Single-particle and cluster excitations
Inelastic scattering: cluster model

- Some nuclei permit a description in terms of two or more clusters: 
  \(d = p + n, \ 6^\text{Li} = \alpha + d, \ 7^\text{Li} = \alpha + 3^\text{H}.\)
- Projectile-target interaction:
  \[V(R, r) = U_1(r_1) + U_2(r_2)\]

**Example:** \(7^\text{Li} = \alpha + t\)

\[
\begin{align*}
 r_\alpha &= R - \frac{m_t}{m_\alpha + m_t}r; \\
r_t &= R + \frac{m_\alpha}{m_\alpha + m_t}r
\end{align*}
\]

**Internal states:**

\[
[T_r + V_{\alpha-t}(r) - \varepsilon_n]\phi_n(r) = 0
\]

**Transition potentials:**

\[
V_{n,n'}(R) = \int d r \phi_n^*(r) [U_1(r_1) + U_2(r_2)] \phi_{n'}(r)
\]
Models for inelastic scattering

## Coupling potentials in the angular momentum basis

- **Projectile states:**

  \[
  \phi_{n\ell j}^m(r) = \frac{u_{n\ell j}(r)}{r} [Y_{\ell}(\hat{r}) \otimes \chi_s]_{jm}
  \]

  (from J.A. Tostevin)

- **So, we need to evaluate:**

  \[
  \langle (\ell_f s) j_f m_f | V(R, \xi) | (\ell_i s) j_i m_i \rangle = \langle \phi_{n_f \ell_f j_f}^{m_f} | V(R, \xi) | \phi_{n_i \ell_i j_i}^{m_i} \rangle
  \]
Inelastic scattering: cluster model

Example: $^7\text{Li}(\alpha + t) + ^{208}\text{Pb}$ at 68 MeV (Phys. Lett. 139B (1984) 150):

$\Rightarrow$ CC calculation with 2 channels ($3/2^-, 1/2^-$)
Application of the CC method to weakly-bound systems

**Example:** Three-body calculation \((p+n+^{58}\text{Ni})\) with Watanabe potential:

\[
V_{dt}(\mathbf{R}) = \int d\mathbf{r} \phi_{gs}(\mathbf{r}) \left( V_{pt}(\mathbf{r}_{pt}) + V_{nt}(\mathbf{r}_{nt}) \right) \phi_{gs}(\mathbf{r})
\]
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\]

\[\frac{d\sigma}{d\Omega} = \frac{d\sigma_R}{d\Omega} \]

Three-body calculations omitting breakup channels fail to describe the experimental data.
Inelastic scattering of weakly bound nuclei

- Single-particle (or cluster) excitations become dominant.
- Excitation to continuum states important.

Exotic nuclei are weakly bound ⇒ coupling to continuum states becomes an important reaction channel
Models for inelastic scattering

Normal versus halo nuclei

How does the halo structure affect the elastic scattering?

- $^4\text{He} + ^{208}\text{Pb}$ shows typical Fresnel pattern → strong absorption
- $^6\text{He} + ^{208}\text{Pb}$ shows a prominent reduction in the elastic cross section due to the flux going to other channels (mainly break-up)
Inclusion of the continuum in CC calculations: continuum discretization

**Quantum Hamiltonian**

**Bound states**
- Discrete
- Finite
- Normalizable

**Unbound states**
- Continuous
- Infinite
- Non-normalizable

**Continuum discretization:** represent the continuum by a finite set of square-integrable states

*True continuum* → *Discretized continuum*

Non normalizable → Normalizable
Continuous → Discrete
Bound versus scattering states

Continuum state:

\[
\phi_{k,\ell,j}^m(r) = \frac{u_{k,\ell,j}(r)}{r} [Y_\ell(\hat{r}) \otimes \chi_s]_{jm}
\]
Models for inelastic scattering

Bound versus scattering states

Unbound states are not suitable for CC calculations:

- Continuous (infinite) distribution in energy.
- Non-normalizable: \( \langle u_{k,\ell_j}(r) | u_{k',\ell_j}(r) \rangle \propto \delta(k - k') \)

**SOLUTION** ⇒ continuum discretization
CDCC formalism: construction of the bin wavefunctions

Bin wavefunction:

\[ u_{\ell sj,n}(r) = \sqrt{\frac{2}{\pi N}} \int_{k_1}^{k_2} w(k) u_{k,\ell sj}(r) dk \]

- \( k \): linear momentum
- \( u_{k,\ell sj}(r) \): scattering states (radial part)
- \( w(k) \): weight function
Select a number of partial waves ($\ell = 0, \ldots, \ell_{\text{max}}$).

- For each $\ell$, set a maximum excitation energy $\varepsilon_{\text{max}}$.
- Divide the interval $\varepsilon = 0 - \varepsilon_{\text{max}}$ in a set of sub-intervals (bins).
- For each bin, calculate a representative wavefunction.
CDCC equations for deuteron scattering

- Hamiltonian:
  \[ H = T_R + h_r(r) + V_{pt}(r_{pt}) + V_{nt}(r_{nt}) \]

- Model wavefunction:
  \[ \Psi(R, r) = \phi_{gs}(r)\chi_0(R) + \sum_{n>0} \phi_n(r)\chi_n(R) \]

- Coupled equations: \[ [H - E]\Psi(R, r) = 0 \]

\[ [E - \varepsilon_n - T_R - V_{n,n}(R)]\chi_n(R) = \sum_{n' \neq n} V_{n,n'}(R)\chi_{n'}(R) \]

- Transition potentials:
  \[ V_{n,n'}(R) = \int dr \phi_n(r)^* \left[ V_{pt}(R + \frac{r}{2}) + V_{nt}(R - \frac{r}{2}) \right] \phi_{n'}(r) \]
What observables can we study with CDCC

- Elastic scattering
- Breakup angular distribution, as a function of excitation energy:
- Breakup energy distribution, as a function of c.m. angle:
Elastic scattering

Breakup angular distribution, as a function of excitation energy:

Breakup energy distribution, as a function of c.m. angle:

From the S-matrices, more complicated breakup observables can be obtained, such as angular/energy distribution of one of the fragments.
Application of the CDCC formalism: $d + ^{58}\text{Ni}$

- $\ell = 0, 2$ continuum
- $p + ^{58}\text{Ni}$ and $n + ^{58}\text{Ni}$ from Koning-Delaroche OMP.
- $V_{pn}(r) = -72.15 \exp[-(r/1.484)^2]$

Coupling to breakup channels has an important effect on the reaction dynamics
Application of the CDCC method: $^6$Li and $^6$He scattering

The CDCC has been also applied to nuclei with a cluster structure:

- $^6$Li = $\alpha + d$
- $^{11}$Be = $^{10}$Be + n
The CDCC has been also applied to nuclei with a cluster structure:

- $^6\text{Li}=\alpha+d$
- $^{11}\text{Be}=^{10}\text{Be}+n$

In Fraunhofer scattering the presence of the continuum produces a reduction of the elastic cross section.