2i\rho) \rightarrow \rho \gg \rho_{tp} \quad e^{i\rho - \pi \eta/2 - i\eta \ln(2\rho)},
\]

(4.2.8)

and, for bound states with \( \rho = ik_l r \), this becomes

\[
W_{-\eta_l, \ell+\frac{1}{2}} (-2k_l r) \rightarrow \rho \gg \rho_{tp} \quad e^{-k_l r + \eta_l \ln(2k_l r)}.
\]

(4.2.9)

For neutral particles this is the familiar exponential decay, and for charged particles the Whittaker function has a logarithmic variation arising from the Coulomb potential.
Neutron single particle states

Fig. 4.1. Neutron eigenstates $n j^\pi$ in a Saxon-Woods binding potential, for spin $j$, parity $\pi$ and number of radial nodes $n$. The chosen potential has $V_r = 56.6$ MeV, $r_r = 1.17$ fm, $a_r = 0.75$ fm, $V_{so} = 6.2$ MeV, $r_{so} = 1.01$ fm and $a_{so} = 0.58$ fm. Both radii scale with $A$ as $R = raA^{1/3}$. The occupation numbers are $n_{occ} = 2j+1$. 
Binding potentials

- important concepts:
  - mean field (no correlations)
  - fermi level $E_F$ (energy of highest occupied level)
    - pickup reactions $(p,d)$ – study occupied levels
    - “stripping” reactions $(d,p)$ – study of unoccupied levels
  - fragmentation of strength (correlations)
Binding potentials - continuum

- when binding potential is too weak it may produce a state with $E>0$
  - resonance or virtual state?
  - resonant continuum and non-resonant continuum?

\[
\phi_{\ell sj}^m (r; k) = [Y_\ell (\hat{r}) \otimes X_s]_{jm} u_{\ell sj} (r; k) / r
\]

\[
u_{\ell sj} (r; k) \rightarrow \frac{i}{2} [H^- - \mathbf{S} H^+ ]
\]

- orthogonality of continuum states

\[
\int_0^\infty u_{\ell sj} (r; k)^* u_{\ell sj} (r; k') dr = \frac{\pi}{2} \delta (k - k')
\]

- completeness relation

\[
\sum_b |u_{\ell sj; b} (r) \rangle \langle u_{\ell sj; b} (r')| + \int_0^\infty dk |\hat{u}_{\ell sj} (r; k) \rangle \langle \hat{u}_{\ell sj} (r'; k)| = \delta (r - r')
\]
Binding potentials and Optical potentials

- states above and below the fermi level are connected
- absorptive component has to be zero around fermi level but increase away from it

Diagram:

- Diagrams showing different potential distributions for surface and volume.
- Notations like $A_s$, $B_s$, $C_s$, $B_{v1}$, and $B_{v2}$ are used.
- Energy scale $E-E_F$ is shown.

Reference: Dickhoff et al.
Binding potentials and Optical potentials

A-1 system
Binding potentials and Optical potentials

A+1 system
single particle excitations

coupling potential for single particle transitions

\[
V_{f\ell}(R) = \langle (L_f J_f) \Lambda | V_{vt} + V_{ct} | (L_i J_i) \Lambda \rangle
\]

Relevant coordinates

\[
\begin{align*}
\mathbf{r}_{vt} &= \mathbf{R} + \frac{m_c}{m_p} \mathbf{r} \\
\mathbf{r}_{ct} &= \mathbf{R} - \frac{m_v}{m_p} \mathbf{r},
\end{align*}
\]

With \( V_{vt} \) and \( V_{ct} \) depending only on the length square roots of

\[
\begin{align*}
r_{vt}^2 &= R^2 + \frac{m_c^2}{m_p^2} r^2 + \frac{2m_c}{m_p} R r z \\
r_{ct}^2 &= R^2 + \frac{m_v^2}{m_p^2} r^2 - \frac{2m_v}{m_p} R r z,
\end{align*}
\]

\[
z = \hat{R} \cdot \hat{r}
\]
single particle excitations

coupling potential for single particle transitions

$$V_{fi}(R) = \langle (L_f J_f) \Lambda | V_{vt} + V_{ct} | (L_i J_i) \Lambda \rangle$$

Two-coordinate multipole expansion

$$V_{vt} + V_{ct} = \sum_{\lambda} (2\lambda + 1) F_{\lambda}(R, r) P_{\lambda}(z)$$

$$= \sum_{\lambda} (2\lambda + 1) F_{\lambda}(R, r) \sum_{m} Y_{\lambda}^m(\hat{r})^* Y_{\lambda}^m(\hat{R})$$

Two-coordinate form factors

$$F_{\lambda}(R, r) = \frac{1}{2} \int_{-1}^{+1} [V_{vt}(r_{vt}) + V_{ct}(r_{ct})] P_{\lambda}(z) dz$$

$$z = \hat{R} \cdot \hat{r}$$
single particle excitation

- When we sum $V_{ct}$ and $V_{xt}$ we still get a function of $(R,r)$
- Need to integrate over $r$ to obtain the coupling potential $V(R)$

If we use our explicit expression for the bound states

$$V_{ji}^\lambda (R) = \int_0^\infty u_{\ell_i s_j} (r) \mathcal{F}_\lambda (R, r) u_{\ell_j s_i} (r) \, dr \, \hat{\lambda} \, \hat{\ell}_i \, \langle \ell_i 0, \lambda 0 | \ell_f 0 \rangle$$
one nucleon transfer: coordinates

\[ r = pR' + qR \text{ and } r' = p'R' + q'R' \]

Initial and final bound states:

\[ [H_p - \varepsilon_p] \phi_p(r) = 0 \text{ where } H_p = T_r + V_p(r) \]

\[ [H_t - \varepsilon_t] \phi_t(r') = 0 \text{ where } H_t = T_{r'} + V_t(r'). \]

\[ Q = \varepsilon_p - \varepsilon_t \]
one nucleon transfer: operator

\[ H = T_r + T_R + V_p(r) + V_t(r') + U_{c'c}(R_c), \]

\[ T_r + T_R = T_{r'} + T_{R'} \]

\[ H = H_{\text{prior}} = T_R + U_i(R) + H_p(r) + V_i(R, r) \]

\[ = H_{\text{post}} = T_{R'} + U_f(R') + H_t(r') + V_f(R', r'), \]

\[ V_i(R, r) = V_t(r') + U_{c'c}(R_c) - U_i(R) \]

or \[ V_f(R', r') = V_p(r) + U_{c'c}(R_c) - U_f(R'). \]
one nucleon transfer: amplitude

\[ V_i(R, r) = V_t(r') + U_{c'c}(R_c) - U_i(R) \]

or \[ V_f(R', r') = V_p(r) + U_{c'c}(R_c) - U_f(R'). \]
One nucleon transfer kernel

Include multiple partitions in the wavefunction

\[
\Psi = \left[ \phi_p(\mathbf{r}) \otimes Y_L(\mathbf{R}) \right]_\Lambda \psi_i(R)/R + \left[ \phi_t(\mathbf{r}') \otimes Y_{L'}(\mathbf{R}') \right]_\Lambda \psi_f(R')/R'
\]

Need to calculate matrix elements of this form (\(o=i/f\))

\[
V_{fi}^o(R', R) = R' \langle [\phi_t(\mathbf{r}') \otimes Y_{L'}(\mathbf{R}')] | \mathcal{V}_o | [\phi_p(\mathbf{r}) \otimes Y_L(\mathbf{R})] \rangle \Lambda R^{-1}
\]

This is non-local integral operator – hard to handle in differential eqns

Can use integral forms!

\[
\Omega_f(R') = \int V_{fi}^o(R', R) \psi_i(R) dR
\]

If bound states are real, then kernel is symmetric

\[
V_{fi}^o(R', R) = V_{if}^o(R, R')
\]
Transfer matrix elements: finite range

Evaluating matrix elements in not trivial!

\[ V_{\ell L'; \ell L}^{\Lambda o} (R', R) = (-1)^{L+L'} \hat{L} \hat{L'} \sum_{TKK'} \begin{pmatrix} K & L & T \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} K' & L' & T \\ 0 & 0 & 0 \end{pmatrix} \]

\[ \times \sum_{\lambda} W(\ell L \ell' L'; \Lambda \lambda) W(KLK' L'; T \lambda) \mathcal{F}_{\lambda, K'KT}^{\ell \ell' \Lambda o} (R', R) \]

○ non local form factor

\[ \mathcal{F}_{\lambda, K'KT}^{\ell \ell' \Lambda o} (R', R) = \frac{|q|^3}{2} \sum_{nn'} RR' (pR')^{\ell-n} (qR)^n (p'R')^{\ell'-n'} (q'R)^{n'} \]

\[ \times (2T+1)(-1)^{\Lambda+T} \hat{\ell} \hat{\ell'} (\ell-n) (\ell'-n') \hat{n'} (2K+1)(2K'+1) \]

\[ \times c(\ell, n)c(\ell', n') \begin{pmatrix} \ell-n & n' & K \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell'-n' & n & K' \\ 0 & 0 & 0 \end{pmatrix} \]

\[ \times (2\lambda+1) \left\{ \begin{array}{ccc} \ell' & \lambda & \ell \\ n' & K & \ell-n \\ \ell'-n' & K' & n \end{array} \right\} q_{\ell', \ell}^{T, o} (R', R). \]

\[ q_{\ell', \ell}^{T, o} (R', R) = \frac{1}{2} \int_{-1}^{+1} \mathcal{V}_o \frac{u_{\ell'}(r')}{r'^{\ell'+1}} \frac{u_{\ell}(r)}{r^{\ell+1}} P_T(z) dz \]
Transfer in the zero-range approximation

Zero range approximation for c+v

\[ V_0 \phi(r) \sim D_0 \delta(r) \]

Radial transfer kernel \( V(R', R) \) simplifies due to delta to 1D

\[
V_{\ell' L':0L}^{L}(R', R) = D_0 \frac{(-1)^{L'-\ell'}}{\hat{L}} \frac{\hat{\ell}' \hat{L}' \hat{L}}{\sqrt{4\pi}} \begin{pmatrix} \ell' & L & L' \\ 0 & 0 & 0 \end{pmatrix} \\
\times \frac{1}{R'} u_{\ell'}(R') \frac{q^2}{p} \delta(pR' + qR).
\]

used for s-shell nuclei: d, t, α
If projectile is s-wave and potential’s range is small, a simple correction to zero-range can be made, improving the accuracy.

Rate of oscillation of the incoming wave within a finite range effective radius $R_{\text{eff}}$ can be estimated from:

$$K_i(R)^2 = \frac{2\mu_p}{\hbar^2} (E_i - U_i)$$

Result of Buttle and Goldfarb’s derivation – b.s. needs to be multiplied by:

$$\left[ 1 + \rho_{\text{eff}}^2 \frac{2\mu_p}{\hbar^2} (U_{cc}(R') + V_{\ell'}(R') - U_f(R') + \varepsilon_p) \right]$$

if you don’t care about the details of the potentials (sub-Coulomb reactions):

$$V_0\phi(r) \sim D_0\delta(r)$$

$$D = (1 + k_p^2 \rho_{\text{eff}}^2)D_0$$
Transfer: D and $D_0$ constants

Zero-range constant:

$$D_0 = \sqrt{4\pi} \int_0^\infty r V_0(r) u_0(r) \, dr.$$  

$D$ reflects the asymptotic strength of the wfn:

$$u_0(r) = r \to \infty \frac{2\mu_p}{\hbar^2} \frac{1}{\sqrt{4\pi}} \text{De}^{-k_p r}.$$  

$$D = \sqrt{4\pi} \int_0^\infty \frac{\sinh(k_p r)}{k_p} V_0(r) u_0(r) \, dr.$$  

as the range of potential goes to zero $D$ goes to $D_0$

$$D = (1 + k_p^2 \rho_{\text{eff}}^2) D_0.$$
Asymptotic normalization coefficients

At large distances a bound state behaves as a Whittaker function:

\[ u_\ell(r) \equiv_{r \to R_n} C_\ell \, W_{-\eta, \ell + \frac{1}{2}}(-2k_p r) \approx C_\ell \, e^{-k_p r} \]

- we also defined the asymptotics in terms of \( D \)

\[ u_0(r) = \lim_{r \to \infty} \frac{2\mu_p}{\hbar^2} \frac{1}{\sqrt{4\pi}} \, De^{-k_p r} \]

\[ C_\ell = \frac{2\mu_p}{\hbar^2} \frac{1}{\sqrt{4\pi}} \, D \]

- ANC corresponds to the strength of the asymptotic tail of b.s. Influences directly all peripheral reactions!
Knockout reactions

Several types:
- removal of a nucleon \( A(B,X)A-1 \) with residual not detected
- quasi-elastic knockout \(^{12}\text{C}(p,2p)^{11}\text{B}\) (high energy)
- knockout of cluster which is replaced by projectile (transfer) \(^{14}\text{C}(p,\alpha)^{11}\text{B}\)

Most common at low energy: knockout-transfer \(^{14}\text{C}(p,\alpha)^{11}\text{B}\)

Triton transfer
\[
\left(^{14}\text{C} = ^{11}\text{B} + ^{3}\text{H}\right) + p \rightarrow ^{11}\text{B} + (\alpha = p + ^{3}\text{H})
\]

Heavy ion transfer
\[
\left(^{14}\text{C} = ^{10}\text{Be} + \alpha\right) + p \rightarrow \alpha + (^{11}\text{B} = p + ^{10}\text{Be})
\]

Amplitude needs to be permuted.

Interaction terms in prior and post:

\[
prior \quad \mathcal{V}_i = V_{p-\text{Be}} + U_{p\alpha} - U_{pC}
\]
\[
post \quad \mathcal{V}_f = V_{\alpha-\text{Be}} + U_{p\alpha} - U_{\alpha B}
\]
Breakup reactions

- single particle excitations into the continuum
  - transition from bound state to continuum state
  - contain complete set of single particle states

![Diagram showing single particle states and breakup reactions]

- $\ell$s$j$ transfer mechanism can populate continuum too
Isospin transitions

$^{12}\text{C}(p,n)^{12}\text{N}$

- heavy-particle transfer of $^{11}\text{C}$ from $^{12}\text{C} = ^{11}\text{C} + ^{1}\text{n}$ to $^{12}\text{N} = ^{11}\text{C} + ^{1}\text{p}$, as discussed in subsection 4.5.2,
- two-step transfers via a $^{11}\text{C} + ^{1}\text{d}$ intermediate state, the first transfer adding a neutron to the projectile and the second step removing a proton, and
- direct conversion of a proton to a neutron, for example by a meson $\pi^+$ being emitted from the proton, and absorbed by the $^{12}\text{C}$ where it changes a neutron there into a proton.

- the optical potential for scattering of protons+nucleus contains $t \cdot T$ term!

$$V(R) = V_0(R) + \frac{t \cdot T}{A} V_T(R)$$
Charge exchange reactions

- Fermi transitions – populates isobaric analogue states

\[ t \cdot T = t_x T_x + t_y T_y + t_z T_z \]

\[ = \frac{1}{2} [t_+ T_- + t_- T_+] + t_z T_z \]

\[ T_{\pm}|T, T_z\rangle = \sqrt{T(T+1) - T_z(T_z \pm 1)}|T, T_z \pm 1\rangle \]

\[ H_F = V_F(R)t \cdot T \]
Charge exchange reactions

- Gamow-Teller transitions
  - populates many spin states over a wide range of energies

\[ H_{GT} = V_{GT}(R) \ (s \cdot S) \ (t \cdot T) \]

- GT single-particle reduced matrix elements for n->p transitions
  \[
  \langle u_{\ell' s j'}(r) p \ | \ s t^- \ | \ u_{\ell s j}(r) n \rangle \\
  = \frac{1}{2} \langle u_{\ell' s j'}(r) \ | \ s \ | \ u_{\ell s j}(r) \rangle \\
  = \frac{1}{2} \int_0^\infty u_{\ell' s j'}(r)^* u_{\ell s j}(r) dr \\
  \times \hat{j j'} (-1)^{\ell+s+j'+1} \left\{ \begin{array}{ccc} \ell & s & j' \\ 1 & j & s \end{array} \right\} \langle s \ | \ s \ | \ s \rangle \delta_{\ell \ell'}
  \]

- Measures the spatial overlap between initial and final states

\[
\langle s \ | \ s \ | \ s \rangle = \sqrt{s(s+1)(2s+1)}
\]
Generalized multipole transitions

- NN interactions have many spatial-spin-isospin-transition components:

\[ H_{LSJT} = V_{LSJT}(R) \left[ Y_L(\hat{R}) \otimes T_S(s,S) \right] T_T(t,T) \]

- S=1 spin-flip
- T=0 isoscalar; T=1 isovector
- L=0 monopole; L=1 dipole; L=2 quadrupole
Capture reactions

- scattered wave at large distances, tunnels through barrier
- penetrability factor

- how can we trap particles permanently?
- after barrier penetration, gamma emission to bound state (direct capture)
- final nucleus pushed up to an excited state (doorway resonances - semidirect)
- capture to a long living resonance of the compound nucleus (resonant capture)
dσ_{el}/dθ always forward peaked! Remember ratio to rutherford...

Coulomb only – above and below the barrier
  - what is the Coulomb barrier for your case?

Optical potential is energy dependent
  - how does it vary from 5 to 50 MeV?

Relation with W and flux removal from elastic:
  - should look at integrated cross sections

Relation of R_r with diffraction pattern

Relation of R_i and L_{max} of absorption S(L)

  - what are reasonable parameter variations?
  - are the results converged? Needs to be checked per energy
  - inputs used? Optical potentials
  - what were the difficulties in the analysis?