PHY862 Accelerator Systems
Beam Measurements and Instrumentation
Interactions and Sensors I

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Introduction

- Measurements, diagnostics, instrumentation
- Interactions and sensors
- Beam generated signals
- Diagnostic architecture
- References and exercises
Role of diagnostics and instrumentation

Beams are composed of 100’s to $10^{8}$’s of individual particles. Getting them all from point A to point B can be a challenge

We need to perform certain operations on the beam
- Tuning, optimization of beam quality
- Targetry, beam collisions
- Monitoring, stability, minimizing losses, machine protection
Measurements, diagnostics, and instrumentation

• **Measurements**
  • Incorporate (incomplete) knowledge of lattice, beam dynamics
  • Point vs. Distributed (lattice-dependent) measurements

• **Diagnostics** are phenomena or techniques involved in performing a measurement
  • Direct vs indirect measurements
  • Correlations – use known dependencies

• **Instrumentation** is the set of particular devices used in the execution of the measurement
  • Set of beam sensors, signal transmission lines, data acquisition and reporting systems, controls and feedback
## Diagnostics vs. Instrumentation examples

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Diagnostic</th>
<th>Instrumentation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam current</td>
<td>Beam wall return current</td>
<td>AC Current Transformer + electronics</td>
</tr>
<tr>
<td>Beam position</td>
<td>Beam E-field distribution @ walls</td>
<td>Capacitive pickups + electronics</td>
</tr>
<tr>
<td>Beam emittance (1DoF)</td>
<td>Quad scan</td>
<td>Quadrupole magnet + view screen + camera</td>
</tr>
</tbody>
</table>

Particular diagnostic/instrumentation methodologies rely on particular types of beams and beam parameters/lattice functions, available beamline space and shielding, required measurement accuracy and precision, cost, . . .
Typical beam measurements

Measurements and associated diagnostics and instrumentation are specific to

- Geometry of beamline
  - linac, synchrotron booster, storage ring, analyzing beamline, injector, final focus, etc.)
- Particle type
  - Hadron, lepton, neutron, neutral atom, rare isotope
- Beam energy
- Beam intensity
- Beam time structure
  - . . .
SIS-18 Synchrotron at GSI: diagnostic suite

<table>
<thead>
<tr>
<th>Device</th>
<th>Purpose</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 BPMs</td>
<td>Position, tune</td>
</tr>
<tr>
<td>3 Phase pick-up</td>
<td>Longitudinal structure</td>
</tr>
<tr>
<td>Quad pick-up</td>
<td>Tune, Quadrupole oscillations</td>
</tr>
<tr>
<td>Schottky pick-up</td>
<td>Schottky diagnostics</td>
</tr>
<tr>
<td>2 DC-CTs</td>
<td>Current</td>
</tr>
<tr>
<td>1 FCT</td>
<td>Bunch structure</td>
</tr>
<tr>
<td>1 ACT</td>
<td>Injected current</td>
</tr>
<tr>
<td>1 IPM</td>
<td>Transverse profile</td>
</tr>
<tr>
<td>1 Wire grid</td>
<td>Transverse profile</td>
</tr>
<tr>
<td>1 Scint. screen</td>
<td>Transverse profile</td>
</tr>
<tr>
<td>2 Beam exciters</td>
<td>Excitation</td>
</tr>
<tr>
<td>15 BLMs</td>
<td>Beam losses</td>
</tr>
</tbody>
</table>

(a) SIS-18 Synchrotron at GSI: diagnostic suite

(b) Table of diagnostic devices and their purposes.
Beam parameters [1] – 1st Order Moments

- Charge/current, charge states, mass states
- Beam energy
- Beam position, orbit, tune

(36x21) Charge/current, charge states, mass states

- Beam energy

- Beam position, orbit, tune

Schottky spectra of stored and cooled rare isotopes from $^{197}$Au$^{79+}$. Spectra of 16th revolution harmonic.

(Schlitt, et al., Nucl. Phys A626 (1997), 315.)
Beam parameters [2] – 2nd Order (+ higher) Moments

- Profile (transverse, longitudinal), envelope, energy spread
- Phase space density, emittance measures
- Beam halo - transverse, longitudinal

Beam halo with wire scanner measurements
Beam parameters [3] – Bunch Trains

- Single bunch vs. many bunch measurements
- Time domain vs. frequency domain
Lattice parameters

- **Betatron tune** $Q_x$, $Q_y$

- **Dispersion function**

  - Chromatic Correction Section
  - BPMs @ A, B, C, D
  - Large energy spread
  - Nominal energy spread

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Beam Measurements and Instrumentation I-II
Lattice parameters

- Loss distributions in SC RF modules
- Beam envelope matching to lattice

- Power density (Gy/s) at 200 MeV

- Matched distributions
- Mismatched distributions
Dynamic parameters

- Emittance growth for mismatched beams
- Instabilities
- Electron cloud effects

Direct phase measurement in resonant BPM configuration

(DeSantis, et al, PAC09 TH5RFP071)
Diagnostic/instrumentation design elements

- **Physics** of beam-sensor interactions
  - EM, Nuclear, AMO, Solid State
  - Charge and mass interception
  - Capacitive, inductive, resonant, thermal field sensing
  - Secondary radiation fields

- **Mechanical** design
  - Thermal, structural, vacuum, actuator

- **Electrical** design
  - Grounding/shielding
  - HV bias and insulation

- **Electronics**
  - Signal acquisition, conditioning, processing
  - Noise, bandwidth, sensitivity, response time
Faraday cups

- Fully intercepting charge measurement
- Sensitivity to 10 pC. ~100 Hz BW (‘slow’, deep cup)
- Beam charges impinge on Collector, are collected by electronics
- Suppressor negatively biased to repel electrons
- Design is to prevent escape of secondary electrons

**Diagram:**

- **Incoming ions**
- **Suppressor**
- **Collector**
- **Transimpedance amplifier**
- **Negative HV Bias**
Viewers

• Based on scintillator and camera
  • Coated screen (reflection mode)
  • Solid, thick (100 μm) scintillator (transmission mode)
• Scintillator can be single- or multi-crystalline, or sintered powder
• Direct 2D measurement
  • Direct digital output
  • Video out and frame grabber
• Resolution depends on scintillator material (grains), CCD size, optics
• Amplitude response depends on field flatness, scintillator dose and aging effects, temperature
Allison Scanner

• Analyzes intensity $J(x \text{ or } y)$ at first slit
• Applied voltage across plates + drift + exit slit analyzes momentum
• Reconstructs phase space density

![Diagram of Allison Scanner](image)

- $\varepsilon_x = 0.0854 \text{ mm-mrad}$
- $\alpha_x = -1.41$
- $\beta_x = 0.54 \text{ m}$

- $x_{cen} = 6.74 \text{ mm}$
- $x_{pcen} = 8.89 \text{ mrad}$
- $x_{rms} = 3.01 \text{ mm}$
- $x_{prms} = 9.69 \text{ mrad}$
- $x_{emitn} = 0.0854 \text{ mm-mrad}$
- $x_{alpha} = -1.41$
- $x_{beta} = 0.54 \text{ m}$
- $x_{gamma} = 5.58 / \text{m}$

- $\varepsilon_y = 0.0834 \text{ mm-mrad}$
- $\alpha_y = 0.36$
- $\beta_y = 2.57 \text{ m}$

- $y_{cen} = 5.71 \text{ mm}$
- $y_{pcen} = 3.38 \text{ mrad}$
- $y_{rms} = 6.49 \text{ mm}$
- $y_{prms} = 2.69 \text{ mrad}$
- $y_{emitn} = 0.0834 \text{ mm-mrad}$
- $y_{alpha} = 0.36$
- $y_{beta} = 2.57 \text{ m}$
- $y_{gamma} = 0.44 / \text{m}$
Pepperpots, slits, and pinholes

- Devices scan a 1D or 2D beam distribution
- Analyze intensity $J(x,y)$
- Analyze transverse velocity over drift
Emittance analysis

\[
\Sigma_4 = \begin{pmatrix}
\langle xx \rangle & \langle xx' \rangle & \langle xy \rangle & \langle xy' \rangle \\
\langle xx' \rangle & \langle x'x' \rangle & \langle x'y \rangle & \langle x'y' \rangle \\
\langle xy \rangle & \langle x'y \rangle & \langle yy \rangle & \langle yy' \rangle \\
\langle xy' \rangle & \langle x'y' \rangle & \langle yy' \rangle & \langle y'y' \rangle \\
\end{pmatrix} = \begin{pmatrix}
\Sigma_x & C \\
C^T & \Sigma_y \\
\end{pmatrix}
\]

\[
\langle fg \rangle = \frac{\sum_i \rho_i f_i g_i}{\sum_i \rho_i}
\]
det \(\Sigma_x = |\Sigma_x| = \langle xx \rangle \langle x'x' \rangle - \langle xx' \rangle^2 = \varepsilon_x^2\]

\[
\gamma x^2 + 2\alpha xx' + \beta x'^2 = \varepsilon
\]

\[
\Delta x_0' = [\varepsilon/\beta - x_0^2/\beta^2]^{1/2}
\]

\[
(x_0', -\alpha/\beta \ x_0)
\]

Aperture, 2a
Hole separation, d
Drift, L
Beam Position Monitors

- Beam positions can be monitored using a 4-electrode array of capacitive pickups on the beampipe circumference.
- Various geometries are employed for sensitivity, compactness, protection from intense radiation.

Figure 8. Field lines and equipotential contours found by conformal mapping.
BPM Position Algorithm

- Positions are estimated from the normalized intensities using the difference over sum algorithm:
  \[\Delta x = \frac{1}{S_x} \frac{V_2 - V_4}{V_2 + V_4}, \quad \Delta y = \frac{1}{S_y} \frac{V_1 - V_3}{V_1 + V_3}\]

- Position sensitivities are proportionality constants between beam displacement and signal strength.
  - \(S_x = \frac{d}{dx} \left(\frac{\Delta x}{\Sigma x}\right) \left[\frac{\%}{mm}\right]\) or \(S_x = \frac{d}{dx} \log \left(\frac{V_2}{V_4}\right) \left[\frac{dB}{mm}\right]\)

- Offset displacements also occur and must be measured and calibrated.

- Button-button capacitive coupling introduces frequency dependent offset and sensitivity variation.

- Intensities at each button can be calculated from the transfer impedance, using the electrode surface area.
Profile monitors

- Interceptive diagnostic onto W, C, Cu-Be wires
- Wide range of wire based geometries
- Biased wire to discourage (or encourage) secondary e⁻’s
- Slit + Faraday cup
Reconstruct beam parameters from profile data

- We observe day-to-day variation of transverse beam parameters
  - Two most significant factors are: ECR setting and beam center matching to the RFQ
Slice envelope properties from slit/slit-cup diagnostic

50 mA Li+

133 kV, 2.2 Tesla

Normalized Emit: [mm mrad]

Current H
Current V
Emit H
Emit V

1.2 - 1.202 usec
Far field EM radiation for high energy charged particle beams

- Bremmstrahlung
- Synchrotron

- Other types found in beam diagnostics
  - Cerenkov
  - Transition
  - Diffraction (electrons)
Synchrotron Radiation

- Relativistic beams, $\gamma >> 1$
- Parasitic, nonintercepting
- Photon image reproduces electron beam distribution
- Optics, coupling
- Impedances, instabilities
- IR $\rightarrow$ Hard X-rays

$$\epsilon_{\text{crit}} = \hbar \omega_c = \frac{3h \gamma^3 c}{2\rho} = 0.665 \frac{E^2 \text{[GeV]}}{B \text{[T]}} \text{[keV]}$$

$$\nu_{\text{crit}} = \frac{1}{\gamma^2 - \left(\frac{v}{c}\right)^2} = \frac{E}{m_0 c^2}$$
Synchrotron radiation diagnostic beamline

- Multiple/simultaneous ways to measure relativistic beams

(Figure courtesy J. Corbett, W. Cheng, A. Fisher, W. Mok)
Transition vs. Cerenkov Radiation

Transition radiation occurs when a charge crosses a boundary of two dielectric media. No acceleration is required, nor is it necessary for charge to move faster than the speed of light as in Cerenkov radiation. In this respect, transition radiation is a least demanding radiation mechanism. Radiation emitted from a charge approaching a conductor is an extreme case of transition radiation with an infinite permittivity, and may be regarded as the inverse process of radiation accompanying $\beta$ decay. Disappearance, rather than creation, of charge is responsible for transition radiation.

Cherenkov radiation occurs when a charged particle travels faster than electromagnetic waves in a material medium. It does not require acceleration of charges and the basic mechanism is very similar to that of sound shock waves in gases. As in the case of $\beta$ decay, we assume a charge travelling along a straight line at a velocity $\beta = v/c(\omega)$, where

$$c(\omega) = \frac{1}{\sqrt{\varepsilon(\omega)\mu_0}}$$

**Coherent formation length**

$$d(\omega) = \frac{2c/\omega}{\left(1/\gamma^2 + \beta^2 + \omega_p^2/\omega^2\right)}$$

**Dielectric constant, $\varepsilon$**

**Metallic or metallized foils, plates**

Mild to ultrarelativistic particles

**Fast particles in background gas or dielectric windows**

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Beam Measurements and Instrumentation I-II  
28
Bunch Shape Monitor

- Used for hadron beamlines
- Scanning wire produces secondary electrons
- Electrons are accelerated in DC field, sorted in RF field
- Time-correlation converted to position on detector

Beam Loss Measurements

• Beam losses provide useful information on
  • Beam orbit deviations
  • Mismatches between beam distributions and lattice design; beam halo
  • Energy and energy spread mismatches to lattice through chromaticity

• Uncontrolled beam losses are potentially harmful to the machine
  • Damage to sensitive components (cryomodules!)
  • Radioactivation of high loss areas of the beamline – affects maintenance and access

• Diagnostics employed to detect losses
  • Beam current/intensity, often in a differential mode to detect changes
  • Secondary radiation production – gammas, neutrons, electrons
  • Others – halo monitors, beamline thermometry, changes to cryo loading
Ionization chambers

Gas-type ionization chambers are in wide use as x-ray and gamma detectors

133 cm³ Ar gas
Typical bias 1 kV
Sensitivity 70 nC/rad
Response time ~1-2 µs

<table>
<thead>
<tr>
<th>Gas</th>
<th>Density, mg cm⁻³</th>
<th>$E_X$, eV</th>
<th>$E_I$, eV</th>
<th>$W_I$, eV</th>
<th>$dE/dx$, keV cm⁻¹</th>
<th>$N_P$, cm⁻¹</th>
<th>$N_T$, cm⁻¹</th>
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<tbody>
<tr>
<td>He</td>
<td>0.179</td>
<td>19.8</td>
<td>24.6</td>
<td>41.3</td>
<td>0.32</td>
<td>3.5</td>
<td>8</td>
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<tr>
<td>Ne</td>
<td>0.839</td>
<td>16.7</td>
<td>21.6</td>
<td>37</td>
<td>1.45</td>
<td>13</td>
<td>40</td>
</tr>
<tr>
<td>Ar</td>
<td>1.66</td>
<td>11.6</td>
<td>15.7</td>
<td>26</td>
<td>2.53</td>
<td>25</td>
<td>97</td>
</tr>
<tr>
<td>Xe</td>
<td>5.495</td>
<td>8.4</td>
<td>12.1</td>
<td>22</td>
<td>6.87</td>
<td>41</td>
<td>312</td>
</tr>
<tr>
<td>CH₄</td>
<td>0.667</td>
<td>8.8</td>
<td>12.6</td>
<td>30</td>
<td>1.61</td>
<td>28</td>
<td>54</td>
</tr>
<tr>
<td>C₂H₆</td>
<td>1.26</td>
<td>8.2</td>
<td>11.5</td>
<td>26</td>
<td>2.91</td>
<td>48</td>
<td>112</td>
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<tr>
<td>C₅H₁₀</td>
<td>2.49</td>
<td>6.5</td>
<td>10.6</td>
<td>26</td>
<td>5.67</td>
<td>90</td>
<td>220</td>
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<tr>
<td>CO₂</td>
<td>1.84</td>
<td>7.0</td>
<td>13.8</td>
<td>34</td>
<td>3.35</td>
<td>35</td>
<td>100</td>
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<tr>
<td>CF₄</td>
<td>3.78</td>
<td>10.0</td>
<td>16.0</td>
<td>54</td>
<td>6.38</td>
<td>63</td>
<td>120</td>
</tr>
</tbody>
</table>

1.5 L volume
100 mbar overpressure N₂
0.5-mm separated Al plates
Bias 1500 V
Sensitivity ~ 54 µC/Gy
Response time ~300 ns e⁻, 80 µs ions
 Ionization chamber schematic

Visualisation of ion chamber operation

Key:
- Ionisation event
- Electron
- +Ve ion

Incident radiation particle
Electric field
Anode
Cathode
DC Voltage Source
Ion Current

+ -
Scintillation based detectors (gammas, neutrons)

- Typically employ photomultiplier tubes for high gain ($10^5$-$10^8$) with applied HV
- Many types of scintillators fluoresce under gamma bombardment
- Li- or B- doped plastic scintillators respond to neutrons
- Additional moderation increases sensitivity at the expense of time response.
- Outside Cd layer provides discrimination against gammas

SNS Fast Detector

Sensitivity tuned with bias

SNS Neutron chamber (SBLM)
FRIB Beam Loss Monitoring Network

- Halo monitoring rings (fast/slow loss)
- Fast beam pipe thermometry (slow loss)
- Differential current monitoring network (fast loss)

**LS1 monitors**
- ICs in warm sections
- Along LS3
- Fast and slow loss detection

**Ion chambers**

**System Wide**
- DBCM
- HMR

**Neutron detectors**
- LS2/3, LS1 (tuning only)
- Mix of fast/slow detection
Exercise 1

• Bias for a Faraday Cup

A 5 eμA beam of Uranium ions with 12 keV/nucleon energy is collected by a Faraday cup for measurement.

The Rutherford differential cross section for scattering between slow heavy ions and free electrons defines a maximum transfer energy, \( W_{\text{max}} = 2m_e c^2 \beta^2 \gamma^2 \). Further, assume that atomic ionization of the Faraday cup collector material requires ~5 eV, and escaping the work function of the cup surface requires 2 eV. Otherwise, assume that all liberated electrons exit normally from the surface of the cup.

a) What is the maximum energy of (in eV) of the emitted electrons? What bias potential needs to be present in order to trap these electrons?

b) Assume that each ion continues to ionize the cup material and produce electrons as it continues traveling into the collector. We will treat this process approximately as a series of discrete events. Assume that at each subsequent event the ion energy has been reduced such that its energy varies as \( W_{\text{max,n}} = W_{\text{max}} e^{-n/5} \), where \( n = 0, 1, 2, 3, \ldots \) indexes the event order. Assume that each event produces a single electron per ion. Plot the measured beam current as a function of bias in the range of -100V to 100V.
Exercise 2

• Pepperpot Design

A pepperpot is to be installed in a 100 keV proton injector beamline. Assume the hole mask is patterned with 0.250 mm diameter holes, spaced 2 mm apart (hole center to hole center) in a rectangular array, and that the mask is 2 mm thick Tungsten.

Assume the beam distribution is Gaussian in x/x’/y/y’, with rms normalized emittances of 0.1 pi mm mrad in x and 0.2 pi mm mrad in y. Assume the x and y phase spaces are uncorrelated. Also assume that the beam has rms extent of 15 mm and 20 mm in x and y, respectively.

If the pepperpot mask location coincides with the beam waist, what drift length between mask and screen is required for 50% intensity modulation between peak and trough on the screen?
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Beam Measurements and Instrumentation
Interactions and Sensors II

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- Beam generated signals
- Diagnostic architecture
- References and exercises
Lienard-Wiechert Fields

\[ E(x, t) = \frac{q}{4\pi\varepsilon_0} \left[ \frac{n - \beta}{\gamma^2 (1 - \beta \cdot n)^3 R^2} + \frac{1}{c} \frac{n \times ((n - \beta) \times \beta)}{(1 - \beta \cdot n)^3 R} \right] \]

\[ B(x, t) = \frac{1}{c} [n \times E]_{\text{ret}} \]

Velocity fields
Acceleration fields
Transverse fields \( \sim \gamma \)
Longitudinal fields \( \sim 1/\gamma^2 \)
Uniform motion, \( \beta \ll 1 \)
Fields of beam bunches (constant velocity)

- Lorentz Transformation of coordinates (here \( n = \beta / \beta \))
  \[
  ct' = \gamma (ct - \beta n \cdot r) \\
  r' = r + (\gamma - 1)(r \cdot n) n - \gamma \beta c t n
  \]

- Lorentz Transformation of fields
  \[
  E_{\parallel}' = E_{\parallel} \\
  B_{\parallel}' = B_{\parallel} \\
  E_{\perp}' = \gamma (E_{\perp} + v \times B) \\
  B_{\perp}' = \gamma \left( B_{\perp} - \frac{1}{c^2} v \times E \right)
  \]

- Lorentz Transformation of charge and current densities
  \[
  c \rho' = \gamma (c \rho - \beta n \cdot J) \\
  J' = J + (\gamma - 1)(J \cdot n) n - \gamma \beta c \rho n
  \]

For a point charge on axis, the extent of the pulse is approximated by (cf. Shafer)

\[
\sigma_t \approx \frac{b}{\sqrt{2} \gamma \beta c}
\]
In the limit of neglecting longitudinal end effects (bunch length >> pipe diameter), we can solve the 2D Laplace equation in the beam rest frame with Doppler shifted spectrum.

Include modulation effects (wavelength >> pipe diameter).

For long pulses that are nonrelativistic or only mildly relativistic, the fields are well approximated by electrostatics.

Intense beams may require self-magnetic field corrections.

Figure 8. Field lines and equipotential contours found by conformal mapping.
Simple beam model

- Assume a beam, carrying current $I$, of radius $a$ centered in a pipe of radius $b$.
  \[ \rho = \frac{1}{\pi a^2} \frac{I}{v} = \frac{\lambda}{\pi a^2} \]
- The radial electric field at the pipe surface is
  \[ E_r = \frac{\rho a^2}{2\varepsilon_0 b} = \frac{\lambda}{2\pi\varepsilon_0 b} \]
- The surface charge density induced at $r=b$ is
  \[ \sigma_s = \varepsilon_0 E_r = \frac{\lambda}{2\pi b} \]
- The azimuthal magnetic field at the pipe surface
  \[ B_\phi = \frac{\mu_0 I}{2\pi b} \]
- With surface current density (longitudinally)
  \[ K_s = \frac{-I}{2\pi b} = -v\sigma_s \]
Wall currents and charges

- Off center beam in pipe creates azimuthal surface charge density distribution

\[ \sigma(\theta) = \frac{-\lambda_b}{2\pi b} \left[ \frac{b^2 - \rho^2}{b^2 + \rho^2 - 2b\rho \cos(\theta - \phi)} \right] \]

\[ \sigma(\theta) = \frac{-\lambda_b}{2\pi b} \left[ 1 + 2 \sum_{n=1}^{\infty} \left( \frac{\rho}{b} \right)^n \cos(n(\theta - \phi)) \right] \]

\[ E_{\perp,\text{lab}}(t) = \gamma \cdot E_{\perp,\text{rest}}(t') \]
Time/Frequency description of beam signals

- RF and beam pulse structure
- Compromise between
  - Cavity rf frequency (aperture, transit factor)
  - Power generation and handling (CW/pulsed)
  - Experimental requirements

Capacitive probe
Why is the signal bipolar?
Fourier Series and Transforms

We define the use of symmetrical transforms

\[
\tilde{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt \ f(t) \ e^{j\omega t}
\]

\[
f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega \ \tilde{f}(\omega) \ e^{-j\omega t}
\]

\[
f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)
\]

\[
a_n = \frac{2}{T_0} \int_{T_0}^{T_0} dt \ f(t) \cos n\omega_0 t
\]

\[
b_n = \frac{2}{T_0} \int_{T_0}^{T_0} dt \ f(t) \sin n\omega_0 t
\]

Time domain          Frequency domain
<table>
<thead>
<tr>
<th>Function name</th>
<th>Function</th>
<th>Fourier transform</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delta function</td>
<td>$\delta(t)$</td>
<td>$\frac{1}{\sqrt{2\pi}}$</td>
</tr>
<tr>
<td>Impulse train (Delta function ‘comb’), period $T$</td>
<td>$\Pi(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$</td>
<td>$\Pi(\omega) = \frac{1}{\sqrt{2\pi}} \sum_{n=-\infty}^{\infty} e^{-j\omega nT}$</td>
</tr>
<tr>
<td>Gaussian</td>
<td>$A e^{-\frac{t^2}{2\sigma^2}}$</td>
<td>$A\sigma e^{-\frac{1}{2}\omega^2\sigma^2}$</td>
</tr>
<tr>
<td>Boxcar</td>
<td>$boxcar(t; A, a, b)$</td>
<td>$\frac{A}{\sqrt{2\pi}} (b - a) e^{-j\omega \frac{a+b}{2}} \text{sinc} \left( \omega \frac{b - a}{2} \right)$</td>
</tr>
</tbody>
</table>

$H(t)$ is the Heaviside step function.
Beam spectra examples

- Beam signals reveal the bunch structure and loading or fill pattern
- Multibunch dynamics are revealed in sidebands and harmonics of underlying carrier frequencies

Below is some raw spectra of a BPM sum signal from the ALS showing coupled bunch oscillations. The measurement was made with 328 bunches (all RF buckets) filled as equally as possible. The number of rotation harmonics from the RF frequency is given in each graph. The upper graph was measured at 20 mA and the lower at 95 mA.
Beam Coupling to its Environment

• Beams carry signals, encoded in their time/frequency structure, transverse position, energy, etc.
  • Time-dependent description (wakefields)
  • Frequency-dependent description (impedances)
• Charged particle beams couple electromagnetically to their environment
  • Noninterceptive means – resistive, capacitive, inductive, resonant, radiative
• Accelerator beams represent ‘nearly perfect current sources’
  • Very high source impedance
• Sometimes the environment drives back on the beam
  • Lorenz reciprocity
  • Beam loading, instabilities!
Assuming Cartesian geometry and variation only along \( n_{12} \) we can show that

\[
B_{\perp}(z) = B_{\perp}(z = 0)e^{-(1+j)\kappa z} \quad J_{\parallel}(z) = J_{\parallel}(z = 0)e^{-(1+j)\kappa z} \quad \text{where } \kappa = 1/\delta_c = \sqrt{\frac{\mu \omega \sigma}{2}}
\]
Surface resistance and Joule losses

- The tangential electric field in the conductor derives from Ohm’s Law (not present in perfect conductor)

\[ E = \frac{1}{\sigma} \nabla \times H \cong \frac{1}{\sigma} \mathbf{n} \times \frac{\partial H}{\partial z} = -Z_s \mathbf{n} \times H_\perp \]

- The transverse electric field satisfies an impedance boundary condition, with surface impedance, \( Z_s \)

\[ Z_s = \frac{1 + js\text{gn}(\omega)}{\sigma \delta_c} = R_s (1 + js\text{gn}(\omega)) \]

- A surface resistance (Ohms) is defined as

\[ R_s = \frac{1}{\sigma \delta_c} = \sqrt{\frac{\mu \omega}{2\sigma}} \]

- Power deposition (W/area) to the surface follows from

\[ \frac{dP_{surf}}{dA} = \mathbf{S} \cdot \mathbf{n} = \frac{1}{2} \mathfrak{N}(\mathbf{E} \times \mathbf{H}^*) \cdot \mathbf{n} = \frac{1}{2} R_s |K_s|^2 \]
Resistive wall impedance

The beam senses the wall through resistive loading

- The longitudinal resistive wall impedance can be defined as

\[
\frac{Z_0^\parallel}{\text{length}} = \frac{Z_s}{2\pi b} = \frac{1}{2\pi b} \sqrt{\frac{\mu \omega}{2\sigma}} (1 + j \text{sgn}(\omega))
\]

- The beam will experience a voltage change

\[
V(\omega) = -I(\omega)Z_0^\parallel(\omega)
\]

- Impedance: \( Z_0^\parallel = ReZ_0^\parallel + j ImZ_0^\parallel \) (longitudinal, monopole)

What is the significance of the of the resistive and reactive components?
Building a detector

- A nonintercepting monitor can be based on monitoring the wall return currents.
- A ceramic break in the beampipe will force the wall current to seek other paths.
- If nothing else is done, the wall currents will find alternative paths.
- The gap impedance is a combination of the gap capacitance and all external parallel elements.
  - At low frequencies the lowest impedance return path can be distant from the gap itself.
- The gap voltage $V_{\text{gap}} = I_{\text{wall}}Z_{\text{gap}} = I_{\text{beam}}Z_{\text{gap}}$ can be generated up to the beam voltage.
Impedance models and behavior

- We model the beam-monitor interaction with an equivalent circuit.
- Beam drive is modeled as a pure current source (infinite input impedance).
- A gap impedance $C_{gap}$ is inevitably present:
  - Electrodes pierce the beam wall with isolated feedthroughs.
  - Typically few – 100s pF.
- The specific signal pickup as well as the signal transmission line and passive analog components are represented by $Z_{mon}$.

\[
V_{mon}(\omega) = i_w(\omega) \frac{Z_{mon}}{1 - j\omega C_{gap}Z_{mon}} = i_b(\omega)Z_t(\omega)
\]
We add a network of $n$ resistors across the gap.

\[ R_{\text{tot}} = \frac{R_{\text{single}}}{n} \]

- Broadband pickup
  - \( V_{\text{mon}}(\omega) = i_b(\omega) \frac{R_{\text{tot}}}{1 - j\omega C_{\text{gap}}R_{\text{tot}}} \)
  - Within passband \( V_{\text{mon}}(\omega) = \left( \frac{R_{\text{single}}}{n} \right) i_b(\omega) \)

- Practical implementations (eg. SPS WCM)
  - Ceramic gap
  - Many resistors (30 – 100) to reduce sensitivity to beam position
  - Ferrite rings to tailor low frequency response \( \sim 10 \) kHz
  - High frequency response to several GHz
  - Shield for ground currents and noise isolation
Wall Current Monitor equivalent circuit and response

- Very high frequency response dominated by gap capacitance
- Very low frequency response dominated by ferrite and shield induction

\[ \omega_{cutoff} = \frac{1}{RC} \]
\[ \omega_{high} = \frac{1}{RC} \]
\[ \omega_{low} = \frac{R}{L} \]
Induced charge densities from capacitive coupling

• Induced charge densities on the beam pipe walls can be approximated or numerically calculated.

• Total induced charges on the electrodes (assuming 2D Laplace solution) for electrodes of length L

\[ Q_{\text{plate}} = Lb \int_{\text{sector}} \sigma(\theta) d\theta \]

\[ \sigma(\theta) = \frac{-\lambda_b}{2\pi b} \left[ \frac{b^2 - \rho^2}{b^2 + \rho^2 - 2b\rho \cos(\theta - \phi)} \right] \]

\[ \sigma (\theta) = \frac{-\lambda_b}{2\pi b} \left[ 1 + 2 \sum_{n=1}^{\infty} \left( \frac{\rho}{b} \right)^n \cos(n \{\theta - \phi\}) \right] \]
Signals from capacitive coupling

- The pickup plate presents a capacitance \( C \) to ground.
- The finite length of the pickup drives a differential current into the monitor.
- \( V_{\text{mon}}(\omega) = i_B(\omega)Z_{\|}(\omega) \)
- The image current on the pickup is related to the beam current
  \[
  i_{\text{im}}(t) = -\frac{A}{2\pi a} \frac{dQ_{\text{beam}}}{dt} = -\frac{A}{2\pi a} \frac{l}{\beta c} \frac{di_B}{dt}
  \]
  \[
  i_{\text{im}}(\omega) = j\omega \frac{A}{2\pi a} \frac{l}{\beta c} i_B(\omega)
  \]
- \( V_{\text{mon}}(\omega) = \frac{R}{1-j\omega CR} i_{\text{im}}(\omega) = \frac{R}{1-j\omega CR} j\omega \frac{A}{2\pi a} \frac{l}{\beta c} i_B(\omega) \)
- \( Z_{\|}(\omega) = \frac{1}{\beta c} \frac{1}{C} \frac{A}{2\pi a} \frac{jRC}{1-j\omega RC} \)
Capacitive pickup response

The transfer impedance was found to be

\[ Z_\parallel(\omega) = \frac{1}{\beta c} \frac{A}{2\pi a} \frac{jRC}{1-j\omega RC} \]

Defining the cutoff frequency as \( \omega_c = 1/RC \)

The magnitude and phase are

\[ |Z_\parallel| = \frac{1}{\beta c} \frac{A}{2\pi a} \frac{\omega/\omega_c}{\sqrt{1 + (\omega/\omega_c)^2}}, \angle Z_\parallel = \tan^{-1}(\omega_c/\omega) \]

High frequency regime, \( \frac{\omega}{\omega_c} \gg 1, Z_\parallel \to 1 \)

\[ V_{mon}(t) = \frac{1}{\beta c} \frac{A}{2\pi a} i_B(t) \]

Direct image of beam current with no phase shift.

Low frequency regime, \( \frac{\omega}{\omega_c} \ll 1, Z_\parallel \to j \frac{\omega}{\omega_c} \)

\[ V_{mon}(t) = \frac{R}{\beta c} \frac{A}{2\pi a} \frac{di_B}{dt} \]

Measured voltage is proportional to time derivative of current.
Spectrum from low-$\beta$ beams in capacitive pickups

Point particle distribution $\sigma_t \equiv \frac{b}{\sqrt{2} \gamma \beta c}$

- Here, $b$ is the distance from the beam to each pickup (or pipe radius for on-axis beam)
- Narrow bandwidth difference measurement must contend with nonlinearities in response.
- Requires calibration against variation in beta for each monitoring frequency and bandwidth.
Beam Position Monitors

• Beam positions can be monitored using a 4-electrode array of capacitive pickups on the beampipe circumference.

• Various geometries are employed for sensitivity, compactness, protection from intense radiation.
BPM Position Algorithm

- Positions are estimated from the normalized intensities using the *difference over sum* algorithm
  \[ \Delta x = \frac{1}{S_x} \frac{V_2 - V_4}{V_2 + V_4}, \Delta y = \frac{1}{S_y} \frac{V_1 - V_3}{V_1 + V_3} \]

- Position sensitivities are proportionality constants between beam displacement and signal strength.
  \[ S_x = \frac{d}{dx} \left( \frac{\Delta x}{\Delta_x} \right) \left[ \frac{\%}{mm} \right] \text{ or } S_x = \frac{d}{dx} \left( \frac{V_2}{V_4} \right) \left[ \frac{dB}{mm} \right] \text{ where } \left( \frac{V_2}{V_4} \right) [dB] = 10 \log \left( \frac{V_2}{V_4} \right) \]

- Offset displacements also occur and must be measured and calibrated.

- Button-button capacitive coupling introduces frequency dependent offset and sensitivity variation.

- Intensities at each button can be calculated from the transfer impedance, using the electrode surface area.
Coupling to the beam’s magnetic field

**Figure 1.** (a) Lines of magnetic induction around circulating beam. (b) Wall currents induced in beam tube attenuate external $B$. (c) Break in tube impedes wall currents permitting external $B$ and appearance of induced voltage. (d) Layout of typical accelerator shows complex and distributed paths available to induced currents.
Inductive coupling

The magnetic field of the beam is used to measure the beam current or intensity.

\[ \mathbf{B} = \mu \frac{I_B}{2\pi r} \phi \]

magnetic field at radius \( r \):
\( B \sim 1/r \)
\( \mathbf{B} \parallel \mathbf{\phi} \)

beam current \( I \)

simplified equivalent circuit

\[ \frac{1}{N} I_{\text{beam}}(t) \]

\( I \)-source represents

beam = primary winding

wire = secondary windings

inductance \( L \)

R

ground

beam = primary winding

torus
Current Transformers

**Ampere’s Law:**
\[
\oint H \cdot dl = N_p I_p + N_s I_s = I_p + N_s I_s \quad \text{with } N_p = 1 \quad \Rightarrow 
\]
\[
H = \frac{(I_p + N_s I_s)}{2\pi r} \tag{1}
\]

**Flux:** (thin toroid approximation)
\[
\Phi = \int B \, dS = \mu H A = \mu A \frac{(I_p + N_s I_s)}{2\pi r} \quad \text{with } A \text{ as area} \tag{2}
\]

**Faraday’s Law:**
\[
V_s = -N_s \cdot \frac{d\Phi}{dt} = I_s \cdot R_s \tag{3}
\]

Combine (2) and (3):
\[
I_s \cdot R_s = -N_s \cdot \frac{\mu A}{2\pi r} \cdot \frac{d(I_p + N_s I_s)}{dt} \quad \text{with } L_s = \frac{N_s^2 \mu A}{2\pi r} \Rightarrow 
\]

**Differential equation:**
\[
\frac{dI_s}{dt} + \frac{R_s}{L_s} I_s = -\frac{1}{N_s} \cdot \frac{dI_p}{dt} \tag{4}
\]
Current Transformer Transfer Impedance

• We will analyze this using Laplace transform pairs instead of Fourier.
  • Fourier (symmetrized)
    \[ \mathcal{F}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-j\omega t} \]
    \[ f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathcal{F}(\omega) e^{j\omega t} \]
  • Laplace (unsymmetrized)
    \[ F(s = \sigma + j\omega) = \mathcal{L}\{f(t)\} = \int_{0-}^{\infty} f(t) e^{-st} \]
    \[ f(t) = \mathcal{L}^{-1}\{F(s)\} = \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} F(s) e^{st} \]
  • The Laplace transform is useful for analyzing linear, time-invariant, causal systems.
    • Differentiation \[ \mathcal{L}\{f'(t)\} = sF(s) - f(0+) \]
    • Integration \[ \mathcal{L}\{\int_{0-}^{t} f(\tau)d\tau\} = \frac{F(s)}{s} \]
    • Linearity, Scaling, Shifting, Translation, Convolution

\[
\frac{dI_s}{dt} + \frac{R_s}{L_s} I_s = -\frac{1}{N_s} \frac{dI_p}{dt} \quad s\tilde{I}_s + \frac{R_s}{L_s} \tilde{I}_s = -\frac{1}{N_s} s\tilde{I}_p
\]

\[
H(s) = \frac{\tilde{V}_s}{\tilde{I}_p} = \frac{R_s\tilde{I}_s}{\tilde{I}_p} = -\frac{R_s}{N_s} \frac{s\tau}{1 + s\tau} \quad \tau = \frac{L_s}{R_s}
\]

In the high frequency limit
\[ s \approx j\omega \gg \frac{1}{\tau} \quad \Rightarrow H(j\omega) = -\frac{R_s}{N_s} \]
\[ V_s = \frac{R_s I_p}{N_s} \quad P_s = I_s^2 R_s = \frac{R_s I_p^2}{N_s^2} \]
Inductance and Capacitance

- The low frequency response is dominated by \( R/L \)
- Practical monitors contain capacitance due to ceramic wall breaks, coupling between windings, etc. This alters the response and determines the device bandwidth.

\[
\frac{1}{Z} = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C \quad \Rightarrow \quad Z(\omega) = \frac{j\omega L}{1 + \frac{j\omega L}{R} - \left(\frac{\omega L}{R}\right)(\omega RC)}
\]

*Low frequency*: \( \omega \ll \frac{R}{L} \quad \Rightarrow \quad Z = j\omega L \quad (B\text{-dot regime})

*Mid frequency*: \( \frac{R}{L} \ll \omega \ll \frac{1}{RC} \quad \Rightarrow \quad Z \approx R

*High frequency*: \( \omega \gg \frac{1}{RC} \quad \Rightarrow \quad Z = \frac{1}{j\omega C} \)
Response from passive transformer

Stray cable capacitance increases risetime.

\[ \tau_{\text{rise}} = \sqrt{L_s C_s} \quad \text{with cable} \]

\[ \tau_{\text{rise}} = R C \quad \text{without cable} \]

\[ \text{rise: } \tau_{\text{rise}} = (L_s C_s)^{1/2} \]

\[ \text{droop: } \tau_{\text{droop}} = \frac{L}{R} \]

\[ R_L \quad L_s \quad C_s \quad R \quad N \text{ windings} \]
We adopt a single mode resonance to calculate the coupling impedance.

The transient cavity-beam-waveguide system can be expressed as an equivalent circuit equation (cf. Whittum)

\[
\left\{ \frac{d^2}{dt^2} + \omega_0^2 \right\} V_c = -\frac{\omega_0}{Q_w} \frac{d}{dt} V_c + \frac{\omega_0}{Q_e} \frac{d}{dt} (V_F - V_R) + \omega_0 \left[ \frac{r}{Q} \right] \frac{d}{dt} I_b
\]

Here \( V_F = nV^+ \), \( V_R = nV^- \) are the normalized forward and reverse waveguide voltages, such that \( V_C = V_F + V_R \). Here, \( n \) is called the transformer ratio for the mode coupling.

We can show that the longitudinal coupling impedance presented by this mode is

\[
Z_{||}(\omega) = \frac{j\omega_0 [r/Q]}{\omega_0^2 - \omega^2 + j\omega_0/Q_L} = Q_L [r/Q] \cos \psi e^{j\psi}
\]

Impedances can be expressed as \( Z = R + jX \) with the reactance \( X = \left( \omega L - \frac{1}{\omega C} \right) \).
Cavity Dipole-mode BPMs (resonant coupling)

- Used mainly for high energy electron beams
- Resonant dipole modes have higher shunt impedance than buttons or striplines
  - High sensitivity
  - Wakefields act back on beam
- Cavity BPMs have been developed to produce sub-\(\mu\)m position resolution, for \(\sim\)mm displacements
- Monopole mode excitation is proportional to beam current
- Antennae pick up combined monopole+dipole signals. Technique requires independent calibration of monopole voltage.
  - Pillbox: \(f_{\text{mono}} \sim 1.2-1.5* f_{\text{dipole}}\)
- \(Q_{\text{load}}\) for both modes \(\sim100 – 1000\)
  - Mode must decay before arrival of next pulse
Relation between longitudinal and transverse effects

Panofsky and Wenzel derived a powerful theorem connecting longitudinal electric field to transverse momentum impulses.

\[
\Delta p = Qe \int_a^b [E + v \times B] dt \\
\Delta W = Qe \int_a^b E \cdot ds
\]

Lorentz force impulse

We can show

\[
\frac{\partial}{\partial t} \Delta p = Qe \int_a^b \frac{\partial E}{\partial t} dt + ds \times \frac{\partial B}{\partial t} = Qe \int_a^b \left[ -\nabla (ds \cdot E) + dE \right]
\]

We can show

\[
\frac{\partial}{\partial t} \Delta p_s = Qe \frac{\partial}{\partial t} \int_a^b E_s dt
\]

The beam trajectory is along \(s\), and \(ds=vdt\)

\[
\frac{\partial}{\partial t} \Delta p_\perp = -Qe \int_a^b \left[ \nabla_\perp (ds \cdot E) - dE_\perp \right] = -\nabla_\perp W + Qe [E_\perp (a) - E_\perp (b)]
\]

We can typically ignore the end effects by taking the integral to field free regions.

\[
\frac{\partial}{\partial t} \Delta p_\perp = -\nabla_\perp W
\]

(Panofsky-Wenzel)
Beam spectrum, impedances and beam loading

• Beam impedance response
  • \( V(\omega) = I_b(\omega) Z(\omega) \)

• Everything that sees the beam can be described in terms of a beam coupling impedance

• Narrowband impedances from resonant structures

• Related to wake functions

• Panofsky-Wenzel relates longitudinal to transverse wake/impedances for ultrarelativistic particles

DAΦNE RF cavity longitudinal impedance

Fundamental mode
damped

undamped

START 300.000 000 MHz
STOP 1 500.000 000 MHz
Beam coupling impedance to broadband device

- Devices may possess narrowband as well as broadband impedance characteristics.
- Impedances can be characterized on benchtop measurement stands.

Figure 18: Wire measurement frequency response of the shielded ceramic break (thin line: outer shield without DCCT; thick line: outer shield with DCCT inside; dotted line: gap shielded by shunting resistors).
Wake fields and wake potentials

- For very high bandwidth impedances, modal and resonant behavior may no longer adequately capture the beam-structure interaction
  - Single and multi-bunch effects (coherence length $>$ bunch length or separation)
  - Regenerative
  - Frequencies above pipe cutoff

- In this class of phenomena, beam wakes fields are a more relevant description
  - Time domain
  - Causal for ultrarelativistic beams

\[
W'_m(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \, e^{j\omega z/c} Z'^{\parallel}_m(\omega)
\]
\[
W_m(z) = -\frac{j}{2\pi} \int_{-\infty}^{\infty} d\omega \, e^{j\omega z/c} Z^\perp_m(\omega)
\]
\[
Z^\perp_m(\omega) = \frac{\omega}{c} Z^\parallel_m(\omega)
\]

\[
\int_a^b dz \, \hat{F} = -\nabla V
\]

\[
V = e I_m W_m(z) r^m \cos m\theta
\]

for an $m$th multipole

\[ W(z) \]

\[ z \]
Synchrotron Radiation

- Relativistic beams, $\gamma >> 1$
- Parasitic, nonintercepting
- Photon image reproduces electron beam distribution
- Optics, coupling
- Impedances, instabilities
- IR -> Hard X-rays
4th generation light sources produce intense electron bunches in the psec to fsec bunch length regime

- THz to X-ray wavelengths
- Can utilize self-fields or radiation → Single shot!

Electro-Optical Spectral Decoding:

- Linear chirped optical pulse
- Polarization variation caused by Coulomb field—laser nonlinear effect
- Polarization → Intensity, by two crossed polarizers
- $I(\lambda) \leftrightarrow I(t)$

$$E_{out} = (0 \ 1) R(\varphi) M_{NW} R(-\varphi) R(\alpha) M_{qw} R(-\alpha) R(\theta) M_{EO} R(-\theta) \left( r_{\text{chirp}}(f) \right)$$

- $R(\theta)$: rotation matrix
- $M_{qw}$: Jones matrix for quarter wave plate
- $M_{NW}$: Jones matrix for half wave plate

Expected temporal resolution

1. Distance between crystal and e-beam
   $$\Delta t \sim \frac{2r}{Y} \sim 10 \text{ fs} \text{ at } r=5 \text{ mm}$$

2. The frequency response of crystal (material and thickness)
   for 1 mm ZnTe: $\sim 333$ fs
   $\sim 1/(3 \text{THz})$

3. EOSS limitation (Laser pulse duration and chirped duration)
   $$\tau_{\text{se}} = \sqrt{\tau_{\text{chirp}} \tau_{\text{pump}}} \sim 550 \text{ fs} \ (100 \text{ fs} \rightarrow 3 \text{ ps})$$

4. Resolution of spectrometer and CCD
   $\sim 40$ fs (512 pixels)
Architecture of a diagnostic measurement

- The complete diagnostic system starts with beamline (or nearby) sensor
- Cabling to transport signals to data acquisition (DAQ) systems
- Processing electronics, and controls/operator interfaces

- Penetrations and racks are laid out for instrumentation
  - Cable runs about 100 ft
  - Diagnostics will use ¼” superflex (Heliax)
    » solid copper jacket provides >120 dB shielding effectiveness

- No electronics in the tunnel!
Many interfaces are needed to deploy diagnostics

- Controls and actuation for interceptive devices
- Global timing and triggering
- Interlocks for machine, device, and personnel protection
- High level controls for data acquisition, analysis, visualization

Allison Scanner Interface Diagram
1. Diagnostics Box
2. Allison Scanner Electronics Rack
3. Current Amplifiers
4. HV Power Supplies
5. Ethernet cable for control interface
Beam Current Monitor System Design

- AC current transformer (baseline sensor) (Bergoz)
- ACCT high-resolution electronics (AFE) (Bergoz)
- Analog-to-digital converter (ADC)
- Transformer digital signal processing algorithms (FPGA, LabView, etc.)
- Connection to accelerator control system, operator interface, etc.

A. ToF delay (Chopper to BCM), ~15 usec
B. Beam “gap”, 50 usec
C. Beam active, 50 usec
Exercise 1 – BPM Sensitivity

A measure of the sensitivity, $S_x$, of a beam position monitor measurement is defined as

$$S_x = \frac{d}{dx} \left( \frac{A}{B} \right) \text{[dB/mm]}$$

where $A$ and $B$ are induced voltage on opposing buttons. In the linear approximation of the BPM response, show that the measurement sensitivity is approximately 13.6 dB/R[mm], where $R$ is the radius of the round BPM aperture (and the distance that the buttons are displaced from the beam axis).

Recall that in the linear approximation, close to the axis, $\Delta x \cong \frac{2R}{\pi} \frac{A-B}{A+B}$

Other helpful relations are $\left( \frac{A}{B} \right) \text{[dB]} = 10 \log \left( \frac{A}{B} \right)$ and $\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$
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