1. Introduction, definitions
2. Electromagnetic transitions
3. Radiative-capture cross sections
4. Radiative capture in nuclear astrophysics
5. Application to the potential model
6. Conclusions
1. Introduction - definitions

Capture reaction = Electromagnetic transition from a scattering state to a bound state

Initial scattering state A+B
$E_i > 0$

Final bound state: $E_f < 0$
In general: several states

Energy conservation: $E_\gamma = E_i - E_f$ (recoil energy of nucleus C is negligible)

Examples:
- $(p,\gamma)$ reactions: $^7\text{Be}(p,\gamma)^8\text{B}, ~ ^{12}\text{C}(p,\gamma)^{13}\text{N}, ~ ^{16}\text{O}(p,\gamma)^{17}\text{F}$
- $(\alpha,\gamma)$ reactions: $^3\text{He}(\alpha,\gamma)^7\text{Be}, ~ ^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$

Typical values: up to a few MeV

Photon wavelength for $E_\gamma = 1$ MeV: $\lambda_\gamma = \frac{2\pi}{k_\gamma} = \frac{2\pi\hbar c}{E_\gamma} \sim 1200$ fm

$\lambda_\gamma \gg$ typical dimension $R$

$k_\gamma R \ll 1$: long wavelength approximation
1. Introduction - definitions

Importance in astrophysics:

1. **Hydrogen burning: CNO cycle, pp chain**

   ![Diagram of the CNO cycle and pp chain reactions]

   - CNO cycle(s)
   - Capture reactions
   - Transfer reactions
   - Beta decays

2. **Helium burning**: $^8\text{Be}(\alpha,\gamma)^{12}\text{C}$, $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$, $^{16}\text{O}(\alpha,\gamma)^{20}\text{Ne}$, ...

3. **Neutron capture reactions**: $^7\text{Li}(n,\gamma)^8\text{Li}$, many others...
1. Introduction - definitions

Difference fusion / radiative capture

Radiative capture: $\gamma$ channel only (electromagnetic)

Fusion:
- all channels with mass higher than the projectile and target
- In general, many channels $\rightarrow$ statistical approach

Example: $^{12}\text{C} + ^{12}\text{C}$ fusion

![Graph showing energy levels and states for $^{12}\text{C} + ^{12}\text{C}$ fusion](image)

Experimental cross section
Satkowiak et al. PRC 26 (1982) 2027
Other example: \(^{12}\text{C}(^{12}\text{C},\gamma)^{24}\text{Mg}\) radiative capture

- Existence of molecular states in \(^{24}\text{Mg}\)?
- Transitions to 3 states of \(^{24}\text{Mg}\)
- Cross section much smaller than fusion

Rad. cap: \(~100\) nb \(* 4\pi \sim 1\) \(\mu\)b (electromagnetic)

Fusion: \(~200\) mb (nuclear)
2. Electromagnetic transitions
2. Electromagnetic transitions

Transition probability

Initial state: energy $E_i$

- system of charged particles: $H = H_0$

- Wave function $\Psi_i$

Final state: energy $E_f$

- system of charged particles + photon: $H = H_0 + H_\gamma$ ($H_\gamma$ small)

- Wave function $\Psi_f$

- Energy conservation: $E_i = E_f + E_\gamma$
- Perturbation theory ($H_\gamma$ small)
- Fermi golden rule

→ Transition probability from an initial state to a final state
→ $w_{i \to f} \sim |< \Psi_f | H_\gamma | \Psi_i >|^2$ : very general definition
2. Electromagnetic transitions

Three types of electromagnetic transitions

Nucleus C=A+B

- Bound → bound: $E_i < 0, E_f < 0$
- Scattering → bound: $E_i > 0, E_f < 0$
  = radiative capture
- Scattering → scattering: $E_i > 0, E_f > 0$
  = bremsstrahlung

- In all cases $w_{i\rightarrow f} \sim |\langle \psi_f | H_\gamma | \psi_i \rangle|^2$
- But: different types of wave functions
- Bremsstrahlung: transition from continuum to continuum $\rightarrow$ strong convergence problems
2. Electromagnetic transitions

Bound $\rightarrow$ bound:

- Initial and final states characterized by spins $J_i \pi_i$ and $J_f \pi_f$
- Transition probability: $w_{i \rightarrow f} \sim |\langle \Psi_{J_f \pi_f} | H_\gamma | \Psi_{J_i \pi_i} \rangle|^2$
- $w_{i \rightarrow f}$ provides the $\gamma$ width $\Gamma_\gamma = \hbar w$, lifetime $\tau = \hbar / \Gamma_\gamma$

Scattering to bound state: radiative capture

- Final state: characterized by spin $J_f \pi_f$
- Initial state: $\Psi(E) = \text{scattering state (expansion in partial waves, all } J_i \pi_i)$
- Transition probability: $w_{i \rightarrow f}(E) \sim |\langle \Psi_{J_f \pi_f} | H_\gamma | \Psi(E) \rangle|^2$
- $w_{i \rightarrow f}$ provides the capture cross section

General property: $H_\gamma$ small $\rightarrow$ electromagnetic processes have a low probability

- Gamma widths small compared to particle widths
- Capture cross sections small compared to elastic or transfer cross sections (nuclear origin)

$\gamma$ decay: $\Gamma_\gamma = 0.5$ eV, $T = 10^{-14}$ s

Proton decay: $\Gamma_p = 32$ keV, $T = 2 \times 10^{-20}$ s

$\Rightarrow \Gamma_\gamma / \Gamma_p << 1$
Comparaison transfer – capture reactions: $^6\text{Li}(p,\alpha)^3\text{He}$ and $^6\text{Li}(p,\gamma)^7\text{Be}$

S- Factor

$$S(E) = \sigma(E)E\exp(2\pi\eta)$$

- factor $10^4$ typical of nucl/elec.
- $\gamma$ channel negligible compared to $\alpha$ channel

S factor proportional to $\sigma$

Same energy dependence (identical entrance channel)

$\sigma_t$ (transfer)$\gg\sigma_c$ (capture) : general property
2. Electromagnetic transitions

**Electromagnetic operator** $H_\gamma$

- Depends on
  - nuclear coordinates (nuclei or nucleons, according to the model)
  - photon properties (energy $E_\gamma$, emission angle $\Omega_\gamma$)

- Deduced from the Maxwell equations (+quantification)

\[
H_\gamma \sim \sum_{\lambda\mu\sigma} k_\gamma^\lambda M_\mu^{\sigma\lambda} (r_1, \ldots, r_A) D_{\mu q}^\lambda (\Omega_\gamma)
\]

- $\sigma = E$ or $M$ (electric or magnetic)
- $\lambda$=order of the multipole (from 1 to $\infty$, in practice essentially $\lambda = 1, 2$)
- $\mu$ is between $-\lambda$ and $+\lambda$
- $D_{\mu q}^\lambda (\Omega_\gamma)$=Wigner function, $\Omega_\gamma$=photon-emission angle
- $M_\mu^{\sigma\lambda}$ = multipole operators (depend on nucleon coordinates $r_i$)
2. Electromagnetic transitions

- **Electric operators**

\[
\tilde{\mathcal{M}}_{\mu}^{E\lambda} = e \sum_i \left( \frac{1}{2} - t_{iz} \right) r_i^\lambda Y_\lambda^\mu (\Omega_{ri})
\]
for a many-nucleon system (sensitive to protons only)

with \( t_{iz} = \) isospin = +1/2 (neutron)  
-1/2 (proton)

\( r_i = (r_i, \Omega_i) = \) nucleon space coordinate

- **Magnetic operators**

\[
\mathcal{M}_{\mu}^{M\lambda} = \frac{\mu N}{\hbar} \sum_i [\nabla (r^\lambda Y_\lambda^\mu (\Omega_r))]_{r=r_i} \cdot \left( \frac{2g_i}{\lambda+1} L_i + g_s S_i \right)
\]

with \( S_i = \) spin of nucleon i  
\( L_i = \) orbital momentum of nucleon i

- **Transition probability** (integral over \( \Omega_\gamma \))

\[
W_{i\rightarrow f} (E) \sim \left| \langle \Psi_J^f \pi_f | H_\gamma | \Psi_{i\pi_i} \rangle \right|^2
\]

- \( H_\gamma \) is expanded in multipoles

\[
W_{J_i\pi_i \rightarrow J_f\pi_f} \sim \sum_{\lambda, \sigma} k_\gamma^{2\lambda+1} \left| \Psi_J^f \pi_f \parallel \mathcal{M}^{\sigma\lambda} \parallel \Psi_{i\pi_i} \right|^2
\]
2. Electromagnetic transitions

- Reduced transition probability

\[ B(\sigma \lambda, J_i \pi_i \rightarrow J_f \pi_f) = \frac{2J_f + 1}{2J_i + 1} |\langle \Psi_{J_f \pi_f} \| M^\sigma \| \psi_{J_i \pi_i} \rangle|^2 \]

- Units: electric (\(\sigma=E\)): \(e^2 \text{fm}^{2\lambda}\)
magnetic (\(\sigma=M\)): \(\mu_N^2 \text{fm}^{2\lambda-2}\)
- Gamma width: \(\Gamma_\gamma (J_i \pi_i \rightarrow J_f \pi_f) = \hbar W_{J_i \pi_i \rightarrow J_f \pi_f}\)
- Total gamma width: sum over final states

Example: \(^{15}\text{O}\)
- From a given state: several transitions are possible
- In practice:
  - Selection rules
  - Factor \(k_\gamma^{2\lambda+1}\) favors large \(E_\gamma\)
2. Electromagnetic transitions

Important properties:

• Hierarchy between the multipoles: \( \frac{w(\sigma,\lambda+1)}{w(\sigma,\lambda)} \sim (k\gamma R)^2 \ll 1 \) (long wavelength approximation)
  \( E_1 \gg E_2 \approx M_1 \gg E_3 \approx M_2, \ldots \)
  \( \Rightarrow \) only a few multipoles contribute (one)

• Selection rules: \( \langle \Psi^{J_f \pi_f} \parallel M^{\sigma \lambda} \parallel \Psi^{J_i \pi_i} \rangle \)
  angular momentum \( |J_i - J_f| \leq \lambda \leq J_i + J_f \)
  parity: \( \pi_i \pi_f = (-1)^\lambda \) for \( \sigma = E \)
  \( \pi_i \pi_f = (-1)^{\lambda+1} \) for \( \sigma = M \)

• \( E_1 \) forbidden in \( N=Z \) nuclei (\( T=0 \))
  \( M^{E_1} \sim \sum \left( \frac{1}{2} - t_{iz} \right) (r_i - R_{cm}) \)

• No transition with \( \lambda = 0 \)

• Examples:
  • transition \( 2^+ \rightarrow 0^+ : E2 \)
  • transition \( 1^- \rightarrow 0^+ : E1 \)
  • transition \( 2^+ \rightarrow 1^+ : E2, M1, M3 \)
  • transition \( 3^- \rightarrow 2^+ : E1, E3, E5, M2, M4 \)
2. Electromagnetic transitions

Capture cross section (for a given final state $J_f$):

$$\sigma(J_f, E) \sim \sum_{\lambda\sigma} k^{2\lambda+1} |< \Psi_{J_f}^\pi \parallel M_{\sigma\lambda} \parallel \Psi(E)>|^2$$

with $\Psi(E) =$ initial state, expanded in partial waves $\Psi(E) = \sum J_i \Psi_{J_i\pi_i}(E)$

E1 (or E2) dominant

$\rightarrow$ picks up a few partial waves

examples: $^{12}\text{C}(p,\gamma)^{13}\text{N}$, $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$

### $^{12}\text{C}(p,\gamma)^{13}\text{N}$

A single final state: $J_f = 1/2^-$ (ground state)

Main multipolarity: E1

$\rightarrow$ Initial states: $J_i = 1/2^+, 3/2^+$

Resonance for $J_i = 1/2^+$ $\rightarrow$ enhances the cross section
2. Electromagnetic transitions

$^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$

- Several final states: $(J_f = 0^+ \text{ dominant})$
- E1 multipolarity: $J_i = 1^-$
- E2 multipolarity: $J_i = 2^+$
- Cascade transitions small (factor $k_{\gamma}^{2\lambda+1}$)

- N=Z=6
  $\Rightarrow$ E1 forbidden (T=0, long wavelength approx.)
- Exp: $E1 \approx E2$ (T=0 not strict)
3. Cross sections for radiative capture
3. Radiative-capture cross sections

We assume a spin zero for the colliding nuclei

\[
\sigma(J_f, E) \sim \sum_{\lambda \sigma} k_{Y}^{2\lambda+1} |< \Psi_{Jf}^{\pi_f} \parallel M^{\sigma\lambda} \parallel \Psi(E)>|^2
\]

- General definition, valid for any model.
  - Potential model (structure neglected)
  - Microscopic models
  - R-matrix, etc.

- Spins zero for the colliding nuclei: \( \Psi(E) = \sum_{J} \Psi_{J}(E) \)
- Essentially astrophysics applications \( \rightarrow \) low energies \( \rightarrow \) low \( J \) values
- At low scattering energies \( J_i = 0^+ \) is dominant (centrifugal barrier)
  \( \rightarrow \) \( J_f = 1^- (E1), \ J_f = 2^+ (E2) \) are expected to be dominant

\[ E \text{ small } \rightarrow \text{ penetrability decreases with } J \]
3. Radiative-capture cross sections

Example 1: $^{8}\text{Be}(\alpha,\gamma)^{12}\text{C}$

- Initial partial wave $J_i = 0^+$ (includes the Hoyle state).
- E2 dominant (E1 forbidden in N=Z)
- $\rightarrow$ essentially the $J_f = 2^+$ state is populated.
3. Radiative-capture cross sections

Example 2: $^{14}\text{N}(p,\gamma)^{15}\text{O}$

- Spin of $^{14}\text{N}$: $I_1=1^+$, proton $I_2=1/2^+$
- Channel spin $I$:
  $$|I_1 - I_2| \leq I \leq I_1 + I_2$$
  $$\Rightarrow I = 1/2, 3/2$$
- Orbital momentum $\ell$
  $$|I - \ell| \leq J_i \leq I + \ell$$
- At low energies, $\ell = 0$ is dominant
  $$J_i = 1/2^+, 3/2^+$$
- Multipolarity $E1$ transitions to $J_f = 1/2^-, 3/2^-, 5/2^-$
- Resonance $1/2^+$ determines the cross section

\[ E (\text{all } J_i \text{ values}) \]
Radiative capture in astrophysics
4. Radiative capture in astrophysics

- Elastic scattering is always possible, but does not affect the nucleosynthesis
- Essentially two types of reactions
  - Transfer
  - Capture
- capture is important only if transfer channels are closed

\[
\begin{align*}
\text{15}N+p \text{ threshold} \\
\text{15}N(p,\gamma)\text{16}O \text{ and } \text{15}N(p,\alpha)\text{12}C \text{ are open} \\
\Rightarrow \text{15}N(p,\gamma)\text{16}O \text{ negligible}
\end{align*}
\]

\[
\begin{align*}
\text{12}C+\alpha \text{ threshold} \\
\text{only possibility: } 12C(\alpha,\gamma)\text{16}O \\
\Rightarrow 12C(\alpha,\gamma)\text{16}O \text{ (very) important}
\end{align*}
\]
4. Radiative capture in astrophysics

Low energies $\rightarrow$

- cross sections dominated by coulomb effects
- Coulomb functions at low energies ($\eta$=Sommerfeld parameter=$Z_1Z_2e^2/h\nu$)
  \[ F(\eta,x) \rightarrow \exp(-\pi\eta)\mathcal{F}(x), \quad G(\eta,x) \rightarrow \exp(\pi\eta)\mathcal{G}(x) \]

$\Rightarrow$ 2 coulomb effects:
  - strong $E$ dependence: factor $\exp(-2\pi\eta)$
  - strong $\ell$ dependence

Astrophysical S factor: $S(E)=\sigma(E)*E*\exp(2\pi\eta)$ (Units: $E*L^2$: MeV-barn)

- removes the coulomb dependence $\Rightarrow$ only nuclear effects
- weakly depends on energy $\Rightarrow \sigma(E)\approx S_0\exp(-2\pi\eta)/E$
Penetration factor $P_\ell$ for d+d (2 different radii, a=5 fm and a=6 fm)

- Typical Coulomb effect
- depends on the radius
- strongly depends on energy $E$, and on angular momentum $\ell$
- $P_\ell(E) \sim \exp(-2\pi\eta)$ for $\ell = 0$
Non resonant: \( S(E) = \sigma(E) \times E \times \exp(2\pi\eta) \) approximately constant

\[ S \text{ (keV\cdot b)} \]

\[ E \text{ (MeV)} \]

\[ \sigma \text{ (barn)} \]

\[ \text{No resonance with low J} \]
4. Radiative capture in astrophysics

**Resonances**: the Breit-Wigner approximation

General properties of a resonance:
- Spin $J_R$
- Energy $E_R$
- Entrance width (here: particle width)
- Output width (here: gamma width)

Breit-Wigner approximation near a resonance: $E \approx E_R$:

$$
\sigma(E) \approx \frac{\pi}{k^2} \frac{(2J_R + 1) \Gamma_y(E) \Gamma_p(E)}{(E - E_R)^2 + \Gamma(E)^2/4}
$$

Valid for the resonant partial wave $J_R \rightarrow$ possible contribution of other partial waves

Two typical examples:
- $^{12}\text{C}(\text{p},\gamma)^{13}\text{N}$: a resonance is present in the dominant partial wave ($\ell = 0$)
- $^{7}\text{Be}(\text{p},\gamma)^{8}\text{B}$: resonance in the $\ell = 1$ partial wave $\rightarrow$ superposition of resonant and non-resonant contributions

\[ J_R = 1/2^+ \]
\[ E_R = 0.42 \text{ MeV} \]
\[ \Gamma_p = 32 \text{ keV} \]
\[ \Gamma_{\gamma} = 0.5 \text{ eV} \]
Example 1: $^{12}\text{C}(p,\gamma)^{13}\text{N}$

- $J_f=1/2^-$
- $E1 \rightarrow J_i=1/2^+, \ell_i=0 \rightarrow$ dominant
  $J_i=3/2^+, \ell_i=2$
- Resonance for $J_i=1/2^+ \rightarrow$ enhancement
- $\rightarrow$ Two effects make $J_i=1/2^+$ dominant

4. Radiative capture in astrophysics
Example 2: \(^7\text{Be}(p,\gamma)^8\text{B}\)

- \(J_f=2^+\)
- \(E1 \rightarrow J_i=1^-, l_i=0,2 \rightarrow 1^-,2^-\) dominant
  \(J_i=2^-, l_i=0,2,4\)
  \(J_i=3^-, l_i=2,4\)
- Resonance for \(J_i=1^+\)
  - only E2 or M1 are possible
  - \(l_i=1,3\)
  \(\rightarrow\) limited effect of the \(1^+\) resonance
  Both contributions must be treated separately
5. Radiative capture in the potential model
5. Radiative capture in the potential model

**Potential model**: two structureless particles (=optical model, without imaginary part)

- Calculations are simple
- Physics of the problem is identical in other methods
- Spins are neglected

• $R_{cm}$=center of mass, $r$ =relative coordinate

\[ r_1 = R_{cm} - \frac{A_2}{A} r \]
\[ r_2 = R_{cm} + \frac{A_1}{A} r \]

• Initial wave function: $\Psi_{i}^{\ell m_i}(r) = \frac{1}{r} u_{\ell i}(r) Y_{\ell i}^{m_i}(\Omega)$, energy $E_{i}^{\ell}$=scattering energy $E$

Final wave function: $\Psi_{f}^{\ell f m_f}(r) = \frac{1}{r} u_{\ell f}(r) Y_{\ell f}^{m_f}(\Omega)$, energy $E_{f}^{\ell}$

The radial wave functions are given by:

\[ -\frac{\hbar^2}{2\mu} \left( \frac{d^2}{dr^2} - \frac{\ell (\ell + 1)}{r^2} \right) u_{\ell} + V(r)u_{\ell} = E^{\ell} u_{\ell} \]
5. Radiative capture in the potential model

- Schrödinger equation: \(-\frac{\hbar^2}{2\mu} \left( \frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} \right) u_\ell + V(r) u_\ell = E_\ell u_\ell\)

- Typical potentials:
  - coulomb = point-sphere
  - nuclear: Woods-Saxon, Gaussian
    with parameters adjusted on important properties (bound-state energy, phase shifts, etc.)

- Potentials can be different in the initial and final states

- Numerical method: Numerov (finite differences, based on a step \(h\) and \(N\) points)
  - \(u_\ell(0) = 0\)
  - \(u_\ell(h) = 1\)
  - \(u_\ell(2h)\) is determined from \(u_\ell(0)\) and \(u_\ell(h)\)
  - \(\ldots \to u_\ell(Nh)\)

- \(u_\ell(r)\) is adjusted on \(u_\ell(r) = F_j(kr) \cos \delta_j + G_j(kr) \sin \delta_j\) at large distances
  - phase shift
  - the wave function is available in a numerical format
  - any matrix element can be computed
5. Radiative capture in the potential model

- Electric operator for two particles:

\[ \mathcal{M}_{\mu}^{E\lambda} = e (Z_1 |r_1 - R_{cm}|^\lambda Y_\lambda^\mu (\Omega_{r_1-R_{cm}}) + Z_2 |r_2 - R_{cm}|^\lambda Y_\lambda^\mu (\Omega_{r_2-R_{cm}})) \]

which provides

\[ \mathcal{M}_{\mu}^{E\lambda} = e \left[ Z_1 \left( - \frac{A_2}{A} \right)^\lambda + Z_2 \left( \frac{A_1}{A} \right)^\lambda \right] r^\lambda Y_\lambda^\mu (\Omega_r) = eZ_{eff} r^\lambda Y_\lambda^\mu (\Omega_r) \]

- Matrix elements needed for electromagnetic transitions

\[ < \Psi J_f m_f | \mathcal{M}_{\mu}^{E\lambda} | \Psi J_i m_i > = eZ_{eff} < Y_{J_f}^{m_f} | Y_\lambda^\mu | Y_{J_i}^{m_i} > \int_0^\infty u_{J_i}(r) u_{J_f}(r) r^\lambda dr \]

- Reduced matrix elements:

\[ < \Psi J_f | \mathcal{M}^{E\lambda} | \Psi J_i > = eZ_{eff} < J_f 0\lambda 0 | J_i 0 > \times \left( \frac{(2J_i+1)(2\lambda+1)}{4\pi(2J_f+1)} \right)^{1/2} \int_0^\infty u_{J_i}(r) u_{J_f}(r) r^\lambda dr \]

\[ \rightarrow \text{simple one-dimensional integrals} \]
Radiative capture in the potential model

Assumptions:

- spins zero: $\ell_i = J_i, \ell_f = J_f$
- given values of $J_i, J_f, \lambda$

Integrated cross section

$$\sigma_\lambda(E) = \frac{8\pi e^2}{k^2 \hbar c} Z_{eff}^2 k_\gamma^{2\lambda+1} F(\lambda, J_i, J_f) \left| \int_0^\infty u_{J_i}(r, E) u_{J_f}(r)r^\lambda dr \right|^2$$

with

- $Z_{eff} = Z_1 \left(-\frac{A_2}{A}\right)^\lambda + Z_2 \left(\frac{A_1}{A}\right)^\lambda$
- $F(\lambda, J_i, J_f) = <J_i\lambda 0 0|J_f 0> (2J_i + 1) \frac{(\lambda+1)(2\lambda+1)}{\lambda(2\lambda+1)!!^2}$
- $k_\gamma = \frac{E-E_f}{\hbar c}$

Normalization

- final state (bound): normalized to unity $u_J(r) \rightarrow C \exp(-k_B r)$
- initial state (continuum): $u_J(r) \rightarrow F_J(kr) \cos \delta_J + G_J(kr) \sin \delta_J$

Test: $\lambda = 0$ provides $\sigma_\lambda(E) = 0$ (orthogonality of the wave functions)
Integrated vs differential cross sections

- **Total (integrated) cross section:**
  \[ \sigma(E) = \sum_{\lambda} \sigma_\lambda(E) \]
  \(\rightarrow\) no interference between the multipolarities

- **Differential cross section:**
  \[ \frac{d\sigma}{d\theta} = \left| \sum_{\lambda} a_\lambda(E)P_\lambda(\theta) \right|^2 \]

  \(P_\lambda(\theta)=\text{Legendre polynomial}\)
  \(\rightarrow\) interference effects
  \(\rightarrow\) angular distributions are necessary to separate the multipolarities (in general one multipolarity is dominant)
Example: $^{12}\text{C}(p,\gamma)^{13}\text{N}$

- First reaction of the CNO cycle
- Well known experimentally
- Presents a low energy resonance (l=0 $\rightarrow$ J=1/2+)

Potential: \[ V = -55.3 \times \exp(-r/2.70)^2 \] (final state)\[ -70.5 \times \exp(-r/2.70)^2 \] (initial state)
5. Radiative capture in the potential model

Final state: $J_f=1/2^-$
Initial state: $l_i=0 \rightarrow J_i=1/2^+$
$\rightarrow$ E1 transition $1/2^+ \rightarrow 1/2^-$

$^{12}\text{C} + p$

$^{13}\text{N}, J=1/2^-$

Final state: $E_f=-1.94 \text{ MeV}$

Initial wave function: $E_i=100 \text{ keV}$

Integrant $E_i=100 \text{ keV}$
5. Radiative capture in the potential model

The calculation is repeated at all energies

\[ S \text{ (MeV-b)} \]

\[ S = \frac{1}{2} u_f(r) \]

S = spectroscopic factor

Other applications: \( ^7\text{Be}(p,\gamma)^8\text{B} \), \( ^3\text{He}(\alpha,\gamma)^7\text{Be} \), etc…

Necessity of a spectroscopic factor \( S \)

Assumption of the potential model: \( ^{13}\text{N}=^{12}\text{C}+p \)

In reality \( ^{13}\text{N}=^{12}\text{C}+p \oplus ^{12}\text{C}^*+p \oplus ^9\text{Be}+\alpha \oplus \ldots \)

→ to simulate the missing channels: \( u_f(r) \) is replaced by \( S^{1/2} u_f(r) \)