Transfer spectroscopy
Knockout spectroscopy
Breakup Spectroscopy
Charge exchange spectroscopy
DWBA 1st order

\[ \mathcal{V}_i(R, r) = V_t(r') + U_{c'c}(R_c) - U_i(R) \]

or

\[ \mathcal{V}_f(R', r') = V_p(r) + U_{c'c}(R_c) - U_f(R'). \]

**binding potential**

**remnant term**

\[ T_{fi}^{\text{DWBA}} = \langle \chi_f^{(-)}(R_f) \Phi_{I_A:I_B}(r_f) | \mathcal{V} | \Phi_{I_b:I_a}(r_i) \chi_i(R_i) \rangle \]

**Fig. 14.1.** Coordinates in the entrance and exit partitions for the transfer \( A(a, b)B \) where \( a = b + v \) and \( B = A + v \).
Standard approach to transfer
\[ d + A \rightarrow p + B \]

Overlap functions
\[ I_{AB}(r) = \langle \psi_A | \psi_B \rangle \]
\[ I_{pd}(r') = \langle \psi_p | \psi_d \rangle \]

Spectroscopic factor
\[ S_{\ell j} = N \langle I_{AB} | I_{AB} \rangle \]

DWBA transition matrix element
\[ M = \langle \chi_f^{(-)} I_{AB} | \Delta V | I_{pd} \chi_i^{(+)} \rangle \]
\[ \Delta V = V_{np} + U_{pA} - U_{pB} \quad \text{(post)} \]

Experimental xs related to DWBA xs
\[ \frac{d\sigma^{\exp}}{d\Omega} = S^{\exp} \frac{d\sigma^{\text{DWBA}}}{d\Omega} \]

- extract optical potentials from elastic
- Calculate single particle orbitals with standard \((r,a)\)
- Compute diff. Cross section and if all well extract \(S^{\exp}\)
Standard method

Elastic scattering of deuteron by $^{40}$Ca

Table III. Parameters for potentials of $Z$ type which give minimum $\chi^2$ at each energy.

<table>
<thead>
<tr>
<th>$E$ (MeV)</th>
<th>$V$ (MeV)</th>
<th>$\sigma_0$ (F)</th>
<th>$a$ (F)</th>
<th>$W_D$ (MeV)</th>
<th>$\sigma_0'$ (F)</th>
<th>$a'$ (F)</th>
<th>$\sigma_A$ (mb)</th>
<th>$\chi^2/N^{1/2}$</th>
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</thead>
<tbody>
<tr>
<td>7</td>
<td>145.1</td>
<td>0.803</td>
<td>0.987</td>
<td>9.6</td>
<td>1.718</td>
<td>0.578</td>
<td>1165</td>
<td>0.894</td>
</tr>
<tr>
<td>7</td>
<td>145.4</td>
<td>0.798</td>
<td>0.977</td>
<td>9.4</td>
<td>1.724</td>
<td>0.595</td>
<td>1175</td>
<td>0.890</td>
</tr>
<tr>
<td>7</td>
<td>140.4</td>
<td>0.832</td>
<td>0.973</td>
<td>9.0</td>
<td>1.723</td>
<td>0.591</td>
<td>1161</td>
<td>0.854</td>
</tr>
<tr>
<td>8</td>
<td>109.4</td>
<td>1.011</td>
<td>0.977</td>
<td>24.4</td>
<td>1.658</td>
<td>0.343</td>
<td>1027</td>
<td>1.794</td>
</tr>
<tr>
<td>8</td>
<td>124.2</td>
<td>0.908</td>
<td>1.007</td>
<td>21.8</td>
<td>1.634</td>
<td>0.371</td>
<td>1045</td>
<td>1.852</td>
</tr>
<tr>
<td>8</td>
<td>118.9</td>
<td>0.949</td>
<td>0.993</td>
<td>20.6</td>
<td>1.657</td>
<td>0.372</td>
<td>1042</td>
<td>1.694</td>
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<tr>
<td>8</td>
<td>114.3</td>
<td>0.974</td>
<td>0.932</td>
<td>17.1</td>
<td>1.611</td>
<td>0.453</td>
<td>1143</td>
<td>2.342</td>
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<tr>
<td>8</td>
<td>119.9</td>
<td>0.945</td>
<td>0.929</td>
<td>17.0</td>
<td>1.599</td>
<td>0.452</td>
<td>1127</td>
<td>2.434</td>
</tr>
<tr>
<td>8</td>
<td>121.1</td>
<td>0.937</td>
<td>0.943</td>
<td>14.8</td>
<td>1.624</td>
<td>0.474</td>
<td>1160</td>
<td>2.342</td>
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<tr>
<td>8</td>
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<td>0.924</td>
<td>0.920</td>
<td>15.4</td>
<td>1.559</td>
<td>0.498</td>
<td>1189</td>
<td>2.878</td>
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<td>8</td>
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<td>0.905</td>
<td>16.1</td>
<td>1.545</td>
<td>0.483</td>
<td>1161</td>
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<tr>
<td>8</td>
<td>134.0</td>
<td>0.876</td>
<td>0.927</td>
<td>13.3</td>
<td>1.562</td>
<td>0.520</td>
<td>1194</td>
<td>2.706</td>
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<tr>
<td>11</td>
<td>120.7</td>
<td>0.966</td>
<td>0.846</td>
<td>16.4</td>
<td>1.479</td>
<td>0.492</td>
<td>1133</td>
<td>2.205</td>
</tr>
<tr>
<td>11</td>
<td>120.8</td>
<td>0.964</td>
<td>0.841</td>
<td>16.5</td>
<td>1.476</td>
<td>0.489</td>
<td>1128</td>
<td>2.128</td>
</tr>
<tr>
<td>11</td>
<td>122.2</td>
<td>0.960</td>
<td>0.836</td>
<td>16.0</td>
<td>1.484</td>
<td>0.521</td>
<td>1147</td>
<td>1.968</td>
</tr>
<tr>
<td>12</td>
<td>122.8</td>
<td>1.021</td>
<td>0.846</td>
<td>19.8</td>
<td>1.471</td>
<td>0.444</td>
<td>1144</td>
<td>3.112</td>
</tr>
<tr>
<td>12</td>
<td>111.6</td>
<td>1.029</td>
<td>0.840</td>
<td>20.8</td>
<td>1.466</td>
<td>0.429</td>
<td>1132</td>
<td>2.982</td>
</tr>
<tr>
<td>12</td>
<td>110.4</td>
<td>1.036</td>
<td>0.840</td>
<td>17.1</td>
<td>1.485</td>
<td>0.462</td>
<td>1169</td>
<td>2.706</td>
</tr>
</tbody>
</table>

* Polarization potential with $\alpha=0.52$ included.

* Vector spin-orbit coupling with $V_s=5$ MeV, $W_s=0$, included.
Table III. Spectroscopic factors.

<table>
<thead>
<tr>
<th>$Q$ (MeV)</th>
<th>Quantity</th>
<th>7 MeV</th>
<th>8 MeV</th>
<th>9 MeV</th>
<th>10 MeV</th>
<th>11 MeV</th>
<th>12 MeV</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.14</td>
<td>Peak (mb/sr)</td>
<td>4.2</td>
<td>4.4</td>
<td>5.15</td>
<td>5.37</td>
<td>6.55</td>
<td>5.65</td>
<td>0.91±0.03</td>
</tr>
<tr>
<td></td>
<td>$S$ (av $Z$)$^a$</td>
<td>0.928</td>
<td>0.866</td>
<td>0.925</td>
<td>0.894</td>
<td>0.943</td>
<td>0.876</td>
<td>0.87±0.07</td>
</tr>
<tr>
<td></td>
<td>$S$ (best $Z$)$^a$</td>
<td>0.742</td>
<td>0.934</td>
<td>0.891</td>
<td>0.831</td>
<td>0.957</td>
<td>0.832</td>
<td>0.86±0.07</td>
</tr>
<tr>
<td></td>
<td>$S$ (best $ZS$)$^b$</td>
<td>0.813</td>
<td>0.888</td>
<td>0.901</td>
<td>0.856</td>
<td>0.959</td>
<td>0.756</td>
<td>0.86±0.07</td>
</tr>
<tr>
<td></td>
<td>$S$ (av $Z$)$^{a,c}$</td>
<td>1.54</td>
<td>1.52</td>
<td>1.70</td>
<td>1.68</td>
<td>1.99</td>
<td>1.63</td>
<td>1.67±0.16</td>
</tr>
</tbody>
</table>

$^a$ Zero-range approximation without spin-orbit coupling. $^b$ Finite-range approximation with spin-orbit coupling. $^c$ Radial cutoff at 4.1 F.
Fig. 14.2. Transfer cross sections for different $Q$-values for $^{12}$C(d,p)$^{13}$C at 20 MeV. The $Q$-value is varied arbitrarily.
Q-value dependence

Fig. 14.2. Transfer cross sections for different $Q$-values for $^{12}$C(d,p)$^{13}$C at 20 MeV. The $Q$-value is varied arbitrarily.
Angular momentum dependence

\[ T_{fi}^{\text{DWBA}} = \langle \chi_f^{(-)}(R_f) \Phi_{I_A: I_B}(r_f) | V | \Phi_{I_b: I_d}(r_i) \chi_i(R_i) \rangle \]

In zero-range approximation

\[ T_{fi}^{\text{PWBA}} = D_0 \int e^{i q \cdot R} \Phi_{I_A: I_B}(R) dR \]

\[ = \sum_{l=0}^{\infty} i^l (2l + 1) \int F_l(0, qR)/(qR) P_l(\cos \theta) \Phi_{I_A: I_B}(R) dR. \]
Fig. 14.3. Dependence of the transfer angular distribution on the transferred angular momentum for $^{58}\text{Ni}(d,p)^{59}\text{Ni}$ at 8 MeV, with data from [2]. Reprinted from [3], with permission.
Angular momentum dependence

Fig. 14.3. Dependence of the transfer angular distribution on the transferred angular momentum for $^{58}\text{Ni}(d,p)^{59}\text{Ni}$ at 8 MeV, with data from [2]. Reprinted from [3], with permission.
Mapping occupation numbers for Zn isotopes

Fig. 2. Excitation energies and the values of $J^+$ or $I$ in the final nuclei. Uncertainties are about $\pm 5$ keV for the lower excitation energies and about $\pm 10$ keV for the higher. The 0.888-MeV level of Zn$^{67}$ with $J^+ = \frac{3}{2}^-$ is taken from Ref. 16. (For details, see the Discussion.)
(d,p) L-dependence

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### Mapping occupation numbers for Zn isotopes

**Table VI.** Distorted-wave Born approximation DWBA parameters used in the calculations.

<table>
<thead>
<tr>
<th>Particle</th>
<th>( V_s ) (MeV)</th>
<th>( W ) (MeV)</th>
<th>( r_{qs} ) (F)</th>
<th>( a_s ) (F)</th>
<th>( r_{qf} ) (F)</th>
<th>( a_f ) (F)</th>
<th>( r_{qe} ) (F)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d )</td>
<td>94.0</td>
<td>19.0</td>
<td>1.15</td>
<td>0.81</td>
<td>1.34</td>
<td>0.68</td>
<td>1.30</td>
<td>12</td>
</tr>
<tr>
<td>( p )</td>
<td>50.5 (Zn(^{67}))</td>
<td>13.4</td>
<td>1.25</td>
<td>0.65</td>
<td>1.25</td>
<td>0.47</td>
<td>1.30</td>
<td>13</td>
</tr>
</tbody>
</table>

**Table IV.** Summary of results from \( {\text{Zn}}^{48}(d,p) \) reactions.

<table>
<thead>
<tr>
<th>( E_x ) (MeV)</th>
<th>( l )</th>
<th>( J^* )</th>
<th>( \sigma_{\text{max}} ) (mb/sr)</th>
<th>( (2J+1)S ) (Ref. 11, 12)</th>
<th>( (2J+1)S ) (Ref. 11, 13)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1</td>
<td>( \frac{1}{2}^- )</td>
<td>4.1</td>
<td>0.80</td>
<td>1.10</td>
</tr>
<tr>
<td>0.438</td>
<td>4</td>
<td>( \frac{3}{2}^+ )</td>
<td>1.2</td>
<td>5.5</td>
<td>9.0</td>
</tr>
<tr>
<td>0.531</td>
<td>3</td>
<td>( \frac{3}{2}^- )</td>
<td>0.42</td>
<td>0.97</td>
<td>1.17</td>
</tr>
<tr>
<td>0.853</td>
<td>1</td>
<td>( \frac{1}{2}^- )</td>
<td>2.2</td>
<td>0.37</td>
<td>0.53</td>
</tr>
<tr>
<td>0.872</td>
<td>2</td>
<td>( \frac{3}{2}^+ )</td>
<td>2.5</td>
<td>0.72</td>
<td>0.99</td>
</tr>
<tr>
<td>1.634</td>
<td>2</td>
<td>( \frac{5}{2}^+ )</td>
<td>1.8</td>
<td>0.45</td>
<td>0.61</td>
</tr>
<tr>
<td>1.696</td>
<td>0</td>
<td>( \frac{1}{2}^+ )</td>
<td>6.3(^{b})</td>
<td>0.19</td>
<td>0.21</td>
</tr>
<tr>
<td>1.831</td>
<td>1</td>
<td>( \frac{3}{2}^- )</td>
<td>0.25</td>
<td>0.04</td>
<td>0.05</td>
</tr>
<tr>
<td>1.937</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
</tr>
<tr>
<td>1.968</td>
<td>1</td>
<td>( \frac{1}{2}^- )</td>
<td>0.15</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td>2.262</td>
<td>0</td>
<td>( \frac{1}{2}^+ )</td>
<td>1.0(^{b})</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>2.293</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
</tr>
<tr>
<td>2.403</td>
<td>2</td>
<td>( \frac{3}{2}^+ )</td>
<td>1.5</td>
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<td>0.43</td>
</tr>
<tr>
<td>2.554(^{a})</td>
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<td>( \frac{1}{2}^+ )</td>
<td>(0.35)</td>
<td>(0.07)</td>
<td>(0.1)</td>
</tr>
<tr>
<td>2.676(^{a})</td>
<td>(1)</td>
<td>( \frac{1}{2}^-, \frac{3}{2}^- )</td>
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<td>(0.07)</td>
<td>(0.1)</td>
</tr>
<tr>
<td>2.669</td>
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<td>( \frac{1}{2}^+ )</td>
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<td>0.16</td>
</tr>
<tr>
<td>2.740</td>
<td>(2)</td>
<td>( \frac{3}{2}^- )</td>
<td>( \ldots )</td>
<td>( \sim -0.02 )</td>
<td>( \sim -0.03 )</td>
</tr>
<tr>
<td>2.837</td>
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<td>( \frac{1}{2}^+ )</td>
<td>1.0(^{b})</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>2.912</td>
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<td>( \frac{3}{2}^+ )</td>
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<td>0.18</td>
</tr>
<tr>
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<tr>
<td>3.025</td>
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</tr>
<tr>
<td>3.063(^f)</td>
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<td>( \frac{1}{2}^+ )</td>
<td>1.0(^{b})</td>
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<td>0.03</td>
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<tr>
<td>(3.193)(^f)</td>
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<td>( \frac{3}{2}^+ )</td>
<td>0.18</td>
<td>(0.04)</td>
<td>(0.05)</td>
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<tr>
<td>3.352(^a)</td>
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<td>( \frac{5}{2}^+ )</td>
<td>(0.18)</td>
<td>(0.03)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>(1)</td>
<td>( \frac{1}{2}^-, \frac{3}{2}^- )</td>
<td>(0.18)</td>
<td>(0.02)</td>
<td>(0.03)</td>
<td></td>
</tr>
<tr>
<td>3.393(^f)</td>
<td>0</td>
<td>( \frac{1}{2}^+ )</td>
<td>4.2(^{b})</td>
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<td>0.15</td>
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<tr>
<td>3.426(^f)</td>
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<td>0.06(^{b})</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>3.457</td>
<td>(2)(^d)</td>
<td>( \frac{3}{2}^- )</td>
<td>( \sim -1.1 )</td>
<td>(0.2)</td>
<td>(0.25)</td>
</tr>
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</table>

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Single particle states with high l

FIG. 1. The upper part of the figure is a schematic level diagram of single-particle states, arising from different oscillator shells. Pairs of states with highest angular momentum \( l \) in each shell are labeled to emphasize the proximity of a high-\((\ell + 1) + \frac{1}{2}\) intruder state to the \( \ell - \frac{1}{2} \) state from the lower shell. The lower part of the figure shows the neutron-excess dependence of the binding energy of the last proton in \( Z = 51 \) nuclei. The stars correspond to the 11/2\(^-\) states and the circles to the 7/2\(^+\) states. The open symbols designate states where spin assignments have been made but there is no information from transfer reactions about the single-particle character of the states. The points for \(^{133}\)Sb are from \([5]\) and those in parentheses are unpublished \([6]\).

FIG. 2 (color online). The upper part of the figure shows a triton spectrum arising from the \((\alpha, t)\) reaction on a \(^{124}\)Sn target. The two relevant peaks are indicated by shading. The lower part of the figure shows angular distributions for the 7/2\(^+\) (dots) and 11/2\(^-\) (stars) states in two Sn isotopes, together with DWBA calculations.

Transfer in inverse kinematics

\(^{124}\text{Sn}(d,p) \, @ \, 4.25 \text{ MeV/A} – \text{Angular Distributions}\)

**Low Excited States**

**2.767 MeV State**

**Spectroscopic Factors**

<table>
<thead>
<tr>
<th></th>
<th>This Work(^1)</th>
<th>Stromich et al.</th>
<th>Bingham and Hillis</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/2^+</td>
<td>0.44(6)</td>
<td>0.53</td>
<td>0.44</td>
</tr>
<tr>
<td>1/2^+</td>
<td>0.33(4)</td>
<td>0.32</td>
<td>0.33</td>
</tr>
<tr>
<td>7/2^-</td>
<td>0.46(5)</td>
<td>0.52</td>
<td>0.54</td>
</tr>
</tbody>
</table>

Transfer in inverse kinematics

example: $^{132}\text{Sn}(d,p)^{133}\text{Sn}$

[K. Jones et al, Nature 2010]
Other transfers

example: $^{132}\text{Sn}(d,p)^{133}\text{Sn}$
Uncertainties in theory for transfer

\[ d + A \rightarrow p + B \]

Overlap functions

\[ I_{AB}(r) = \langle \psi_A | \psi_B \rangle \]
\[ I_{pd}(r') = \langle \psi_p | \psi_d \rangle \]

Spectroscopic factor

\[ S_{\ell j} = N \langle I_{AB} | I_{AB} \rangle \]

DWBA transition matrix element

\[ M = \langle \chi_f(\downarrow) I_{AB} | \Delta V | I_{pd} \chi_i(\uparrow) \rangle \]
\[ \Delta V = V_{np} + U_{pA} - U_{pB} \quad (post) \]

Experimental xs related to DWBA xs

\[ \frac{d\sigma^{exp}}{d\Omega} = S^{exp} \frac{d\sigma^{DWBA}}{d\Omega} \]

- Optical pot (\( U_{\text{opt-in}} \) & \( U_{\text{opt-out}} \))
- Validity of DWBA
- Overlap function \( I_{AB}(r) \)
Dependence on optical potential

Fig. 1. Comparison of the measured cross sections for the elastic scattering of 12.8-MeV deuterons from $^{40}$Ca with the predictions of the three optical potentials given in Table III. The two sets of data points correspond to targets of different thicknesses; the filled circles are for a target with 5.4 mg/cm$^2$, the open circles are for 4.6 mg/cm$^2$.

Fig. 3. Comparison of the measured inelastic cross sections for $^{40}$Ca($d,d'$) with the predictions of potentials No. 2 and 3, using complex coupling. The deformation parameters are given in Table IV.
Dependence on optical potential

**Table V.** Spectroscopic factors. Nonlocal and finite-range effects are included.

<table>
<thead>
<tr>
<th>(Q) (MeV)</th>
<th>(l,j)</th>
<th>Deuteron potential No. 2</th>
<th>Deuteron potential No. 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.14</td>
<td>7/2</td>
<td>0.95</td>
<td>0.65</td>
</tr>
<tr>
<td>4.19</td>
<td>5/2</td>
<td>0.44</td>
<td>0.34</td>
</tr>
<tr>
<td>3.67</td>
<td>3/2</td>
<td>0.22</td>
<td>0.17</td>
</tr>
</tbody>
</table>

**Fig. 5.** Comparison of the measured cross sections for \(^{40}\text{Ca}\) \((d,p)^{41}\text{Ca}\) with the distorted-wave predictions using potentials No. 2 and 3. Nonlocality and finite-range effects are included, and no cutoff is used.
Dependence on d optical potential

Fig. 14.6. Dependence of the spectroscopic factor on optical parameters for $^{40}\text{Ca}(d,p)^{41}\text{Ca}(\text{g.s.})$ at 11 MeV. Optical potentials are taken from [12]. All cross sections are renormalized to emphasize the different angular dependence.
Reducing optical potential uncertainties

$^{12}\text{C}(d,p)^{13}\text{C}$: systematic extraction

![Graph showing spectroscopic factors vs. $E_{\text{beam}}$ (MeV)]

- $^{13}\text{C}(p,d)^{12}\text{C}_{\text{gs}}$
- $^{12}\text{C}(d,p)^{13}\text{C}_{\text{gs}}$

Published

Reducing optical potential uncertainties

$^{12}\text{C}(d,p)^{13}\text{C}$ : systematic extraction

**FIG. 2.** (Color online) Angular distributions for $^{12}\text{C}(d,p)^{13}\text{C}$ reactions for beam energy from 7 to 56 MeV. Each distribution is displaced by factors of 10 from adjacent distributions. The overall normalization factor is 1 for the 19.6 MeV data. References are listed in Table I.

Optical potential uncertainties

$^{12}\text{C}(d,p)^{13}\text{C}$ : systematic extraction
DWBA: dependence on optical potential

$^{10}\text{Be}(d,p)^{11}\text{Be} @ 12-21 \text{ MeV}$

Standard approach to transfer

\[ d + A \rightarrow p + B \]

Overlap functions

\[ I_{AB}(r) = \langle \psi_A | \psi_B \rangle \]

\[ I_{pd}(r') = \langle \psi_p | \psi_d \rangle \]

Spectroscopic factor

\[ S_{\ell j} = N \langle I_{AB} | I_{AB} \rangle \]

DWBA transition matrix element

\[ M = \left\langle \chi_f^{(-)} I_{AB} \right| \Delta V \left| I_{pd} \chi_i^{(+)} \right\rangle \]

\[ \Delta V = V_{np} + U_{pA} - U_{pB} \quad (\text{post}) \]

Experimental xs related to DWBA xs

\[ \frac{d\sigma^{\text{exp}}}{d\Omega} = S^{\text{exp}} \frac{d\sigma^{\text{DWBA}}}{d\Omega} \]

- Optical pot \( (U_{\text{opt-in}} \& U_{\text{opt-out}}) \)
- **Validity of DWBA**
- Overlap function \( I_{AB}(r) \)
Optical potential uncertainties

$^{12}\text{C}(d,p)^{13}\text{C}$ : systematic extraction

Reaction mechanism: target excitations

\[ ^{12}\text{C}(d,p)^{13}\text{C}: \text{couplings} \]

- 2+ \[ \rightarrow \] 2-way transfer
- 0+ \[ \rightarrow \] \[ ^{12}\text{C} \]
- \[ ^{1/2+} \]
- \[ ^{13}\text{C} \]
- \[ ^{1/2-} \]

Delaunay et al, PRC 72 (2005) 014610

Graphs showing CCBA/DWBA and CRC/CCBA couplings.
transfer reactions for spectroscopy

inverse kin $^{11}\text{Be}(p,d)^{10}\text{Be}$ at $E_{\text{beam}} = 35$ MeV/A

[Fortier et al., PLB 461 (1999) 22; Winfield et al., NPA 83 (2001) 48]

- assuming direct transfer DWBA analysis w no remnant $\Rightarrow$ 30% d-wave in $^{11}\text{Be}_{\text{g.s.}}$
- dependence on opt. pot. could be reduced by consistent elastic data analysis

$$R_{ce} = \frac{S(2^+)/[S(0^+)+S(2^+)]}{1}$$
transfer for spectroscopy

2-step processes have large effect (FR-DW2) but full CC reduce this effect (FR-CRC)

remnant term introduces additional uncertainty

removal of breakup ??
Reaction mechanism: target excitations

FIG. 1. Transfer differential cross section for $^{13}\text{C}(^{3}\text{He},d)^{14}\text{N}$, comparing prior-DWBA with prior-CCBA. The core-core interaction is set to the nuclear part of the entrance optical potential.

FIG. 2. Transfer differential cross section for $^{10}\text{Be}(^{7}\text{Be},^{8}\text{B})^{9}\text{Be}$, comparing post-DWBA with post-CCBA. The core-core interaction is set to the nuclear part of the exit optical potential.

Reaction mechanism: including breakup

Johnson and Soper potential is based on the zero-range+adiabatic approx

\[ U_{ad}(R) = U_{ct}(R) + U_{vt}(R) \]

Tandy and Johnson reformulated the adiabatic model without the zero-range approx and obtained a new adiabatic potential for the deuteron including breakup effect and finite-range:

\[ V_{TJ}(R) = \frac{\langle \phi_d | V_{np}(U_n + U_p) | \phi_d \rangle}{\langle \phi_d | V_{np} | \phi_d \rangle} \]

Effective diffuseness increases!
Beyond DWBA: importance of breakup

$^{10}\text{Be}(d,p)^{11}\text{Be} @ 12\text{-}21\text{ MeV}$

DWBA entrance channel

DWBA exit channel

ADWA

FIG. 1: Angular distributions for $^{11}\text{Be}(p,d)^{10}\text{Be}$: (a) $E_p = 5$ MeV, (b) $E_p = 10$ MeV, and (c) $E_p = 35$ MeV.
FIG. 2: Angular distributions for $^{12}$C$(d,p)^{13}$C: (a) $E_d = 7.15$ MeV, (b) $E_d = 12$ MeV and (c) $E_d = 56$ MeV
FIG. 3: Angular distributions for $^{48}\text{Ca}(d,p)^{49}\text{Ca}$: (a) $E_d = 19.3$ MeV, (b) $E_d = 56$ MeV and (c) $E_d = 100$ MeV.
systematic comparison: FR-ADWA vs Faddeev

FIG. 4: Ratio of Faddeev prediction for the cross section at the peak of the angular distribution versus the Adiabatic counterpart plotted in term of the deuteron energy in the c.m. over the Coulomb Barrier.
Uncertainties in theory for transfer

\[ d + A \rightarrow p + B \]

Overlap functions
\[ I_{AB}(r) = \langle \psi_A \middle| \psi_B \rangle \]
\[ I_{pd}(r') = \langle \psi_p \middle| \psi_d \rangle \]

Spectroscopic factor
\[ S_{\ell j} = N \langle I_{AB} \middle| I_{AB} \rangle \]

DWBA transition matrix element
\[ M = \langle \chi_f^{(-)} I_{AB} \middle| \Delta V \middle| I_{pd} \chi_i^{(+)} \rangle \]
\[ \Delta V = V_{np} + U_{pA} - U_{pB} \quad (post) \]

Experimental xs related to DWBA xs
\[ \frac{d\sigma^{exp}}{d\Omega} = S^{exp} \frac{d\sigma^{DWBA}}{d\Omega} \]

- Optical pot (\( U_{opt-in} \& U_{opt-out} \))
- Validity of DWBA
- Overlap function \( I_{AB}(r) \)
The overlap function for $^{19}\text{C} \rightarrow \text{n}+^{18}\text{C}$ in arbitrary units. The radial sensitivity of the $^{18}\text{C}(d,p)^{19}\text{C}$ cross section is represented by the colored bars for different beam energies.
single particle approximation

\[(T_r + V_{nA} + \varepsilon)\varphi_{nlj}(r) = 0\]

nucleons feel mean field generated by core nucleons \(V_{nA}\)

- specific \(n,l,j\) and separation energy
- assumptions about mean field geometry \((r,a)\)
- single particle states normalized to 1
single particle approximation (neutrons)

\[
S_{nlj}^B = A_{nlj}^2 = \frac{C_{lj}^2}{b_{nlj}^2}
\]

Same radial dependence at large distances:

\[
I_{AB}(r)_{r>R_N} \rightarrow C_{lj} i\kappa \quad h_l(ikr) \quad \bm{\varphi}_{nlj}(r)_{r>R_N} \rightarrow b_{nlj} i\kappa \quad h_l(ikr)
\]

Extend that assumption within the range of the interaction:

\[
I_{AB}(r) = A_{nlj} \bm{\varphi}_{nlj}(r)
\]
Overlap function, SF and ANC

**S (spectroscopic factor) – volume property**

\[ S_{\ell j} = N\langle I_{AB} | I_{AB} \rangle \]

**C (asymptotic normalization coefficient) – asymptotic property**

\[ I_{AB}(r) \xrightarrow{r > R_N} C_{\ell j} i\kappa \ h_l(i\kappa r) \]
Sensitivity to parts of the overlap function

\[ S_{nlj}^B = A_{nlj}^2 = \frac{C_{lj}^2}{b_{nij}^2} \]

Fig. 14.4. Dependence of the spectroscopic factor and the asymptotic normalization coefficient on the single-particle parameters as a function of \( b \) for fits to the transfer cross section \(^{48}\text{Ca}(d,p)^{49}\text{Ca}(\text{g.s.})\) at 13 MeV. The line is an inverse quadratic to guide the eye.
extracting SF or ANC?

Fig. 14.1. Coordinates in the entrance and exit partitions for the transfer \(A(a,b)B\) where \(a = b + c\) and \(B = A + c\).

\[
T = A(M_{in}(b) + bM_{out}) = Ab\left(\frac{M_{in}(b)}{b} + M_{out}\right)
\]

\[
\sigma = Sb^2\left(\frac{M_{in}(b)}{b} + M_{out}\right)^2 = C^2\left(\frac{M_{in}(b)}{b} + M_{out}\right)^2
\]

Typically \(M_{out}\) is large so if ANC is unknown

large error in SF

If \(M_{in}\) is negligible then one cannot extract SF unambiguously

one can only extract ANC

Mukhamedzhanov et al., PRC72 (2005) 017602

(d,p) reactions: combined method

From sub-Coulomb transfer reaction obtain ANC

\[ \sigma^{th}(b) = C^2 M_{out}^2 \]

From higher energy transfer reaction obtain SF consistent with ANC

\[ \sigma^{th}(b) = (S M_{in}(b) + C M_{out})^2 \]

\[ C^2 = Sb^2 \]

The overlap function for \(^{19}\text{C} \rightarrow \text{n}^{+^{18}}\text{C}\) in arbitrary units. The radial sensitivity of the \(^{14}\text{C}(d,p)^{19}\text{C}\) cross section is represented by the colored bars for different beam energies.

Combined method provides a handle on single particle parameters!
From peripheral process extract ANC

$$C^2 = Sb^2$$
From higher energy, extract SF with fixed ANC

SFs and ANCs from $^{48}$Ca(d,p)$^{49}$Ca and $^{48}$Ca(n,$\gamma$)$^{49}$Ca

\[ SF = 0.55 \pm 0.25 \]

\[ SF = 0.53 \pm 0.11 \]

The question:

Are the analyses of transfers to extract SF consistent with independent ANC measurements?

Requirements:
- data for elastic+transfer at an energy above the barrier
- data from which to extract ANC independently

Three test cases:

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Q Value (MeV)</th>
<th>BE Value (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{14}\text{C}(d,p)^{15}\text{C}$ @ 14 MeV</td>
<td>-1.0</td>
<td>1.2</td>
</tr>
<tr>
<td>$^{16}\text{O}(d,p)^{17}\text{O}$ @ 15 MeV</td>
<td>+1.9</td>
<td>4.1</td>
</tr>
<tr>
<td>$^{40}\text{Ca}(d,p)^{41}\text{Ca}$ @ 11 MeV</td>
<td>+6.1</td>
<td>8.4</td>
</tr>
</tbody>
</table>

Independent ANC

Knockout data on $^9$Be and $^{12}$C $\sim$50 MeV/u was used to extract the $^{15}$C ANC

$$C^2_{\text{(exp)}} = 1.48(18) \text{ fm}^{-1}$$

Maddalena et al., NPA 682 (2001) 332.

Sub-Coulomb heavy ion transfer reactions can be used to extract ANCs

$^{16}$O($^{17}$O, $^{16}$O)$^{17}$O @ 22 MeV

$$C^2_{\text{(exp)}} = 0.69(03) \text{ fm}^{-1}$$


$^{17}$O($^{17}$O, $^{16}$O)$^{17}$O @ 22 MeV

$$C^2_{\text{(exp)}} = 1.48(18) \text{ fm}^{-1}$$

Franey et al., NPA324 (1979) 193.

Only $^{40}$Ca(d,p)$^{41}$Ca@2.5 MeV to extract ANC

$$C^2_{\text{(exp)}} = 8.36(42) \text{ fm}^{-1}$$

Kocher et al., NPA172 (1971) 652.
Fitting elastic

$^{14}\text{C}(d,p)^{15}\text{C} @ 14 \text{ MeV}$

$^{16}\text{O}(d,p)^{17}\text{O} @ 15 \text{ MeV}$

$^{40}\text{Ca}(d,p)^{41}\text{Ca} @ 11 \text{ MeV}$

SF=1.0

$r_0=1.65 \text{ fm}$

$C^2=2.65 \text{ fm}^{-1}$

$C^2_{(exp)}=1.48(18) \text{ fm}^{-1}$

SF=0.7

$r_0=1.1 \text{ fm}$

$C^2=0.62 \text{ fm}^{-1}$

$C^2_{(exp)}=0.67(5) \text{ fm}^{-1}$

SF=0.7

$r_0=1.1 \text{ fm}$

$C^2=2.7 \text{ fm}^{-1}$

$C^2_{(exp)}=8.36(42) \text{ fm}^{-1}$

summary

1) Global pot
2) Fit to elastic
3) Deuteron breakup
4) Couplings
5) Overall estimate

transfer reactions for spectroscopy

inverse kin $^{11}\text{Be}(p,d)^{10}\text{Be}$ at $E_{\text{beam}} = 35$ MeV/A

[Fortier et al., PLB 461 (1999) 22; Winfield et al., NPA 83 (2001) 48]

• assuming direct transfer DWBA analysis
  $\rightarrow$ 30% d-wave in $^{11}\text{Be}_{g.s}$.

• dependence on opt. pot. could be reduced by consistent elastic data analysis

$$R_{ce}=S(2^+)/[S(0^+)+S(2^+)]$$
Reaction mechanism: projectile beyond single particle

→ 1-step DWBA analysis w CC $^{11}\text{Be}$

structure models for $^{11}\text{Be}$:

- Core Excited Model
  [NPA 596(1996) 171]

- VIBrational model
  [NPA 592(1995) 592]

no normalisation to the data indicating that $\sim 16\%$ d-wave in $^{11}\text{Be}_{g.s.}$
Extracting ANCs from transfer for astrophysics

\[ \frac{d\sigma}{d\Omega} = C_{8B,P3/2}^2 \left[ \frac{C_{14N,p1/2}^2}{b_{14N,p1/2}^2} \frac{\sigma_{p_{1/2}^{-1/2}}} {b_{8B,p3/2}^2} + \frac{C_{14N,p3/2}^2}{b_{14N,p3/2}^2} \frac{\sigma_{p_{3/2}^{-1/2}}} {b_{8B,p3/2}^2} \right] \\
+ C_{8B,P1/2}^2 \left[ \frac{C_{14N,p1/2}^2}{b_{14N,p1/2}^2} \frac{\sigma_{p_{1/2}^{-1/2}}} {b_{8B,p1/2}^2} + \frac{C_{14N,p3/2}^2}{b_{14N,p3/2}^2} \frac{\sigma_{p_{3/2}^{-1/2}}} {b_{8B,p1/2}^2} \right] \\
+ \frac{C_{14N,p3/2}^2}{b_{14N,p3/2}^2} \frac{\sigma_{p_{3/2}^{-1/2}}} {b_{8B,p1/2}^2} \right], \]

(1)

FIG. 4. Observed \(^7\)Be elastic scattering angular distribution. Statistical errors are smaller than the plotted data points. The overall normalization uncertainty is \(\pm 5.0\%\). The dashed curve is the predicted angular distribution, summed over \(^{14}\)N, \(^{12}\)C, and \(^1\)H target nuclei, while the solid curve shows the same distribution corrected for finite angular resolution.

\[ T = \langle \chi_f^{(-)} | I_{8B7Be} \Delta V | I_{14N13C} \chi_i^{(+)} \rangle \]

FIG. 6. Measured \(^{14}\)N\((^7\)Be, \(^8\)B\)\(^{13}\)C angular distribution for those events in Fig. 5 that have \(Q > -9.5\) MeV. The dashed and dotted curves show DWBA calculations of the dominant contributions from the \(p_{1/2}^{-1/2}p_{3/2}\) and \(p_{1/2}^{-1/2}p_{1/2}\) channels, respectively. The solid curve shows the total predicted angular distribution, normalized to the total cross section inferred from the Q-value fit in Fig. 5. All three curves have been corrected for finite angular resolution.

Azhari et al, PRC 60, 055803 (1999)
FIG. 2. Angular distributions for the ground and two excited states of $^{14}$N from the $^{13}$C($^3$He,$d$)$^{14}$N reaction. The solid lines represent DWBA calculations using optical potential combination B–E in Table I, and the dashed lines represent combination B–D.
The relation between the $^8$B ANC's and $S_{17}(0)$, which was derived in [6], is

$$ S_{17}(0) = \frac{38.6 \text{ eV b}}{\text{fm}^{-1}} (C_{8B,p_{3/2}}^2 + C_{8B,p_{1/2}}^2). \quad (2) $$

Thus, we conclude that $S_{17}(0) = 16.6 \pm 1.9 \text{ eV b}$ for $^7$Be($p,\gamma)^8$B. This is in very good agreement with the results obtained from our previous ANC measurement [17] and the three smaller direct measurements of the $^7$Be($p,\gamma)^8$B cross section [3–5] and is more than $3\sigma$ below the two larger direct measurements [1,2]. This result provides strong, independent confirmation of the procedure in Ref. [5], which chose to disregard the two larger measurements and calculate $S_{17}(0)$ based on a weighted mean of the three smaller direct measurements alone.

Azhari et al, PRC 60, 055803 (1999)
Extracting ANCs: peripherality

Fig. 14.1. Coordinates in the entrance and exit partitions for the transformation $\alpha = b + v$ and $B = A + v$.

FIG. 1. Calculated reaction cross section in the partial wave $L$, for the $^7\text{Be}(d,n)^8\text{B}$ at 5.8, 15.6, and 38.9 MeV c.m. energies (a) and corresponding differential cross section (b).
Transfer for ANCs: Higher order effects

reaction measured with the aim of extracting $S_{17}(0)$

$^8\text{B continuum effect}$

Moro et al.,
PRC66(02)024612
Difficulties in transfer with unstable nuclei

- Extended tails: finite range more important!

- $U_{\text{opt}}$ very different from $U_{\text{core}}$
  remnant more important!

- Continuum! Continuum! Continuum!
  Which continuum, deuteron or exotic nucleus?
Transfer to the continuum

$^9\text{Li}(d,p)^{10}\text{Li}$

Fig. 4. (Color online.) Relative contribution of $s_{1/2}$ and $p_{1/2}$ final states in the $^9\text{Li}+n$ system to the excitation energy spectrum for the reaction $^9\text{Li}(^2\text{H},p)^{10}\text{Li}$.

Theory still catching up...
Knockout spectroscopy
How to measure spectroscopic factors?

For stable nuclei

Knockout using stable beams (e,e’p)

Transfer reactions using light beams (d,p), (³He,d)
or using heavy ions (¹²C,¹³C) or (¹⁶O,¹⁷O)

For radioactive beams

Knockout using radioactive beams

Transfer reactions using exotic beams (inverse kinematics)

Larger combination of initial and final states
How to measure spectroscopic factors?

For stable nuclei

Knockout using stable beams (e,e’p)

Transfer reactions using light beams (d,p),
\((^3\text{He},d)\) or using heavy ions \((^{12}\text{C},^{13}\text{C})\) or
\((^{16}\text{O},^{17}\text{O})\)

Easier to measure
Requires less pps

For radioactive beams

Knockout using radioactive beams

Transfer reactions using exotic beams
(inverse kinematics)
transfer vs \((e,e'p)\)

\((e,e'p)\) probes the deep interior

transfer probes the surface

Fig. 3. (a) The BSWF as obtained from the present \((e,e'p)\) experiment for the 1\(d_{3/2}\) and 2\(s_{1/2}\) orbitals. (b) The sensitivity \(P\) (see text) of the \((e,e'p)\) momentum distributions to these BSWF. (c) The sensitivity \(P\) (see text) of the \((d,^3\text{He})\) differential cross sections to these BSWF.
Fig. 1. Spectroscopic strength relative to the independent-particle shell-model limit for valence orbitals as a function of the mass number according to literature values for \((d,^3\text{He})\) and \((e,e'p)\) experiments. For references see Table 3.
transfer vs (e,e’p)

Kramer et al, NPA679(2001)267

non locality & finite range corrections

Fig. 7. Summed spectroscopic strength of the valence orbit as function of the mass number normalized to the independent-particle shell-model limit according to literature values from (e,e’p) experiments and from a reanalysis, as discussed in the text, of the (d,\(^3\)He) data presented in Fig. 1.
Fig. 14.9. Schematic of a nuclear knockout reaction. Reprinted from [3] with permission.

Includes elastic and inelastic breakup as well as transfer
Stripping of a nucleon – nucleon ‘absorbed’

\[ \sigma_{\text{strip}} = \int \mathrm{d}b \left\langle \phi_0 \left| S_c \right|^2 (1 - \left| S_1 \right|^2) \right| \phi_0 \right\rangle \]
Diffractive (breakup) removal of a nucleon

\[ \sigma_{\text{diff}} = \int \, db \left\{ \langle \phi_0 | S_c S_v | \phi_0 \rangle^2 - \langle \phi_0 | S_c S_v | \phi_0 \rangle \right\} \]
Connection with overlap functions

\[(\Phi_X^{b\beta}, A - 1|\Phi_Y^{a\alpha}, A) \rightarrow F_{YX}^{a\alpha b\beta}(\vec{r})\]

Assumption: the removal reactions perturbs the motion of just a single nucleon – but not the degrees of freedom of A-1.

and e.g. \[\mathcal{O}(1) = (1 - |S_n(1)|^2)\]

\[(\Phi_X^{b\beta}|\mathcal{O}(1)|\Phi_Y^{a\alpha}) = \mathcal{O}(1)(\Phi_X^{b\beta}|\Phi_Y^{a\alpha}) = \mathcal{O}(1)F_{YX}^{a\alpha b\beta}(1)\]

Starting point for the stripping term is therefore

\[\sigma_{str} = \frac{N}{2a + 1} \sum_{\alpha} \int d\vec{b} \langle \Phi_Y^{a\alpha}|(1 - |S_n(1)|^2)|S_X|^2|\Phi_Y^{a\alpha}\rangle\]

\[|S_X|^2 = |S_X(2, \ldots A)|^2\] determines the (survival probability) for the A - 1 nucleons comprising X
Connection with overlap functions

Manipulating \( \mathcal{M} = \langle \Phi_Y^{a\alpha} | (1 - |S_n(1)|^2) S_X | \Phi_Y^{a\alpha} \rangle \)

\[ \mathcal{M} = \langle \Phi_Y^{a\alpha} | (1 - |S_n(1)|^2) S_X S_X^* | \Phi_Y^{a\alpha} \rangle \]

on inserting complete sets of states of nucleus X at the points shown - and assuming in addition that \( S_X \) does not couple/excite the states of \( X \)

\[ 1 = \sum_{b\beta} |\Phi_X^{b\beta}\rangle \langle \Phi_X^{b\beta}| \]

\[ (\Phi_X^{b\beta} | S_X | \Phi_X^{b'\beta'}) = S_b(b_X) \delta_{bb'} \delta_{\beta\beta'} \]

\[ \mathcal{M} = \sum_{b\beta} \langle \Phi_Y^{a\alpha} | \Phi_X^{b\beta} \rangle | (1 - |S_n(1)|^2) S_b(b_X) |^2 \langle \Phi_X^{b\beta} | \Phi_Y^{a\alpha} \rangle \]

where we recognise the overlap functions and the eikonal S-matrix for the scattering of nucleus X in state b from the target

\[ S_b(b_X) = (\Phi_X^{b\beta} | S_X | \Phi_X^{b\beta}) \]
Connecting to overlap functions

\[ \mathcal{M} = \sum_{b\beta} \langle \Phi_{Y}^{a\alpha} | \Phi_{X}^{b\beta} \rangle (1 - |S_n(1)|^2) |S_b(b_X)|^2 \langle \Phi_{X}^{b\beta} | \Phi_{Y}^{a\alpha} \rangle \]

Using our earlier definition for the spectroscopic factor, that

\[ \langle \Phi_{X}^{b\beta} | \Phi_{Y}^{a\alpha} \rangle = \sum_{jm} (b\beta jm|a\alpha) \sqrt{S(\ell j : a \rightarrow b)} \frac{m}{N} \phi_{n\ell j}^m(1) \]

and that

\[ \sigma_{str} = \frac{N}{2a + 1} \sum_{\alpha} \int d\tilde{b} \mathcal{M}(b) \]

\[ \sigma_{str} = \sum_{b\beta} S(\ell j; a \rightarrow b) \sigma_{str}^{sp}(b) \]

\[ \sigma_{str}^{sp}(b) = \frac{1}{2j + 1} \sum_{m} \int d\tilde{b} \langle \phi_{n\ell j}^m | (1 - |S_n|^2)|S_b|^2 | \phi_{n\ell j}^m \rangle \]

Tostevin
Eikonal S-matrix dependence on impact parameter

\[ \left| S_{22}(b_{22}) \right|^2, \left( 1 - \left| S_n(b_n) \right|^2 \right) \left| S_{22}(b_{22}) \right|^2 \]

impact parameters \( b_n, b_{22} \) (fm)
Contributions are from nuclear surface and beyond

\[ {^{12}\text{Be}} + {^{9}\text{Be}} \rightarrow {^{11}\text{Be}}(\text{gs}) + X, \text{ 80A MeV} \]

\[ b \geq R_C + R_T \]
Probing the surface

Viewed from the rest frame of the projectile

$b \approx R_C + R_T$

Interaction with the target probes wave functions at surface and beyond

Mass A-1 residue will be left in the ground state or an excited state

Tostevin
Residue momentum distributions after knockout

\[
\sigma_{str} = \frac{1}{2l + 1} \sum_m \int d^2b \langle \psi_{lm} | S_c(b_c)|^2 (1 - |S_n(b_n)|^2) |\psi_{lm}\rangle
\]

\[
= \frac{1}{2l + 1} \sum_m \int d^2b_n (1 - |S_n(b_n)|^2) \langle \psi_{lm} | S_c^* S_c |\psi_{lm}\rangle \frac{1}{(2\pi)^3} \int d\vec{k}_c |\vec{k}_c\rangle \langle \vec{k}_c | = 1
\]

In projectile rest frame:

\[
\frac{d\sigma_{str}}{d^3k_c} = \frac{1}{(2\pi)^3} \frac{1}{2l + 1} \sum_m \int d^2b_n [1 - |S_n(b_n)|^2]
\]

\[
\times \left| \int d^3r e^{-i\vec{k}_c \cdot \vec{r}} S_c(b_c) \psi_{lm}(r) \right|^2
\]

\[
\frac{d\sigma_{str}}{dk_z} = \frac{1}{2\pi} \frac{1}{2l + 1} \sum_m \int_0^\infty d^2b_n [1 - |S_n(b_n)|^2] \int_0^\infty d^2\rho |S_c(b_c)|^2 \times \left| \int_{-\infty}^{\infty} dz \exp[-ik_z z] \psi_{lm}(r) \right|^2
\]
Dependence on properties of bound state wavefunction

consider momentum components $p_\parallel$ of the core parallel to the beam direction, in the projectile rest frame

$$\Delta p \Delta z > \frac{\hbar}{2}$$
Dependence on properties of bound state wavefunction

Consider momentum components $p_\parallel$ of the core parallel to the beam direction, in the projectile rest frame.

$$\Delta p \Delta z > \hbar / 2$$
Dependence on angular momentum

Residue momentum $^{11}\text{Be} \rightarrow ^{10}\text{Be} –$ halo case

T. Aumann et al. PRL 84 (2000) 35

$Z = 4, N = 7$

$^{11}\text{Be}$
Dependence on angular momentum

\[ {^{15}\text{C}}{^9}\text{Be} \]

\[ p_{t,({^{14}\text{C}})} \text{ (GeV/c)} \]

Cross section (arb. units)

\[ 1^{-} \]
\[ 6.09 \text{ MeV} \]
\[ 1p_{3/2} \]
\[ 0^{+}, \text{ gs} \]
\[ 2s_{1/2} \]

\[ {^{22}\text{O}} \text{ residues} \]

\[ 1/2^{+} (0^{+}) \]
\[ 5/2^{+} (2^{+}) \]
Dependence on projection

(a) small $m$

(b) large $m$

Graph: Cross section (arbitrary units) vs. residue parallel momentum (MeV/c)

- $|m|=1$
- $|m|=2$
- $m=0$

Tostevin
### TABLE I. Cross sections for the reactions

| \(^{12}\text{C}\)\(^{12}\text{C},^{11}\text{B})\text{X},^{12}\text{C}({}^{12}\text{C},^{11}\text{C})\text{X},^{12}\text{C}({}^{16}\text{O},^{15}\text{N})\text{X}\) and \(^{12}\text{C}({}^{16}\text{O},^{15}\text{O})\text{X}\). |
|---|---|---|---|---|---|
| \(^{11}\text{B}\) | 250 | a | E* Strip. Diffr. | \(\sigma_{sp}\) (mb) | \(\sigma_{th}\) (mb) | \(\sigma_{expt}\) (mb) | \(R_S\) |
| 1050 | a | 21.9 | 1.8 | 100.5 | 65.6(26) \(^b\) | 0.65(3) |
| 2100 | a | 20.6 | 2.0 | 96.1 | 53.8(27) \(^c\) | 0.56(3) |
| \(^{11}\text{C}\) | 250 | a | 21.4 | 1.7 | 98.2 | 56.0(41) \(^b\) | 0.57(4) |
| 1050 | a | 20.2 | 1.8 | 93.4 | 44.7(28) \(^c\) | 0.48(3) |
| 2100 | a | 20.1 | 1.9 | 93.3 | 46.5(23) \(^c\) | 0.50(3) |
| \(^{15}\text{N}\) | 2100 | 0 | 15.40 | 1.77 | 6.324 | 12.95 | 1.30 | 80.2 | 54.2(29) \(^b\) | 0.68(4) |
| Sum | | | | | | | | | |
| \(^{15}\text{O}\) | 2100 | 0 | 14.63 | 1.61 | 6.176 | 12.54 | 1.23 | 76.9 | 42.9(23) \(^c\) | 0.56(3) |

\(^a\)The single-particle cross sections are those for the ground state. The values for the two excited states are 5–6% smaller. The \(\sigma_{th}\) listed are the sum of all contributions. The energies and spectroscopic factors are given in the text.

\(^b\)Reference [25].

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Brown et al, PRC65(2002)061601

Gade et al.
Quenching in knockout reactions

FIG. 3. Measured reduction factors $R_s$ as a function of nucleon separation energy. The points, taken from the left, use data from $^8$B, $^9$C, $^{15}$C, $^{57}$Ni, $^{12}$C, and $^{16}$O [4,5,26]. The labeled $N = 14$ nuclei are discussed in the present Letter.

Gade et al, PRL93(2004)042501
Proton versus neutron removal

Removal probes single-nucleon wave functions

Interaction with the target probes wave functions at surface

$S_n = 5.29$ MeV
$S_p = 20.64$ MeV

$b \approx R_p + R_T$
Comparison of knockout and transfer

transfer versus knockout

FIG. 3: (Color online) Reduction factors $R_s = SF(ADWA − FR)/SF(LB − SM)$ as a function of the difference between the neutron and proton separation energies $\Delta S$. The squares and circles correspond to values extracted using transfer or knockout respectively. The bars correspond to the total uncertainty including both experimental and theoretical errors evaluated for the transfer reactions.
TABLE II. Measured and theoretical diffraction (including Coulomb) components of the one-proton knockout cross section for $^9$C and $^8$B. The calculated diffraction component agrees very well with the observation in both reactions. The large error bars in [9] come from the inclusive method used in that experiment. The theoretical inclusive cross sections are shown, which include center-of-mass correction factors $A/(A - 1)$. The deduced reduction factors $R_S = \sigma_{\text{exp}}/\sigma_{\text{th}}$ are also shown, as are the results from previous measurements.

<table>
<thead>
<tr>
<th>Proj.</th>
<th>%$_{\text{diff}}^a$</th>
<th>%$_{\text{diff}}^b$</th>
<th>%$_{\text{diff}}[9]$</th>
<th>$\sigma_{\text{th}}$ (mb)</th>
<th>$R_S$ $^a$</th>
<th>$R_S[9]$</th>
<th>$R_S[11]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^9$C</td>
<td>25(2)</td>
<td>26.8</td>
<td>26(10)</td>
<td>62.90</td>
<td>0.84(5)</td>
<td>0.82(6)</td>
<td>-</td>
</tr>
<tr>
<td>$^8$B</td>
<td>38(3)</td>
<td>37.1</td>
<td>28(14)</td>
<td>144.28</td>
<td>0.88(4)</td>
<td>0.86(7)</td>
<td>0.88(4)</td>
</tr>
</tbody>
</table>

$^a$This work
$^b$Calculated (from Table I)

FIG. 3 (color online). Proton scattering angular distributions obtained for $^9$C (top) and $^8$B (bottom) elastic breakup events, after subtraction of the inelastic contamination using the fitted energy sum spectra (see Fig. 2). Also shown are the results from a CDCC calculation of the same quantity. The CDCC distributions are used to deduce the amount of unobserved cross section due to the lack of angular coverage between 0° and 10°, after (minor) normalization to the data.
Knock-out

\[ ^9\text{Be}(^{17}\text{C}, ^{16}\text{C}\gamma)X \]
\[ (E_{\text{beam}} = 60 \text{ MeV/A}) \]

(a) 8\% s + 92\% d
(b) 26\% s + 74\% d
(c) 100\% d

SM calculation predict no \(^{16}\text{C}(0^+)\) in the \(^{17}\text{C}(\text{g.s.})\). Experiment measured a 20\% branch into \(^{16}\text{C}(0^+)\).

Higher order processes?

Maddalena et al., PRC63(01)024613
Testing the spectator approximation

<table>
<thead>
<tr>
<th>Model</th>
<th>Couplings</th>
<th>$\sigma_0^+$</th>
<th>$\sigma_2^+$</th>
<th>$\sigma_4^+$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPIS</td>
<td>×</td>
<td>6</td>
<td>12</td>
<td>-</td>
<td>18</td>
</tr>
<tr>
<td>CCSE</td>
<td>✓</td>
<td>8</td>
<td>14</td>
<td>4</td>
<td>26</td>
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<tr>
<td>CCDE</td>
<td>✓</td>
<td>8</td>
<td>15</td>
<td>5</td>
<td>28</td>
</tr>
<tr>
<td>SPIS</td>
<td>×</td>
<td>5</td>
<td>11</td>
<td>-</td>
<td>16</td>
</tr>
<tr>
<td>CCDE</td>
<td>✓</td>
<td>4</td>
<td>10</td>
<td>5</td>
<td>19</td>
</tr>
</tbody>
</table>

TABLE II: $^{17}$C breakup cross sections leading to $^{16}$C{$0^+, 2^+, 4^+$} + n, on $^9$Be at 62 MeV/nucleons. The abbreviations in the first row are the same as in Table I. Calculations are repeated with and without continuum-continuum couplings (labeled here by cont).

<table>
<thead>
<tr>
<th>Model</th>
<th>Couplings</th>
<th>$\sigma_0^+$</th>
<th>$\sigma_2^+$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPIS</td>
<td>×</td>
<td>174</td>
<td>46</td>
<td>220</td>
</tr>
<tr>
<td>CCSE</td>
<td>✓</td>
<td>144</td>
<td>9</td>
<td>153</td>
</tr>
<tr>
<td>CCDE</td>
<td>✓</td>
<td>159</td>
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<td>169</td>
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<tr>
<td>SPIS</td>
<td>×</td>
<td>111</td>
<td>1</td>
<td>112</td>
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<tr>
<td>CCSE</td>
<td>✓</td>
<td>110</td>
<td>8</td>
<td>118</td>
</tr>
<tr>
<td>CCDE</td>
<td>✓</td>
<td>125</td>
<td>10</td>
<td>135</td>
</tr>
</tbody>
</table>

TABLE III: $^{11}$Be nuclear and Coulomb breakup cross sections for $^{10}$Be{$0^+, 2^+$} + n, on $^9$Be at 60 MeV/nucleons. The abbreviations in the first row are the same as in Table I.
Breakup spectroscopy
Spectroscopy with breakup reactions

Coulomb Breakup Alder and Winther Theory

\[
\frac{d\sigma(\theta_D, E_x)}{d\Omega} \propto \frac{dN_{E1}(\theta_D, E_x)}{d\Omega} B(E1; E_x)
\]

Photo dissociation

\[
\frac{d\sigma_{\text{dis}}(E\lambda)}{dE} = \frac{(2\pi)^3 (\lambda + 1)}{\lambda((2\lambda + 1)!!)^2} \left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda - 1} dB(E\lambda) \frac{dB(E\lambda)}{dE}
\]
Spectroscopy with breakup reactions

(E_{\text{beam}} = 70 \text{ MeV A})

FIG. 1. (a) Dissociation cross sections as a function of relative energy $E_{\text{rel}}$ for Pb (circles) and C (diamonds) targets. (b) Coulomb dissociation cross section for the Pb target, obtained by subtracting the nuclear contribution scaled from the C target spectrum in (a). The spectrum is compared with the calculations for the possible single-particle configurations described in the text.
Spectroscopy with breakup reactions

![Graph showing angular distributions](image)

**TABLE III.** Comparison of spectroscopic factors obtained from different reactions. For the Coulomb breakup, QM (quantum mechanical) stands for the ECIS analysis, while SC (semiclassical) stands for the semiclassical equivalent photon method. For transfer reactions, Ref. [40] is a reanalysis of the experiment of Ref. [39]. For the knockout reaction, Ref. [42] is a reanalysis of the experiment of Ref. [6] with a corrected eikonal model.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>(E/A) (MeV)</th>
<th>References</th>
<th>(a^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coulomb breakup</td>
<td>69</td>
<td>Present</td>
<td>0.72±0.04 (QM)</td>
</tr>
<tr>
<td></td>
<td>72</td>
<td>[9,17]</td>
<td>0.69±0.04 (SC)</td>
</tr>
<tr>
<td></td>
<td>520</td>
<td>[31]</td>
<td>0.80±0.20 (SC)</td>
</tr>
<tr>
<td>Transfer reaction</td>
<td>12.5</td>
<td>[39]</td>
<td>0.61±0.05 (SC)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[40]</td>
<td>0.77</td>
</tr>
<tr>
<td>Knockout reaction</td>
<td>35.3</td>
<td>[41]</td>
<td>0.60±0.36</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>[6]</td>
<td>0.67–0.80</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[42]</td>
<td>0.79±0.12</td>
</tr>
</tbody>
</table>
Spectroscopy with breakup reactions

In the present calculation, the wave function for the halo configuration, $^{14}\text{C}(0^+) \otimes \nu 2s_{1/2}$, was obtained using a two-body potential model for $^{14}\text{C}$-neutron interaction. A Woods-Saxon potential with a radius parameter $r_0 = 1.223$ fm and a diffuseness parameter $a = 0.50$ fm was used. The same parameters were used by Terry et al. [16] for extracting the spectroscopic factor in the neutron removal reaction of $^{15}\text{C}$ on a $^9\text{Be}$ target. They extracted these parameters to adjust the radius obtained in mean field Hartree-Fock (HF) calculation. The potential depth necessary to bind an s-wave neutron by 1.218 MeV is $V_0 = 55.4$ MeV. The same potential well is used to calculate the outgoing distorted wave in the continuum and ultimately the matrix element of Eq. (4). A spin-orbit potential $V_{SO} = 6.5$ MeV was included in that case.

An excellent agreement between the direct breakup calculation and the spectra, selected for the smaller angular region $0^\circ \leq \theta \leq 2.1^\circ$, is obtained using $\alpha^2 = 0.91(6)$. This value was

FIG. 1. Breakup cross section as a function of relative energy, $E_{rel}$, of $^{14}\text{C} + n$ in the reaction $^{15}\text{C} + \text{Pb}$. Solid squares represent the data for scattering angles up to 6 degrees, and open circles represent the data for selected scattering angles up to 2.1 degrees. Solid curves are calculations for a direct breakup model with a spectroscopic factor $\alpha^2 = 0.91$, for the halo configuration ($a = 0.5$ fm and $r_0 = 1.223$ fm).
1.5 MeV. The same $\alpha^2$ can also describe the overall shape and amplitude of the spectra for $0^\circ \leq \theta \leq 6.0^\circ$, although some deviation appears in that case. This could be attributed to higher order effects in the reaction mechanism as well as to some nuclear breakup contribution, as found for the case for $^{11}$Be [23].

We also repeated the calculations using a larger diffuseness parameter $a = 0.7$ fm and $r_0 = 1.25$ fm, which were used in the primary analysis for the Coulomb breakup at GSI [11]. In this case, we found about 20% smaller value of $\alpha^2 = 0.12(5)$, consistent with the value derived from the GSI experiment. Note that the calculated spectral shapes are found to be almost identical, independent of the choice of potential parameters.

The higher $\alpha^2$ value ($\alpha^2 = 0.91(6)$), obtained with $a = 0.50$ fm, is in better agreement with the value of Terry et al. [16], as can be seen in Table I. This spectroscopic factor is also consistent with the value obtained from the $(d, p)$ reaction [29,30]. The Coulomb breakup reaction at GSI also reproduces a larger spectroscopic factor when the analysis is done using similar $a$ and $r_0$ parameters. We note that in comparing $\alpha^2$ values, it is natural to use the same potential parameters. Hereafter, we adopt the higher value of $\alpha^2 = 0.91$ for consistent discussion with the results of Terry et al., obtained with the same $a$ and $r_0$ values.

FIG. 1. Breakup cross section as a function of relative energy, $E_{\text{rel}}$, of $^{14}$C + n in the reaction $^{15}$C + Pb. Solid squares represent the data for scattering angles up to 6 degrees, and open circles represent the data for selected scattering angles up to 2.1 degrees. Solid curves are calculations for a direct breakup model with a spectroscopic factor $\alpha^2 = 0.91$, for the halo configuration ($a = 0.5$ fm and $r_0 = 1.223$ fm).
Figure 4. The $B(E1)$ distribution of $^{11}$Li obtained in the current Coulomb breakup experiment (solid circles) [15] is compared with the previous ones observed at 28 MeV/nucleon at MSU (dotted-dashed curve) [11], at 43 MeV/nucleon at RIKEN (solid histogram) [12], and at 280 MeV/nucleon at GSI (zone between dashed curves) [13].

Figure 5. The $B(E1)$ distribution of $^{11}$Li obtained in the current Coulomb breakup experiment (solid circles) [15] is compared with the calculations [17] with a three-body model with full n-n correlations and final state interactions (solid curve), n-n correlation only in the ground state (dotted-dashed curve), and no n-n correlation (dotted curve).
TABLE I. Single-neutron removal cross sections \((\sigma_{-1n})\) for \(^{31}\text{Ne}\) and \(^{19}\text{C}\) on Pb and C targets at the incident energies shown. The ratio of the measured cross sections and the deduced Coulomb breakup cross sections are also listed.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>(\tilde{E}/A) (MeV)</th>
<th>(\sigma_{-1n}) (mb)</th>
<th>(\frac{\sigma_{-1n}(\text{Pb})}{\sigma_{-1n}(\text{C})})</th>
<th>(\sigma_{-1n}(E1)) (mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{31}\text{Ne} + \text{Pb})</td>
<td>234</td>
<td>712(65)</td>
<td>9.0(1.1)</td>
<td>540(70)</td>
</tr>
<tr>
<td>(^{31}\text{Ne} + \text{C})</td>
<td>230</td>
<td>79(7)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(^{19}\text{C} + \text{Pb})</td>
<td>243</td>
<td>969(34)</td>
<td>7.4(4)</td>
<td>690(70)</td>
</tr>
<tr>
<td>(^{19}\text{C} + \text{C})</td>
<td>238</td>
<td>132(4)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

FIG. 2 (color online). The Coulomb breakup cross section for \(^{31}\text{Ne}\) \((\sigma_{-1n}(E1) = 540(70)\text{ mb, hatched area})\) is compared with calculations for possible configurations of the valence neutron for the sum-rule limits of \(C^2S\) as a function of \(S_n\). The solid curves are for the negative parity states, \(2p_{3/2}\) and \(1f_{7/2}\) coupled to \(^{30}\text{Ne}(0^+_1)\), while the dot-dashed curves are for the positive parity states, \(2s_{1/2}\) and \(1d_{3/2}\). The blue lines labeled with an asterisk are for the configurations coupled to \(^{30}\text{Ne}(2^+_1)\).
Charge exchange
Spectroscopy with charge exchange

\[ H_F = V_F(R) t \cdot T \]

\[ H_{GT} = V_{GT}(R) (s \cdot S) (t \cdot T) \]

\[ T_{fi}^{DW} = \int \chi_f^{(-)}(k_f, r_p)^* \ U_{fi} \ \chi_i(k_i, r_p) \ dr_p \]

\[ U_{fi} = \langle I_f M_f, \frac{1}{2} m_f | \sum_j V_{pj}(1 - P_{pj}) | I_i M_i, \frac{1}{2} m_i \rangle, \]

\[ \frac{d\sigma}{d\Omega}(q = 0) = N \frac{E_i E_f}{(\pi \hbar c^2)^2} |J_{\sigma \tau}|^2 B(F/GT) \]

FIG. 5. The differential cross sections for the energy bin $1.75 < E_x(^{58}\text{Co}) < 2.0$ MeV and the result of the MDA (solid line) using a linear combination of $1^+$ (dashed line) and $2^+$ (dotted line) components. The error bars in the data are of statistical nature only.
FIG. 6. (Color online) (a) Large-scale shell-model calculations for the Gamow-Teller strength distribution in $^{58}$Co using the KB3G [69] interaction [37,38,70] (black) and GXPF1 [71,72] interaction [38] (gray, green online). (b) Gamow-Teller strength distribution extracted from the $^{58}$Ni($t$, $^3$He) data compared with the theoretical distributions that were folded with the experimental resolution (250 keV) and binned in the same manner as the data.

FIG. 7. (Color online) (a) Comparison of the results of the $^{58}$Ni($d$, $^3$He) and $^{58}$Ni($t$, $^3$He) experiments and the theoretical predictions. A binning of 1 MeV was applied and the theory was folded with the experimental resolution of the ($t$, $^3$He) experiment (250 keV) before binning. (b) Comparison of the results of the $^{58}$Ni($n$, $p$) and $^{58}$Ni($t$, $^3$He) experiments and the theoretical predictions. A binning of 1 MeV was applied. Note the 0.5-MeV shift relative to (a). The ($t$, $^3$He) data set and theory were folded with the experimental resolution of the ($n$, $p$) experiment (1.3 MeV) before binning.