PHY862 Accelerator Systems
Introduction to Accelerators

Peter N. Ostroumov
Professor of Physics
Michigan State University

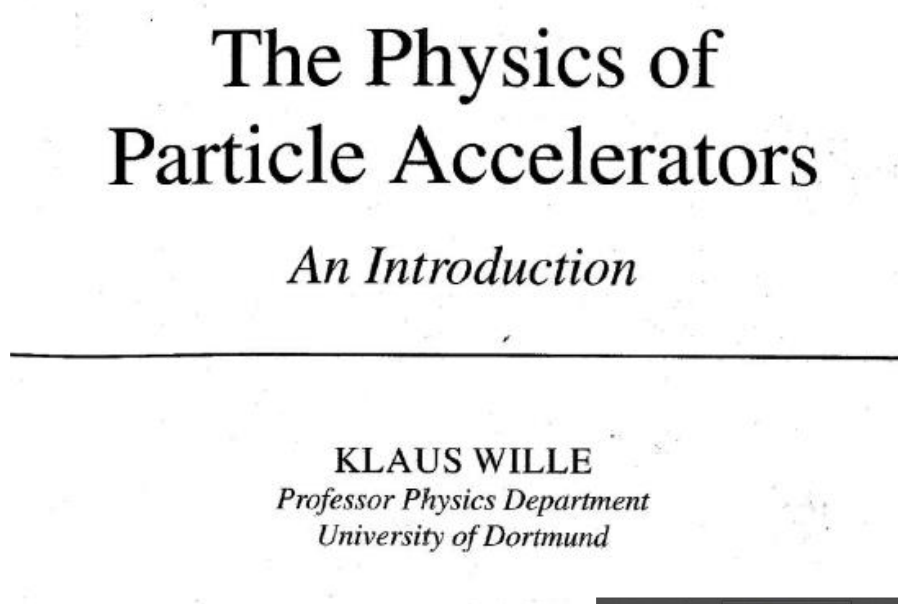
This material is based upon work supported by the U.S. Department of Energy Office of Science under Cooperative Agreement DE-SC0000661, the State of Michigan and Michigan State University. Michigan State University designs and establishes FRIB as a DOE Office of Science National User Facility in support of the mission of the Office of Nuclear Physics.

Content

- Logistics of the course
- Concept of charge particle acceleration
- Brief history of accelerators
- Types of accelerators:
  - DC
  - Betatron
  - Cyclotron
  - Linear accelerators
  - Microtrons
  - Circular accelerators
  - Autophasing
  - Storage rings
  - Colliders
  - Advanced concepts
  - Application of accelerators.

Literature

- Many pictures in this lecture are from this textbook.
- Appreciate very much Klaus Wille's contribution to accelerator physics



The Physics of Particle Accelerators

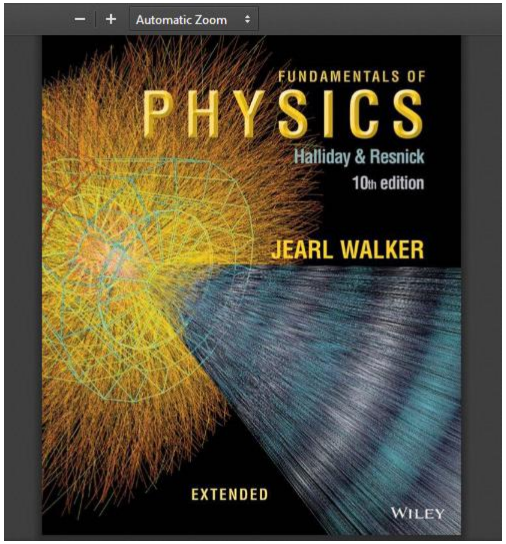
An Introduction

KLAUS WILLE

Professor Physics Department

University of Dortmund

- https://people.nscl.msu.edu/~ostroumo/PHY183-184/



- A. Sessler, E. Wilson, Engines of Discovery: A Century of Particle Accelerators, World Scientific Publishing.
  - History of particle accelerators
  - Applications of accelerators

Web-site

- https://people.nscl.msu.edu/~ostroumo/MSU/
- https://people.nscl.msu.edu/~ostroumo/MSU/Schedule_2023/

Homework includes 2 parts: problem sets and project report

- Two-three problems per lecture will be assigned
- Homework will be assigned by each instructor at their final lecture. The homework is due in a week after each topic.
- Please e-mail your homework to the instructor and Prof. Ostroumov
- Deadline to receive homework problems is December
- List of projects to prepare for the final exam is on website. You should select your project topic.
- Final exam on December 14: 10 min presentation on the project topic

Units

- SI, MKSA – meter, kilogram, second, Ampere

Schedule

- Schedule
  - https://people.nscl.msu.edu/~ostroumo/MSU/Schedule_2023/
- Homework and the list of projects to select https://people.nscl.msu.edu/~ostroumo/MSU/Problems/
  - You can start to work on your project any time
- Lectures are in 1309
  - Some room changes may happen due to FRIB Review meetings

Accelerator Science and Engineering Traineeship (ASET) Program at MSU

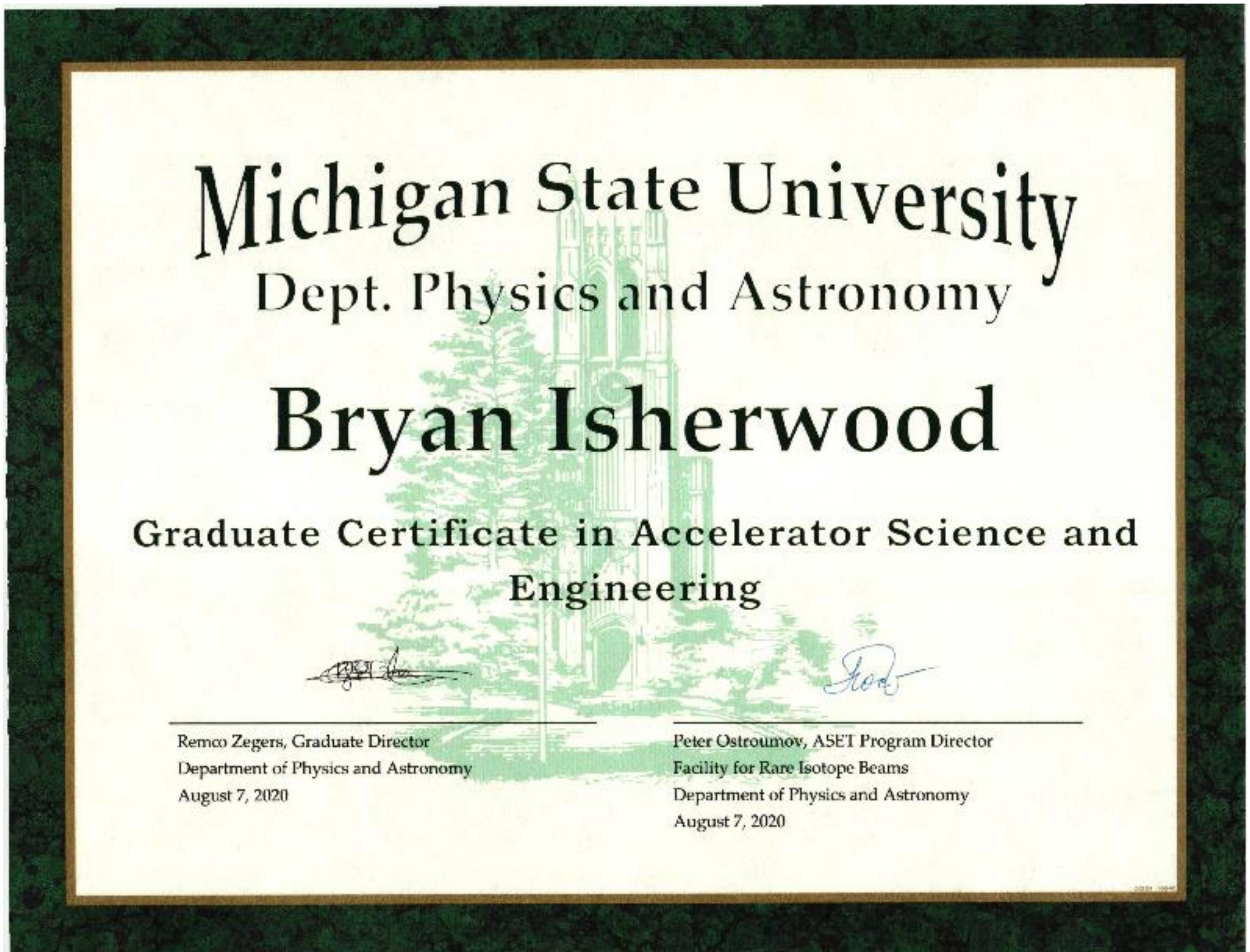
- Funded By DOE Office of Sciences, High Energy Physics
- Web-site https://frib.msu.edu/science/ase/index.html
- Degree certificate in Accelerator Science and Engineering: 9 credits
  - **Core course: PHY 862 Accelerator Systems**
- Three or more additional courses from the following list

– PHY 905	Accelerator Physics	3 credits
– PHY 905	Accelerator Technology	3 credits
– PHY 861	Introduction to Beam Physics	3 credits
– PHY 961	Nonlinear Beam Dynamics	3 credits
– PHY 962	Particle Accelerators	3 credits
– PHY 964	Seminar in Beam Physics Research	3 credits
– PHY 963	US Particle Accelerator School	3 credits
– ECE 802-607	RF Power Engineering	3 credits
– ECE 802	Plasma Simulation	3 credits
– ECE 837	Computational Methods in Electromagnetics	3 credits
– ECE 850	Electrodynamics of Plasmas	3 credits
– ECE 989	Advanced Applications of Plasmas	3 credits

  - All 800 courses related to ASET can be counted against AS&E Certificate

Accelerator Science and Engineering Certificate

- https://reg.msu.edu/ACADEMICPROGRAMS/Programs.aspx?PType=GC



Michigan State University

Dept. Physics and Astronomy

Bryan Isherwood

Graduate Certificate in Accelerator Science and Engineering

Remco Zegers, Graduate Director
Department of Physics and Astronomy
August 7, 2020

Peter Ostroumov, ASET Program Director
Facility for Rare Isotope Beams
Department of Physics and Astronomy
August 7, 2020

High energy electron, proton beams for fundamental research

- Wavelength $\lambda < 10^{-15}$ m is required in elementary particle physics
  - Visible light ~ 500 nm
- We need very high energy photons or particle beams
- MKSA- Joule is energy unit
- *h* is Planck's constant
- High energy electrons are required to produce photons via bremsstrahlung
- Particle beams should have very short de Broglie wavelength

Photon energy for $\lambda \sim 10^{-15}$m

$$E_\gamma = h\nu = \frac{hc}{\lambda} = 2\times 10^{-10}\ J$$

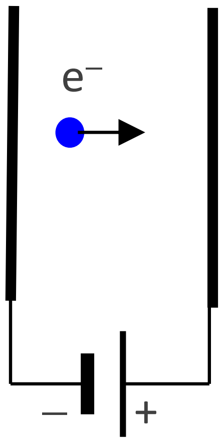
$$E_e = eV,\ E_e > E_\gamma$$

$$\lambda_B = \frac{h}{p} = \frac{hc}{E}$$

For relativistic particles, the velocity is close to the speed of light, *c*

- Particle beam energy is measured in electron-volts

Electron-Volt



- Elementary charge, e=1.602×10^{-19} C.
- ΔU=1 V
- 1 eV =1.602×10^{-19} J
- keV, MeV, GeV, TeV

- The amount of energy required to produce an elementary particle is

$$E = m_0 c^2$$

- Electron positron pair production by gamma-ray in the vicinity of a nucleus

$$\gamma = e^- + e^+$$

- γ-ray energy

$$E_\gamma > 2m_e c^2 = 2 \times 9.1 \cdot 10^{-31} \cdot 9 \cdot 10^{16} = 1.637 \cdot 10^{-13} J = 1.02 MeV$$

Particle's rest mass

		Rest Energy, MeV
Electron	e	0.511
Proton	p	938.272
b quark	b	4735
Vector boson	Z_0	91190
t quark	t	173 000

- Since the velocity of high energy particle is close to c, the particle energy should be written in relativistic invariant form
- In most cases in accelerator physics we refer to the kinetic energy of particles unless it is specifically noted

$$E = m_0 \gamma c^2 = m_0 c^2 + E_k$$

$$E_k = E - E_0 = m_0 c^2 (\gamma - 1)$$

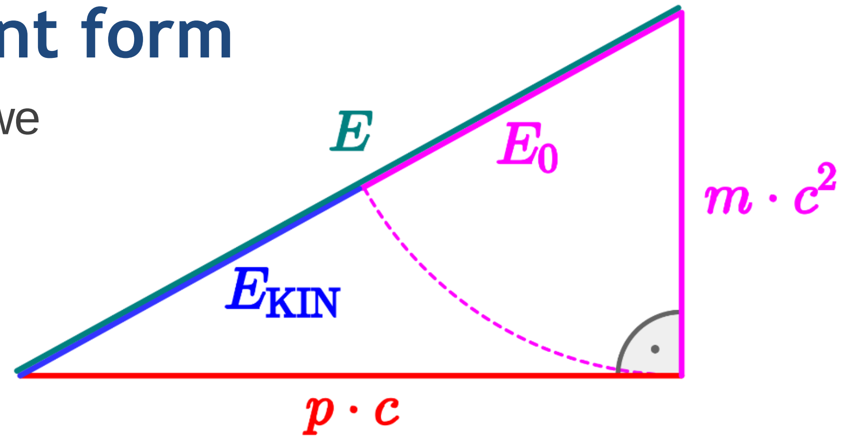
$$\vec{p} = m_0 \gamma \vec{v}$$

$$\beta = v / c,$$

$$c = 2.997925 \cdot 10^8 \; m/s$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}, \quad \beta = \sqrt{1 - \frac{1}{\gamma^2}}$$

Energy in relativistic invariant form



- Once we deal with relativistic velocities, we must use total energy in relativistic invariant form, total energy-momentum relation

$$E^2 = \left(m_0 c^2\right)^2 + \left(pc\right)^2$$

- Energy can be increased only by increasing the momentum or by acceleration
- We can show that

$$E^2 = (m_0 c^2 + E_k)^2 = \left(m_0 c^2\right)^2 + \left(pc\right)^2 \qquad E_k = m_0 c^2(\gamma - 1)$$

$$E^2 = \left(m_0 c^2\right)^2 + 2\left(m_0 c^2\right)^2(\gamma - 1) + \left(m_0 c^2\right)^2(\gamma - 1)^2 =$$

$$= \left(m_0 c^2\right)^2 + 2\left(m_0 c^2\right)^2\gamma - 2\left(m_0 c^2\right)^2 + \left(m_0 c^2\right)^2\gamma^2 - 2\left(m_0 c^2\right)^2\gamma + \left(m_0 c^2\right)^2 =$$

$$= \left(m_0 c^2\right)^2 + \left(m_0 c^2\right)^2\gamma^2 - \left(m_0 c^2\right)^2 = \left(m_0 c^2\right)^2 + \left(m_0 c^2\right)^2(\gamma^2 - 1)$$

$$\left(pc\right)^2 = \left(m_0 \gamma v c\right)^2 = \left(m_0 c^2\right)^2\left(\beta\gamma\right)^2 = \left(m_0 c^2\right)^2\gamma^2\left(1 - \frac{1}{\gamma^2}\right) = \left(m_0 c^2\right)^2\left(\gamma^2 - 1\right)$$

Energy increase

- To increase energy, we need to increase momentum which can be changed by action of force only

$$\dot{\boldsymbol{p}} = \boldsymbol{F}$$

Four different forces

Force	Relative strength	Range [m]	Particles affected
gravity	6×10^{-39}	∞	All particles
electromagnetism	1/137	∞	Charged particles
Strong force	1	10^{-15} - 10^{-16}	hadrons
Week force	10^{-39}	$<<10^{-16}$	Hadrons and leptons

Energy gain in electromagnetic field

- Lorenz force $\vec{F} = e(\left[\vec{v}\vec{B}\right] + \vec{E})$ - electrons, protons
-
- $\vec{F} = q(\left[\vec{v}\vec{B}\right] + \vec{E})$ - multi-charge ions
- As a charged particle moves from $\vec{r}_1$ to $\vec{r}_2$ in electromagnetic field

energy gain is

$$\Delta E = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} d\vec{r} = e\int_{\vec{r}_1}^{\vec{r}_2} (\left[\vec{v}\vec{B}\right] + \vec{E}) d\vec{r}$$

During acceleration under the action of

Lorentz force, the path element $d\vec{r}$ is parallel to the velocity vector

Magnetic fields do not contribute to the energy of the particles

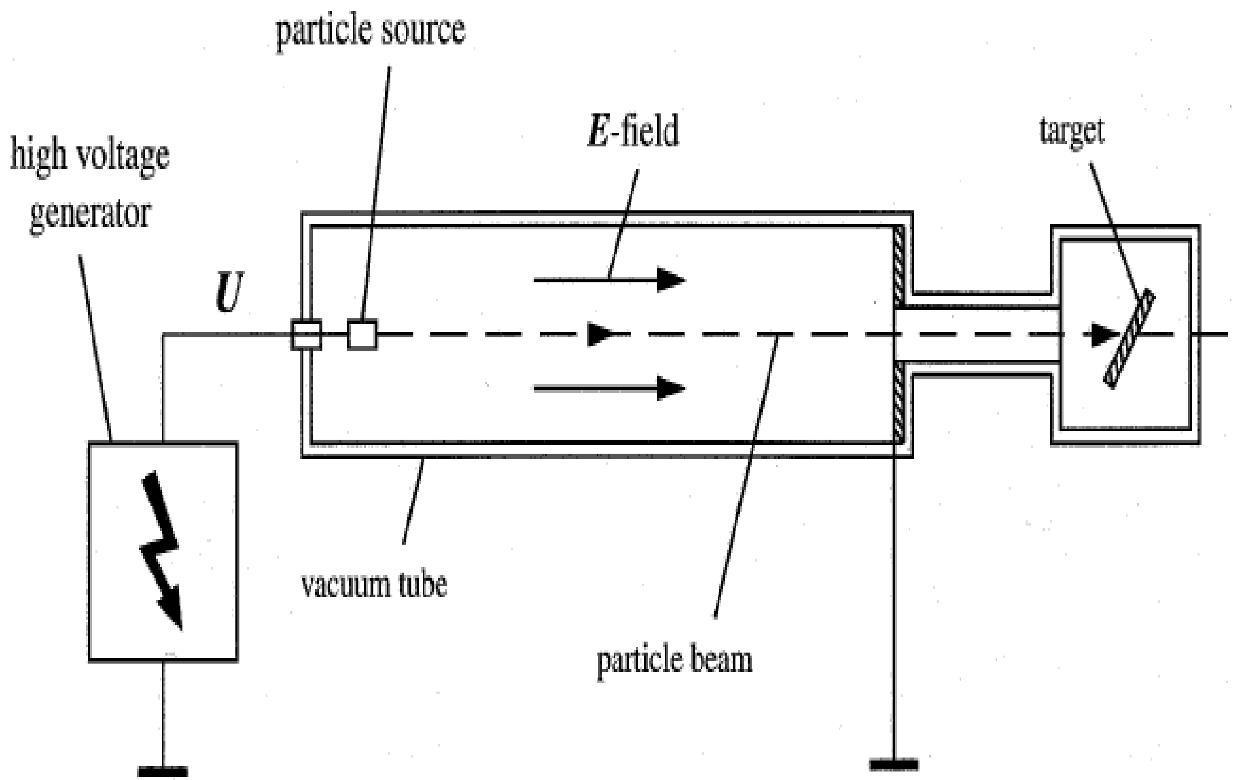
$\left[\vec{v}\vec{B}\right] \cdot d\vec{r} \equiv 0$ therefore $\Delta E = e\int_{\vec{r}_1}^{\vec{r}_2} \vec{E} d\vec{r} = eV$

U is the voltage crossed by the particle

Acceleration and beam steering are based on Maxwell's equations and special theory of relativity. Magnetic field: steering, bending, focusing

Direct-voltage accelerator

- Energy of particle is defined by the applied voltage
- Voltage is limited by breakdowns
- Beam power $P_b = IV$; example: 10 mA, 1 MV, P_b=10 kW
- Power supply power must be higher than beam power to compensate for parasitic currents

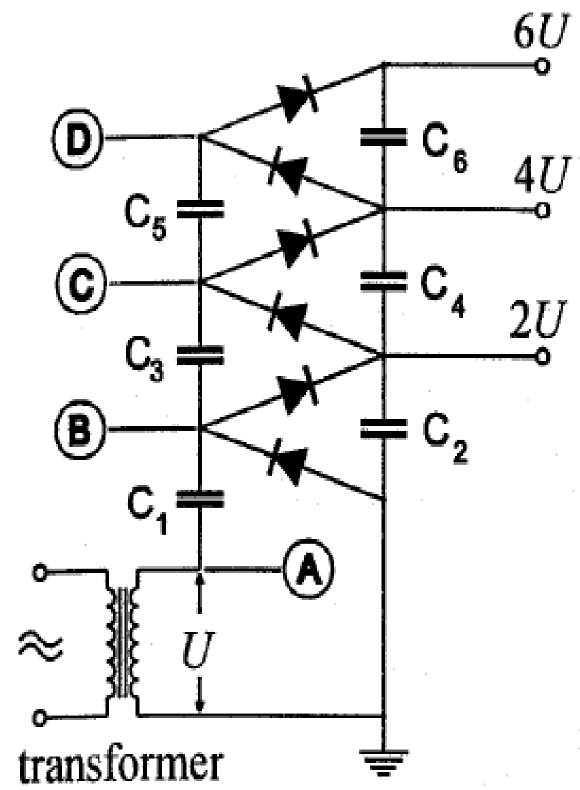


Direct-voltage acceleration

- Charged particles (protons, ions) are produced in plasma
- Electric field is created inside the vacuum tube
  - Avoid particle collisions with the molecules of residual gas
- The particle energy is directly proportional to the applied voltage
- The power supply should compensate beam current, ohmic losses
- Maximum achievable voltage is limited by breakdowns
  - avalanche increase of current through either residual gas or corona current in atmosphere
- Several MeV can be achieved with this technology
- Empirical numbers for DC voltage
  - Breakdown in dry air ~30 kV/cm
  - Breakdown in vacuum ~80 kV/cm

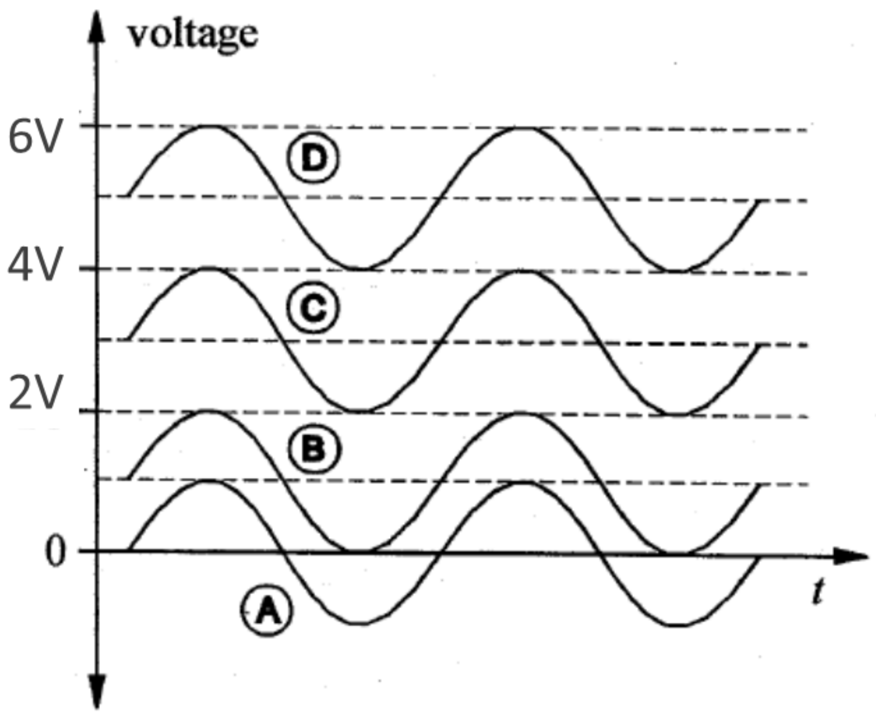
Cockroft-Walton cascade generator

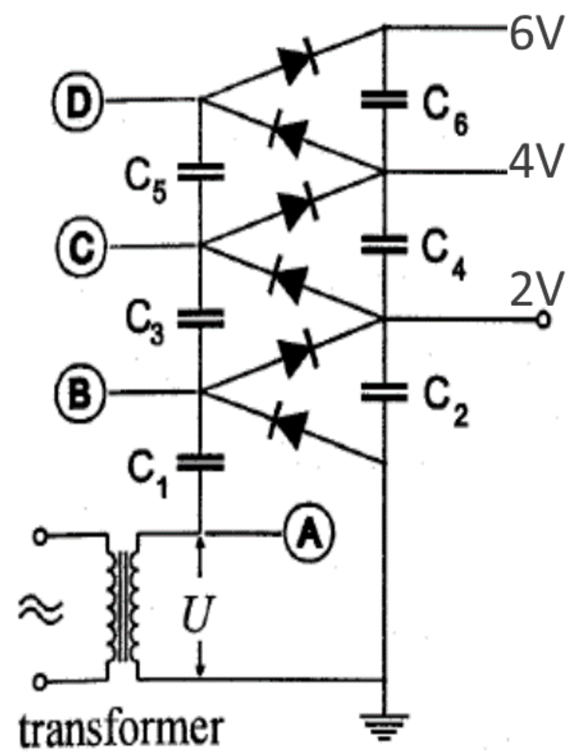
- Significant problem is the production of high voltage
- Cockroft and Walton developed a high voltage cascade generator in 1930s
- Up to ~ 4MV
- Current is available from the generator via the transformer
- In pulsed mode can generate ~n·100 mA beams



Voltage multiplication

- Point A: sinusoidal voltage $V\cos\omega t$
- Point B: The diode ensures that the voltage never goes negative and oscillates between 0 and *2V*, capacitor C_1 charges up to V (AC voltage)
- Then, the capacitor C_2 charges to *2V* (DC voltage) via the second diode
- Point C: The 3rd diode ensures that the voltage never goes below 2V



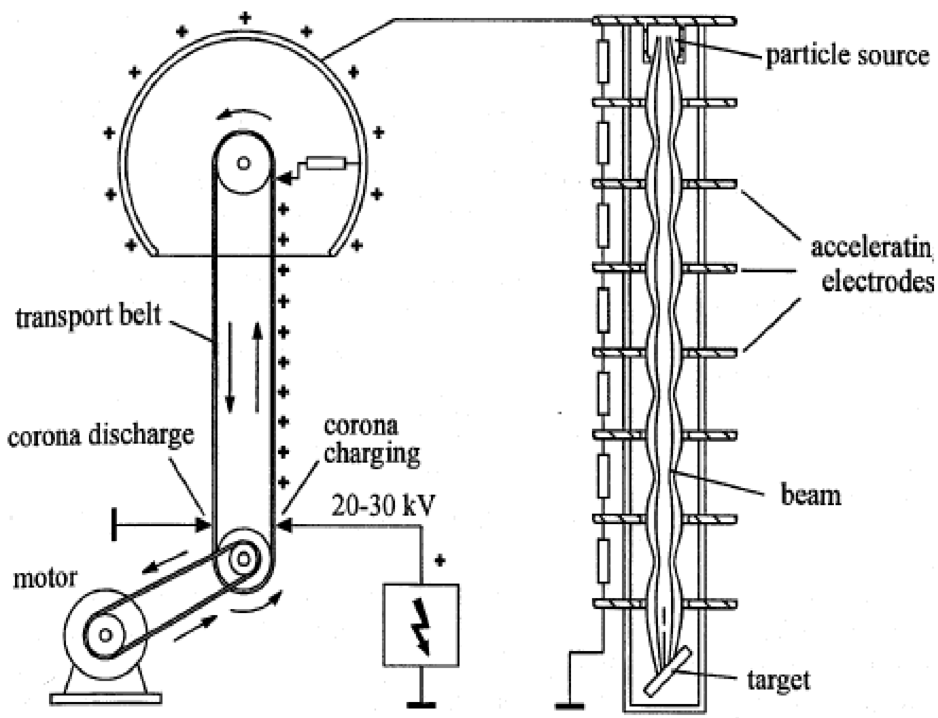


Van de Graff accelerator

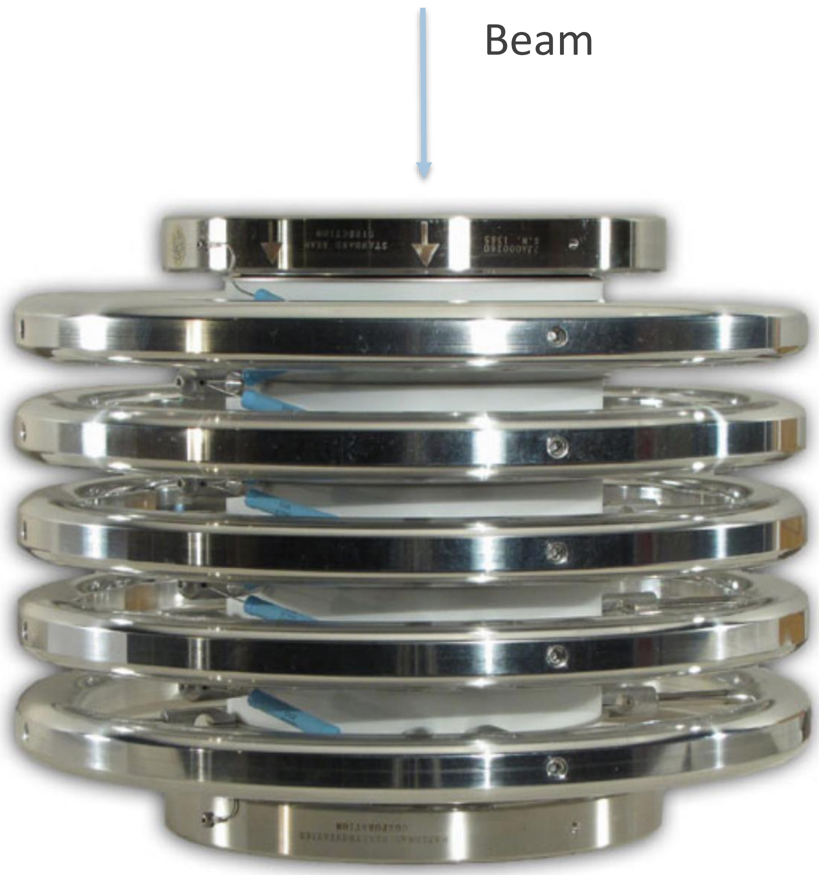
- Belt made of isolating material
- Charge formed around corona is transferred into the belt
- Belt carries charge up to the isolated conductive dome
- The dome is charged until some critical voltage
- Particles are accelerated along the accelerating tube
- High value resistors create uniform potential distribution
- The electrodes can provide beam focusing due to introducing radial electric field
- ~2MV in vacuum
- Up to 10 MV in sulfur hexafluoride, SF_6, at 4-5 atm pressure
  - Increased resistance to breakdowns due to pressurized gas

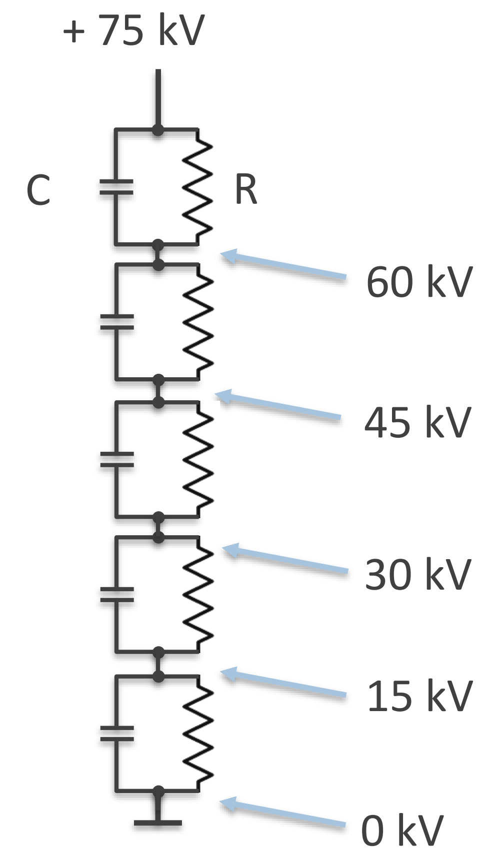
$$V = \frac{q}{4\pi\varepsilon_0 R}$$

$$E = \frac{q}{4\pi\varepsilon_0 r}, \quad r \geq R$$



Accelerating tube





Courtesy of National Electrostatics Corp.

Electrostatic accelerators

Accelerating tube

High Voltage
(HV) Line

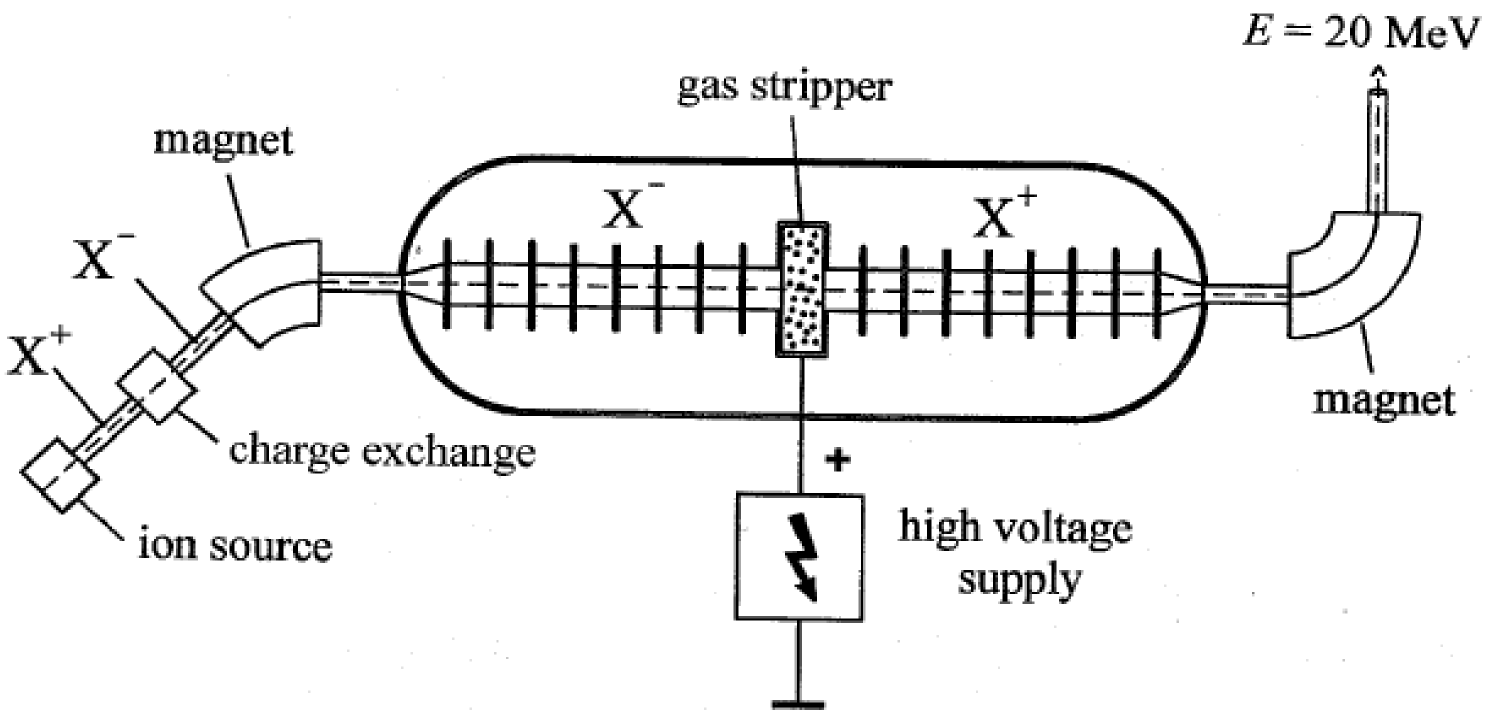
750 kV
High Voltage Source
(Cockcroft–Walton voltage multiplier)

Isolators

Courtesy of Fermilab

Tandem accelerator

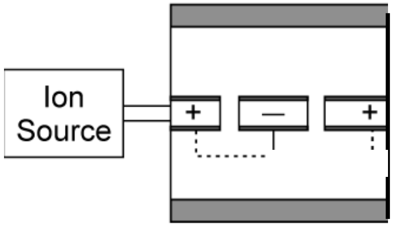
- In 1936 Van de Graff and collaborators built the first tandem accelerator
- Can provide total ~10 + 10=20 MV
- Negative ions created at ground potential
- Stripping at high voltage

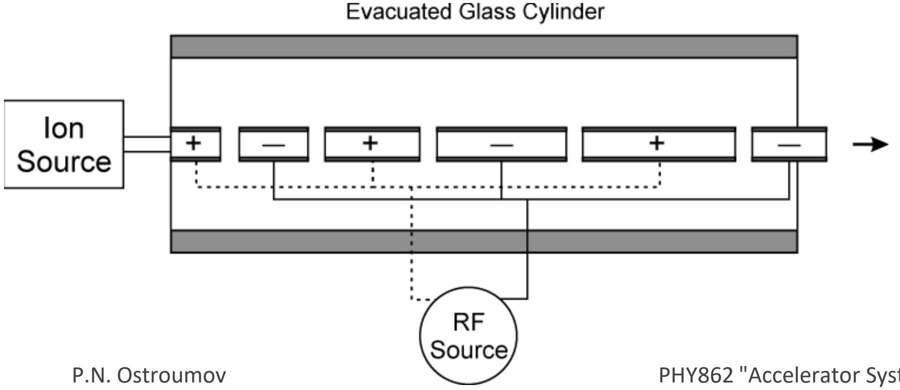


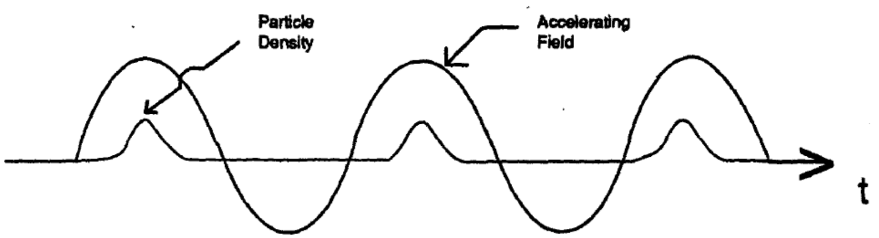
Tandems were very popular in 1960th for nuclear physics research

RF acceleration

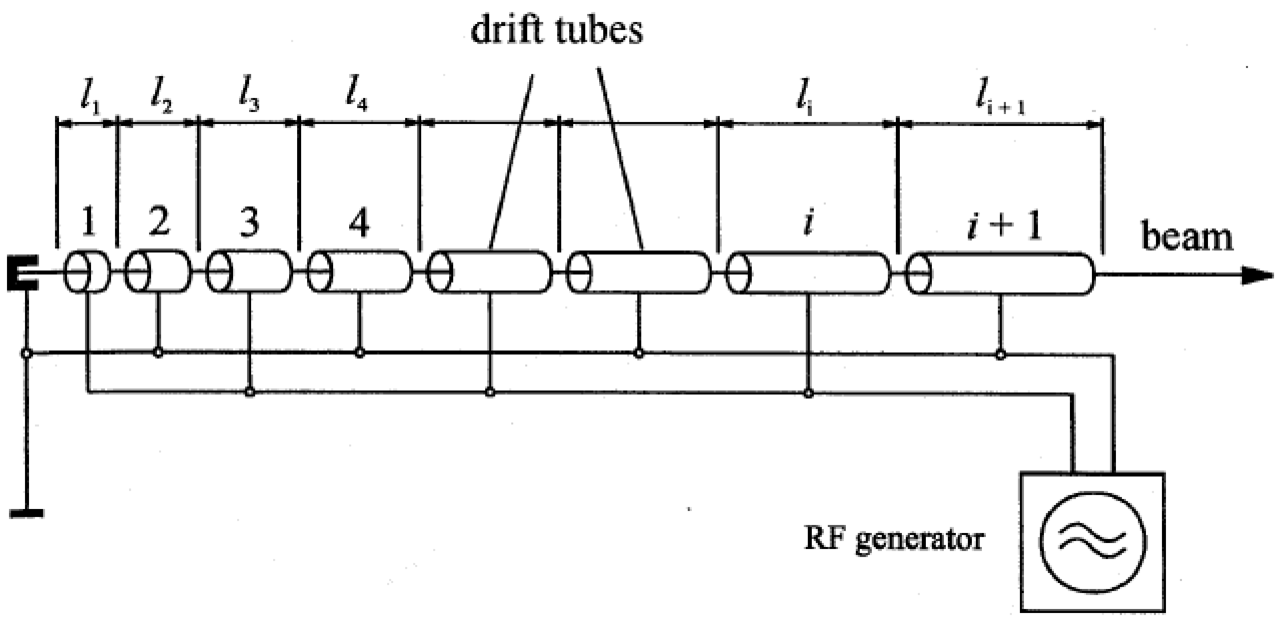
- Demonstrated in 1927 by Wideroe
- 25 kV 1-MHz RF voltage over 2 accelerating gaps
- 50 keV energy gain
  - Acceleration takes place during ½ period of RF
- Multi-gap linac: 1931 by Sloan and Lawrence
  - 42 kV at 10 MHz, 30 drift tubes
  - 1.26 MeV of 1 μA
- Only bunched beam can be accelerated
- Or if DC beam is injected, more than 50% of particles can not be accelerated (decelerated)







Wideroe linear accelerator for ion beams



$$\Delta E = qU_0 \cos\varphi_0, \; E_{k,i} = iqU_0 \cos\varphi_0 = \frac{1}{2} m_0 v_i^2$$ Non-relativistic approximation

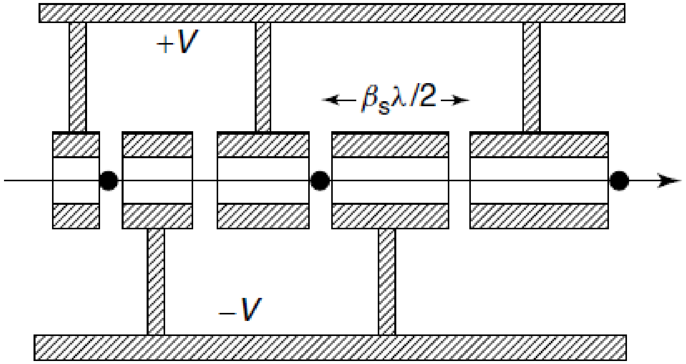
- $E_{k,i}$ kinetic energy after *i*-th accelerating gap
- Time of flight, *f* is RF frequency

$$T_i = \frac{l_i}{v_i} = \frac{1}{2f}, \; l_i = \frac{v_i}{2f}$$

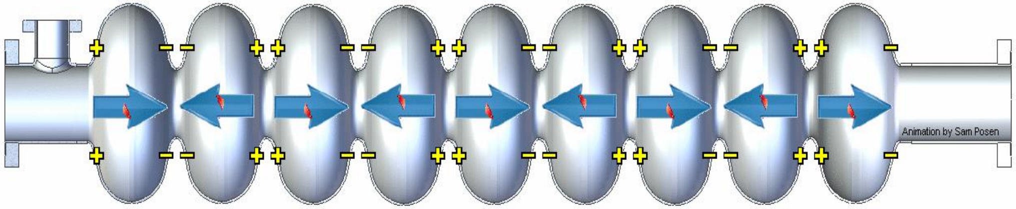
$$l_i = \frac{1}{f}\sqrt{\frac{iqU_0 \cos\varphi_0}{2m_0}}$$

Wideroe or interdigital structure (IH), standing wave structure

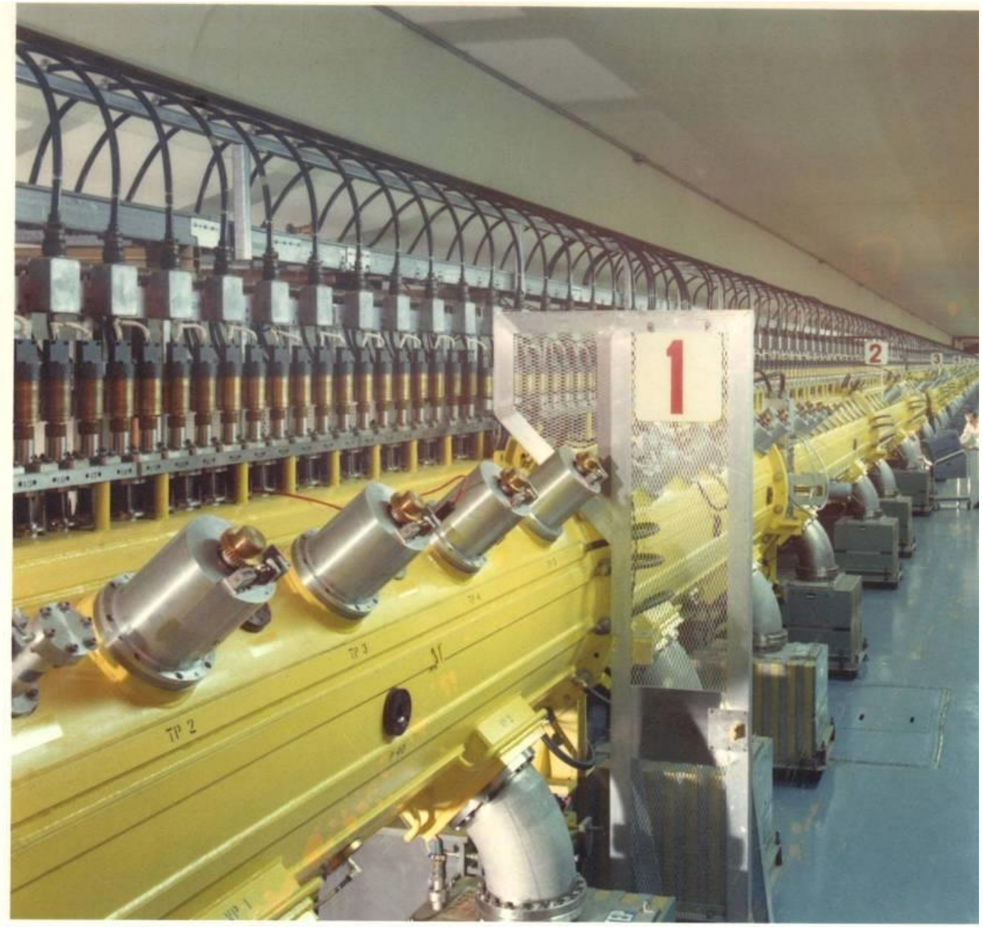
Phase shift between the cells is 180° or π



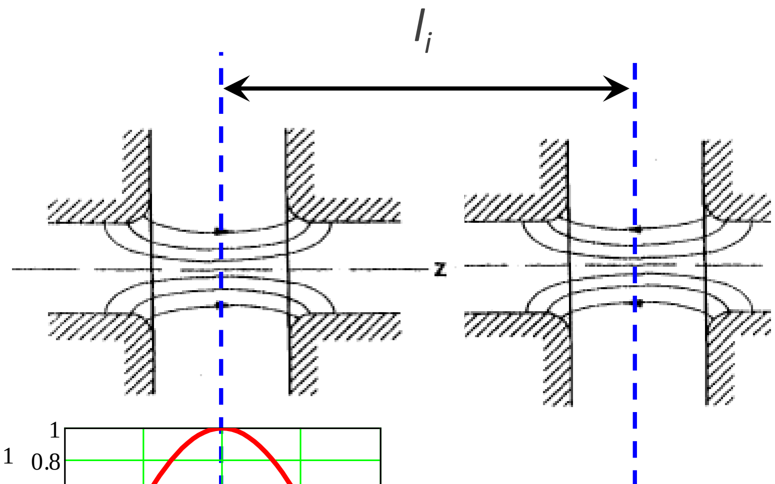
Superconducting structure on π-mode

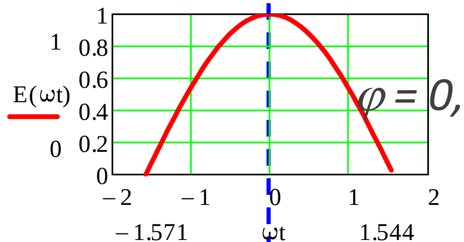


Drift Tube Linac (DTL)

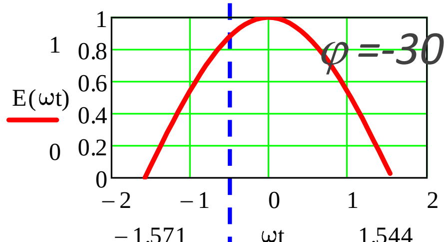


Concept of autophasing in standing wave structure

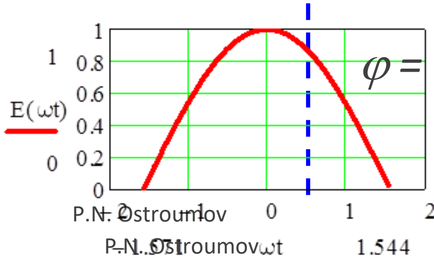




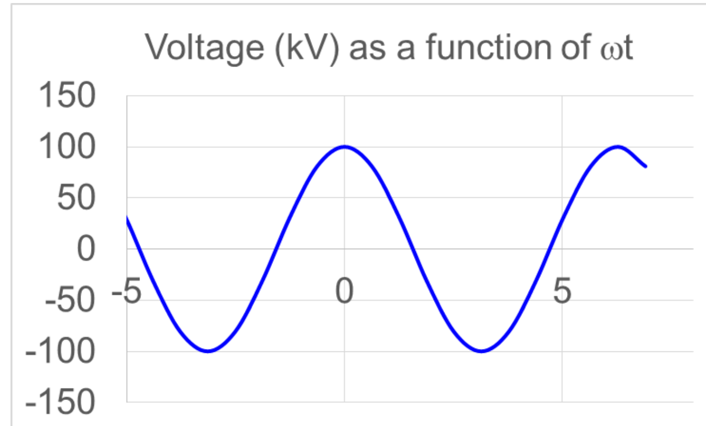
φ = 0, maximum acceleration



φ =-30°, stable acceleration



φ = 30°, unstable acceleration



$$\omega t_{flight} = \pi, \; t_{flight} = \frac{1}{2} T_{period}$$

$$l_i = \frac{1}{2}\beta_{average} c T_{period} = \frac{1}{2}\beta_{average}\lambda$$

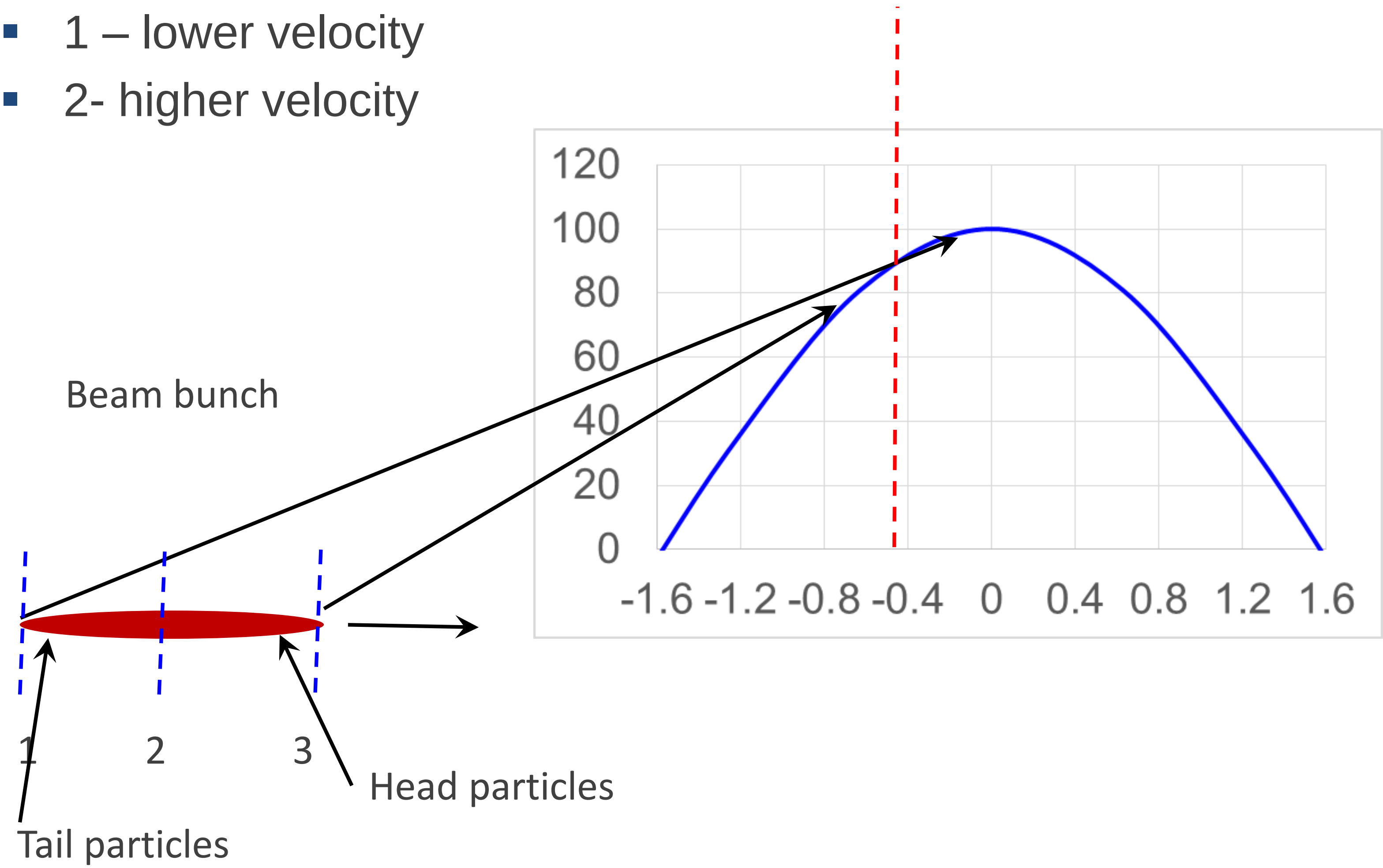
$$\Delta E = qV_0 \cos\phi_s$$

P.N. Ostroumov

Autophasing

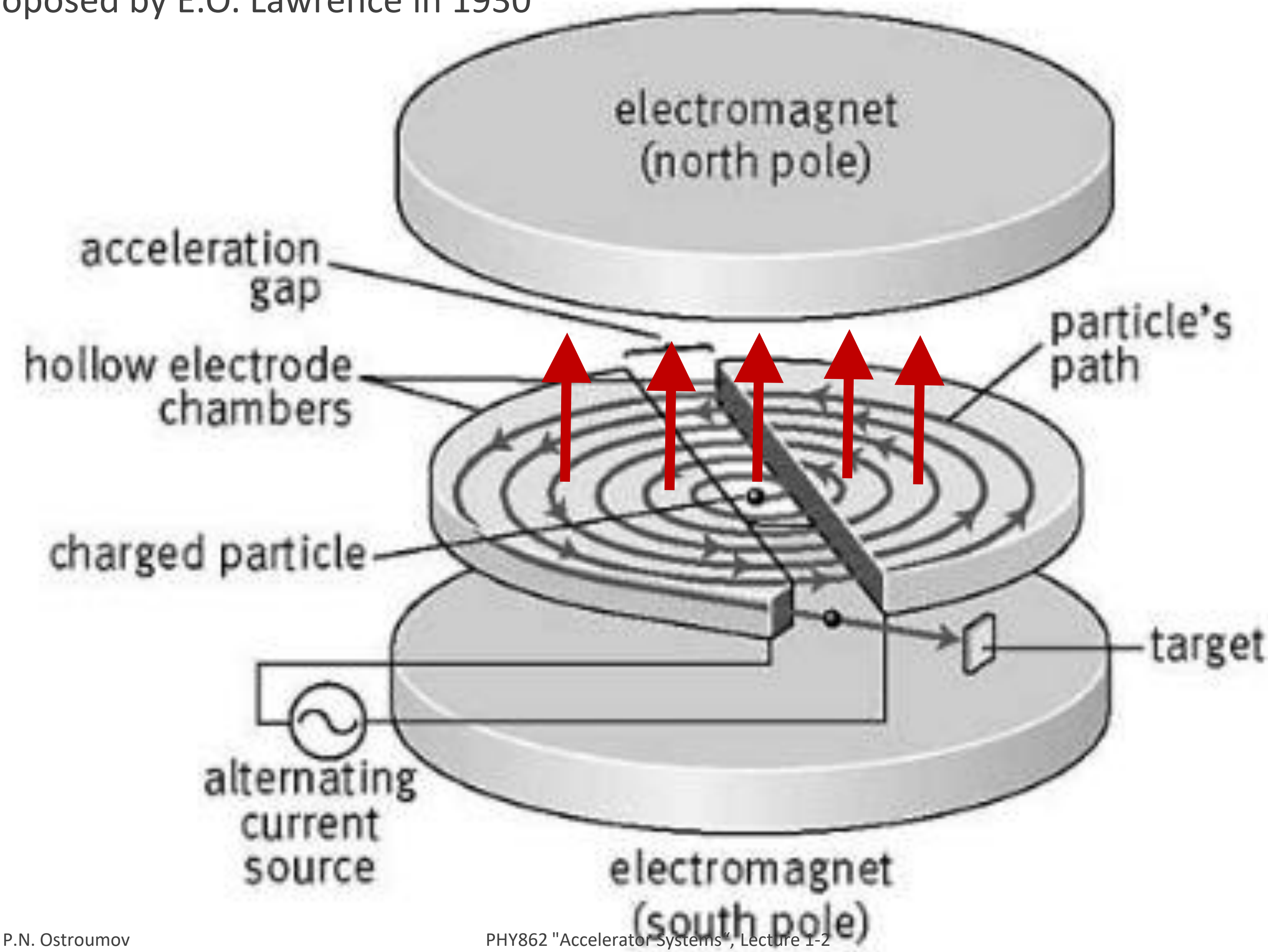
- 2 – synchronous velocity
- 1 – lower velocity
- 2- higher velocity

$\varphi=-30°$,



Cyclotron

Proposed by E.O. Lawrence in 1930

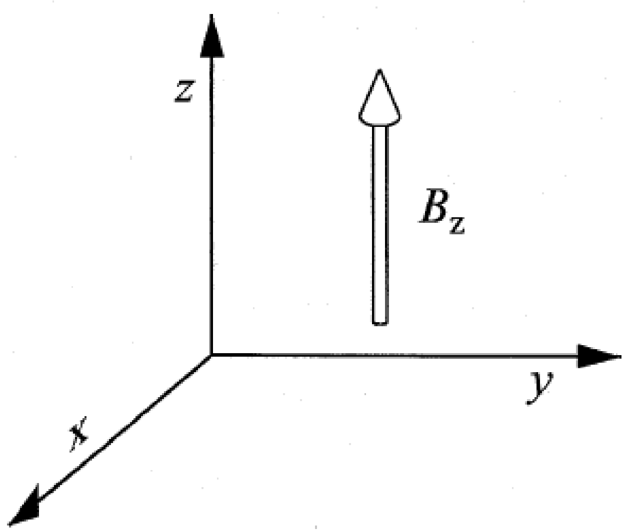


Cyclotron

- First cyclotron, 1.2 MeV, 1932
  - Main idea: provide one RF system (~ 20 MHz) and one RF gap
  - Inject particles between the magnetic poles with B_z
  - Particles orbit in the magnetic field and pass through RF gap many times synchronously
  - Acceleration due to multiple pass through the same RF gap

$$\dot{\vec{p}} = \frac{d}{dt}(m\vec{v}) = q\left[\vec{v} \times \vec{B}\right]$$

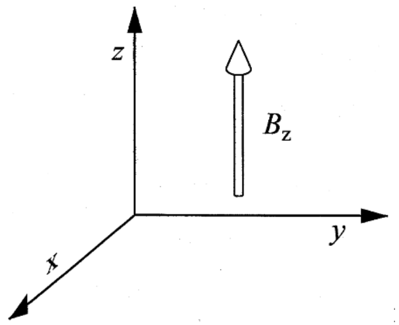
$$\vec{B} = \begin{pmatrix} 0 \\ 0 \\ B_z \end{pmatrix}$$



Equation of motion in a cyclotron

- Motion is confined in X-Y plane

$$\vec{p} = \begin{pmatrix} p_x \\ p_y \\ 0 \end{pmatrix} = m \begin{pmatrix} v_x \\ v_y \\ 0 \end{pmatrix},$$



- Equation of motion in cyclotron (there is no energy gain in constant magnetic field, therefore γ=const, $m=m_0\gamma$)

$$\dot{p}_x = m\dot{v}_x = qv_y B_z$$

$$\dot{p}_y = m\dot{v}_y = -qv_x B_z$$

- Differentiate these equations with respect to time

$$m\ddot{v}_x = q\dot{v}_y B_z = qB_z \frac{-qv_x B_z}{m}$$

$$m\ddot{v}_y = q\dot{v}_x B_z = qB_z \frac{-qv_y B_z}{m}$$

$$A\gamma m_u \dot{v}_x = qev_y B_z$$

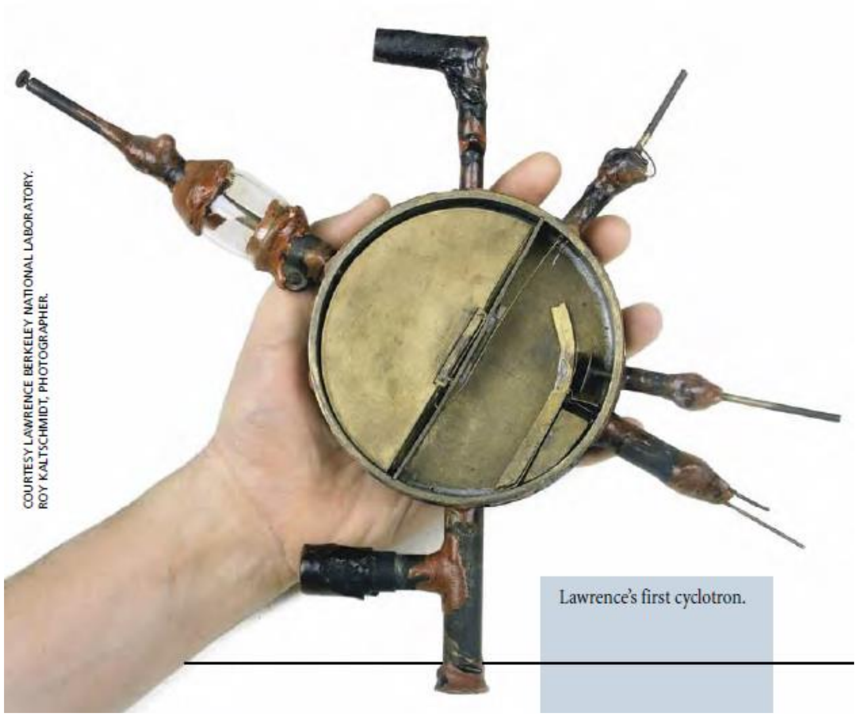
$$A\gamma m_u \dot{v}_y = -qev_x B_z$$

A – mass number
m_u – atomic mass unit =931.5 MeV
qe - ion charge

$$m = \gamma m_o = A\gamma m_u$$

Cyclotron

- Or $$\ddot{v}_x + \frac{q^2 B_z^2}{m^2} v_x = 0$$
  $$\ddot{v}_y + \frac{q^2 B_z^2}{m^2} v_y = 0$$
- Solution is $$v_x(t) = v_0 \cos \omega_z t$$
  $$v_y(t) = v_0 \sin \omega_z t$$



Lawrence's first cyclotron.

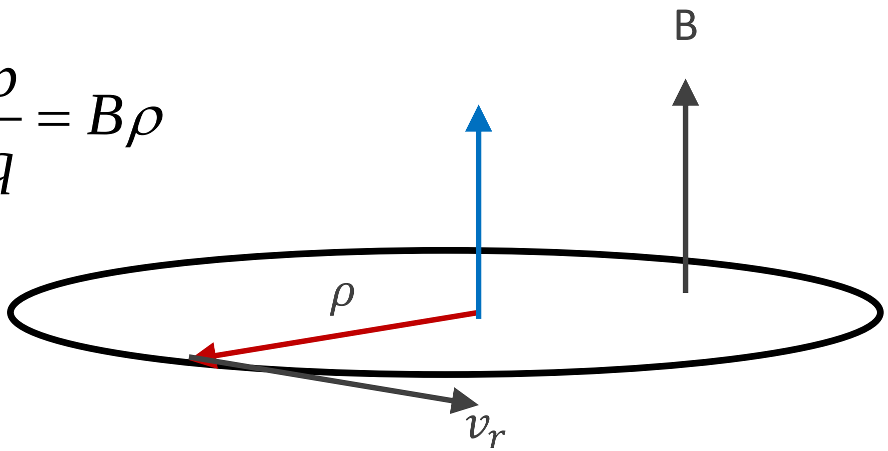
- Particles follow circular motion between the poles with the revolution frequency
- $$\omega_z = \frac{q}{m} B_z = \frac{q}{\gamma m_0} B_z$$ cyclotron frequency
- The revolution frequency does not depend from the velocity: with energy increased particles follow orbit with larger diameter

Cyclotron frequency, another approach

- The centripetal force = Lorenz force

$$\frac{mv_r^2}{\rho} = qv_r \times B_z \Rightarrow \frac{mv_r}{\rho} = qB_z \Rightarrow \frac{p}{q} = B\rho$$

$$\omega_z = \frac{v_r}{\rho} = \frac{qB_z}{\gamma m_0}$$



v_r Is tangential velocity

- Rigidity
- Beam $\frac{p}{q} = (B\rho)$ Accelerator magnets, geometry

$$p[GeV/c] = Am_u c\beta\gamma = A\frac{m_u c^2}{c}\beta\gamma$$

- Useful practical expression

$$\frac{p[GeV/c]}{q[e]} \approx 0.3B[T]\rho[m]$$

q is expressed in number of electrons

- Rigidity of FRIB uranium beam: $B\rho = 238 \cdot 0.931 \cdot 0.7/(0.3 \cdot 78) = 6.63T \cdot m$

Classical cyclotrons

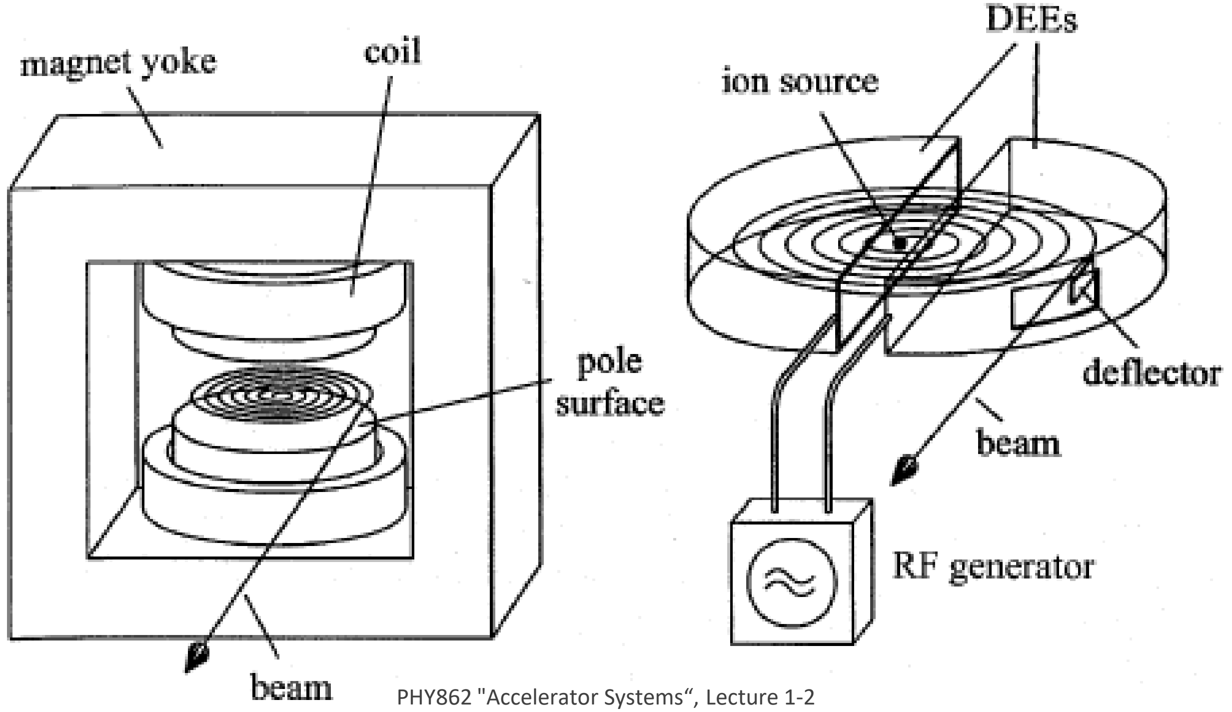
- B ≈ 2 T, $m=m_0$=938 MeV; for protons q=e

$$f=\frac{1.6\cdot 10^{-19}\cdot 2}{2\pi\cdot 1.67\cdot 10^{-27}}\approx 30MHz$$

$\omega_z=\omega_{RF}$ Angular frequency [rad/sec]

$f=\frac{\omega_{RF}}{2\pi}$ Cycles per second [1/sec], [Hz]

- Can accelerate up to $v\approx 0.15c$



Synchrocyclotron

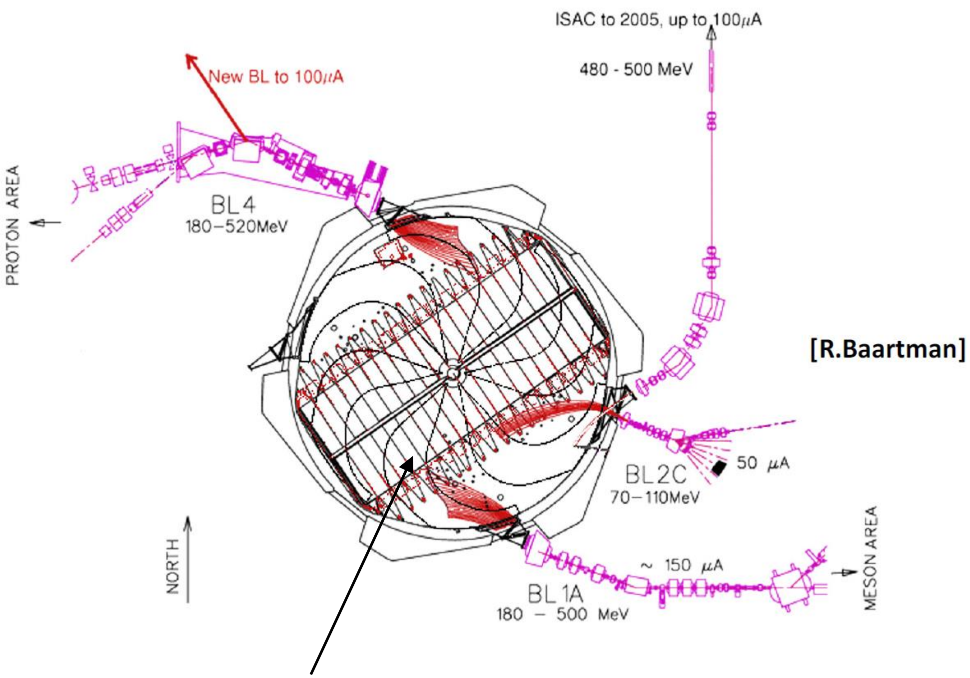
- $\omega_z = \frac{q}{m} B_z$, particle mass is a function of energy $m = \gamma m_0$
- If *B=const*, revolution frequency decreases as particles are accelerated
- Synchrocyclotrons:
  - Injection of a beam pulse, acceleration while ω_{RF} decreases synchronously with cyclotron frequency
  - Extraction
  - Injection of a new beam pulse
- Synchrocyclotron is a pulsed accelerator

The first synchrocyclotron,
Gatchina, Russia, 1950s
1 GeV protons
10,000 metric tons

Isochronous cyclotron

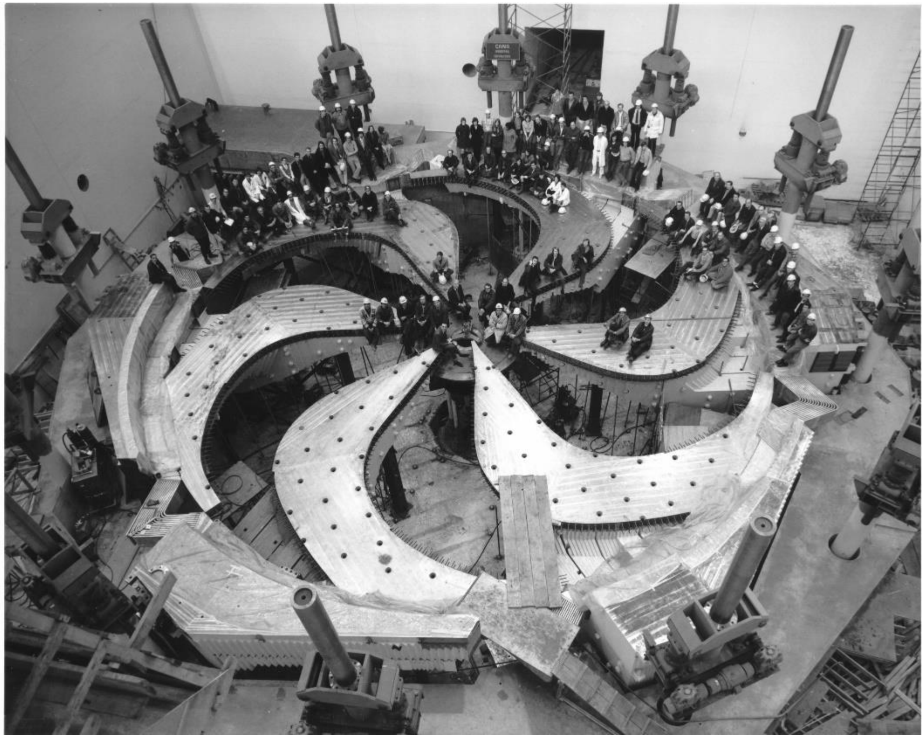
- Increase magnetic field as beam orbit radius increases
  - The RF frequency remains const
- Continuous wave (CW) acceleration

$$\omega_z = \frac{qB_z(r(E))}{m(E)} = const$$



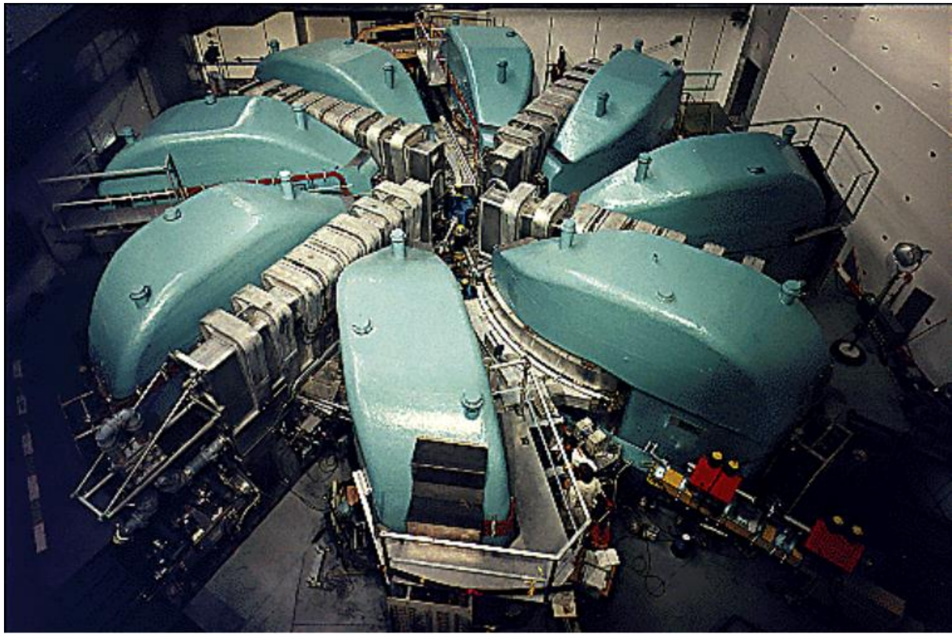
Accelerating gap, 200 kV

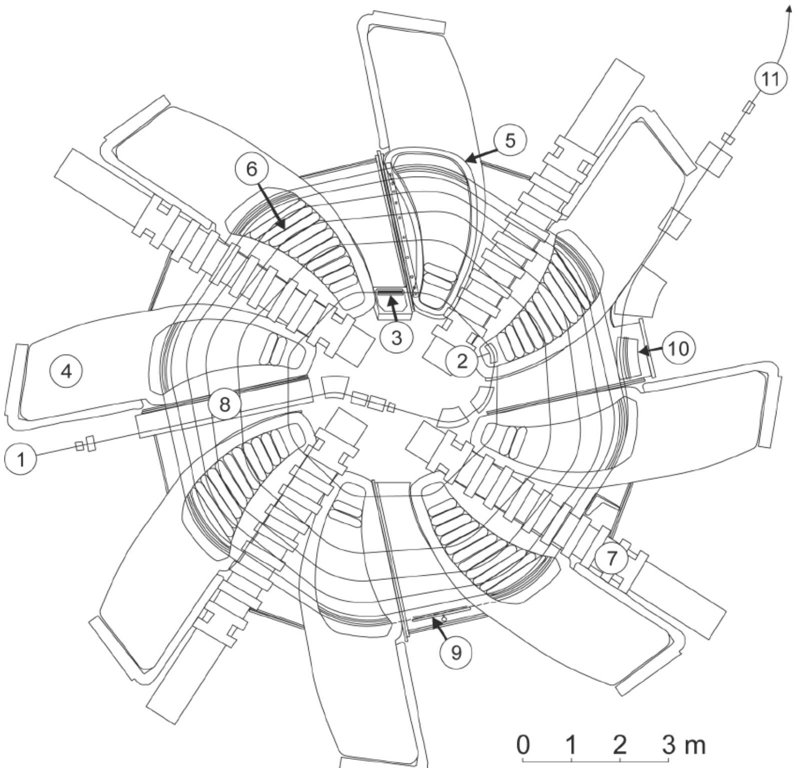
[R.Baartman]

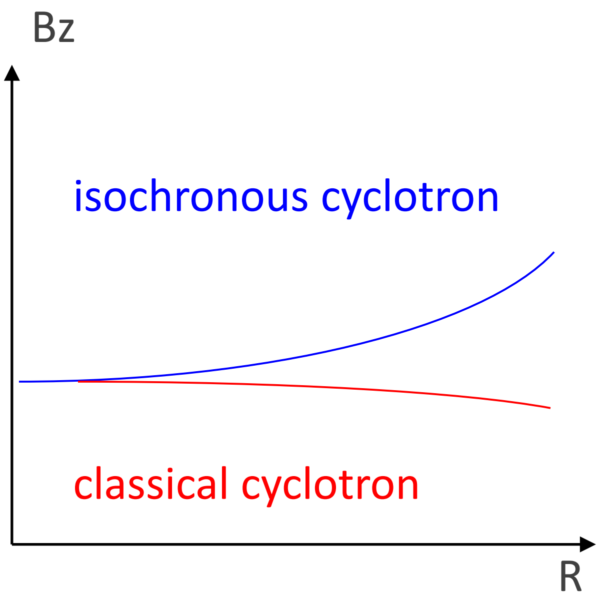


Isochronous cyclotron

- Higher field at higher radius introduces vertical defocusing
- Use bending magnet edge focusing
- 590 MeV proton cyclotron in PSI, Switzerland
- 1.4 MW beam power – world record







Focusing in classical cyclotron

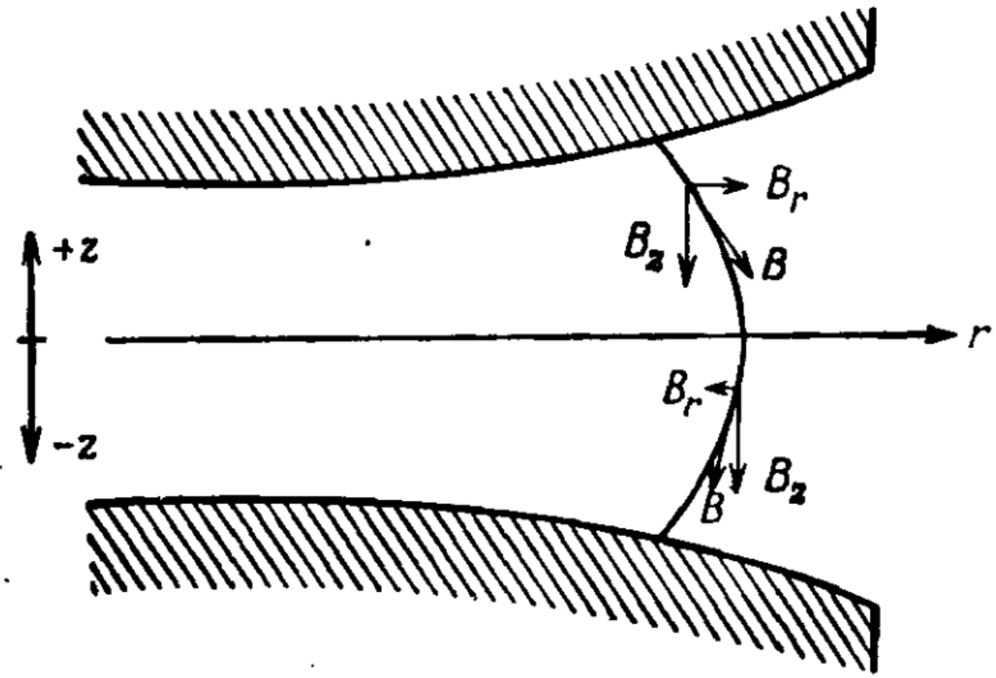
- Lorentz force, electric field is 0 along the orbit except the RF gap

$$\vec{F} = q\left(\left[\vec{v}\vec{B}\right] + \vec{E}\right) \Rightarrow \vec{F} = q\left[\vec{v}\vec{B}\right]$$

- There are 2 components of magnetic field: B_z, B_r
- B_z is responsible for orbital motion with cyclotron frequency
- B_r is responsible for the vertical focusing

$$F_z = qv_\theta B_r$$

- Focusing in radial direction is provided automatically due to the orbital motion

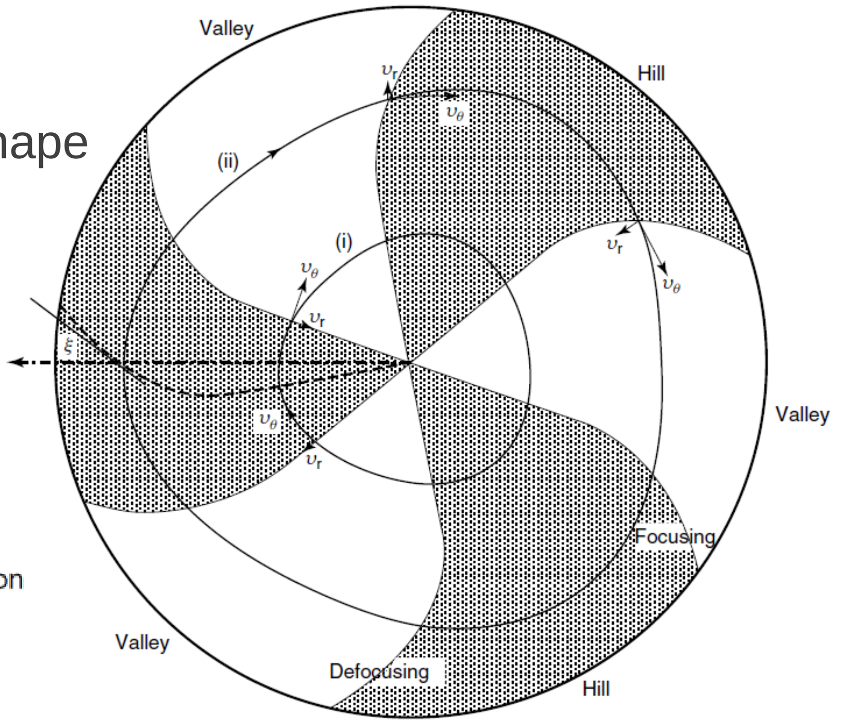


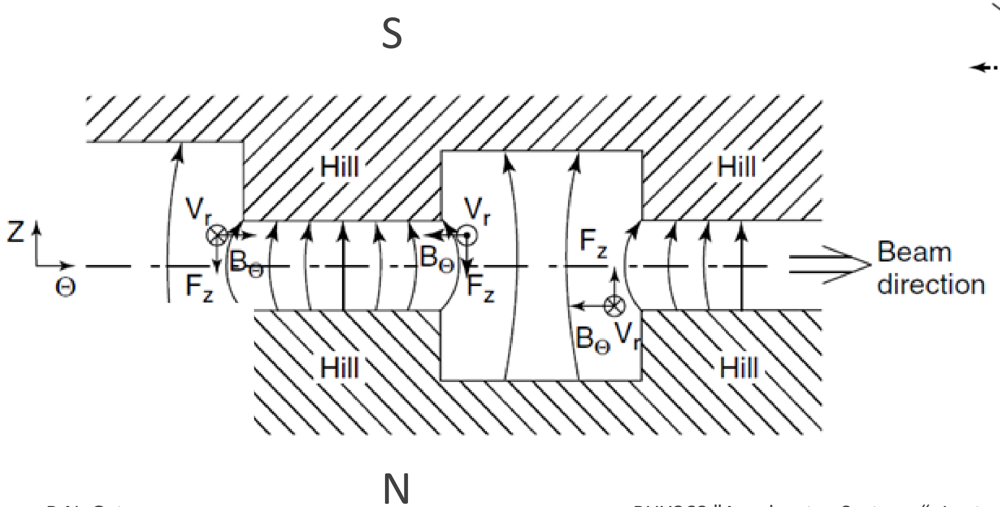
Field profile along the particle path in isochronous cyclotron

- The orbit includes sectors with small and large curvature radii due to "valleys" (low B_z) and "hills" (high B_z)
- The radial component of the magnetic field produces "edge" focusing and particle motion is stable in both radial and vertical directions

$$F_z = qv_r B_\theta$$

- Radial velocity v_r appears due to the shape of trajectory in the magnetic field

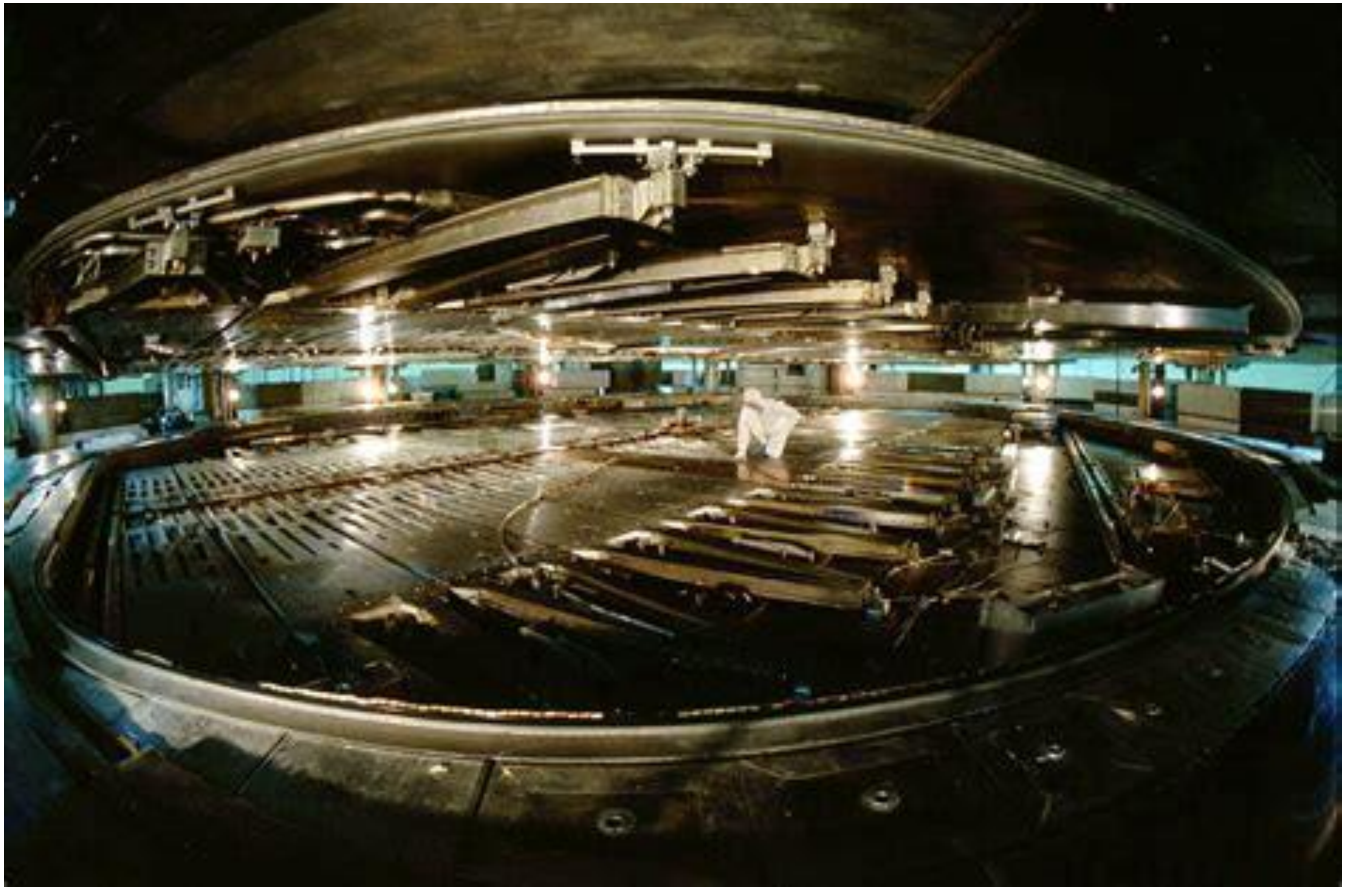




(b)

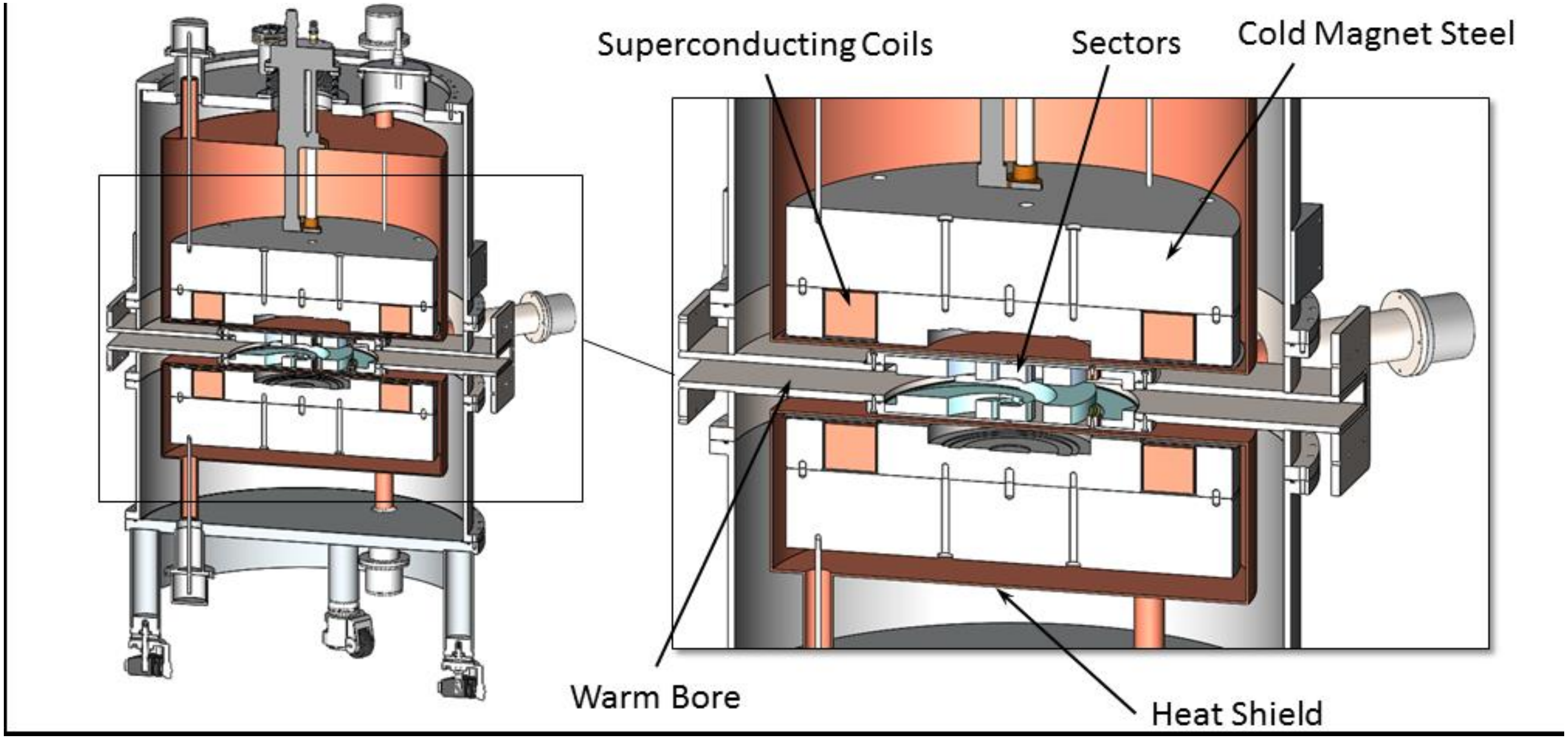
TRIUMF cyclotron

The worker is checking the accelerating structure for worn or damaged components



Superconducting cyclotrons

- Cyclotron can be made compact by increasing magnetic field up to ~4.5 Tesla
- Example of sector focused, isochronous 12 MeV proton cyclotron
  - ~1.5 m height and ~0.6 m diameter



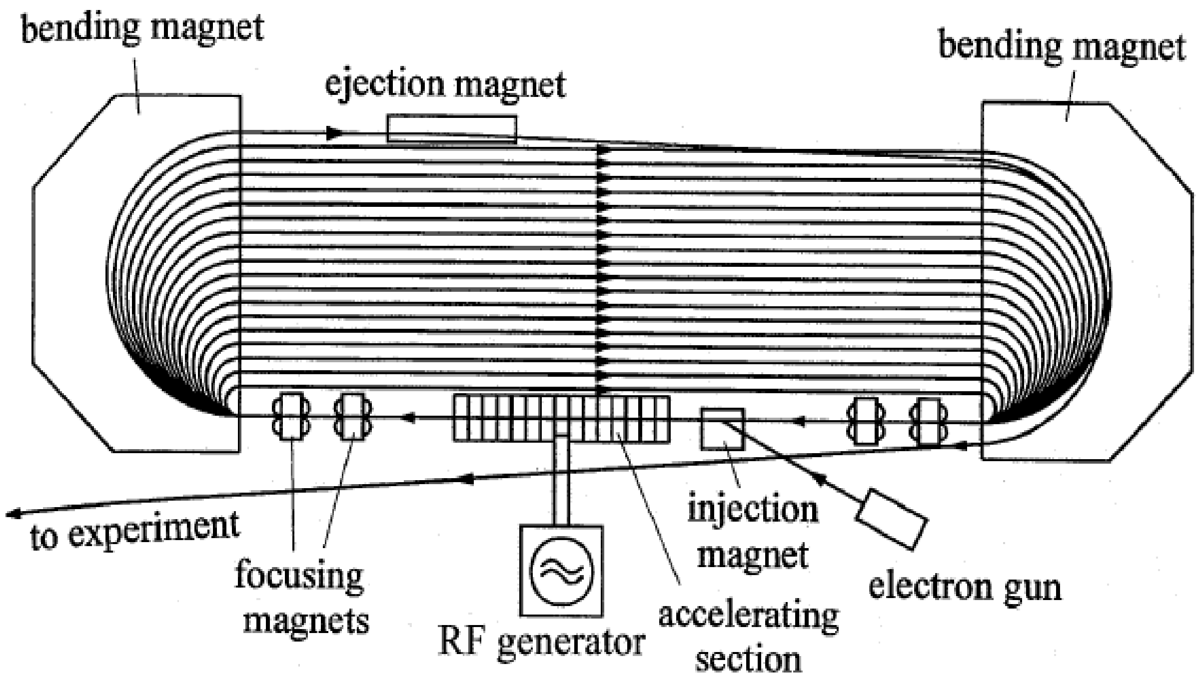
Commercial superconducting synchrocyclotron

Rotco: Rotating Capacitor

Figure 2: The assembled cyclotron placed in the shielded beam-vault. It can be opened in the median plane as well as at the top of the cryostat. Also visible are the rotco-shield (left), the cryocooler-shield (middle) and the vacuum station (right).

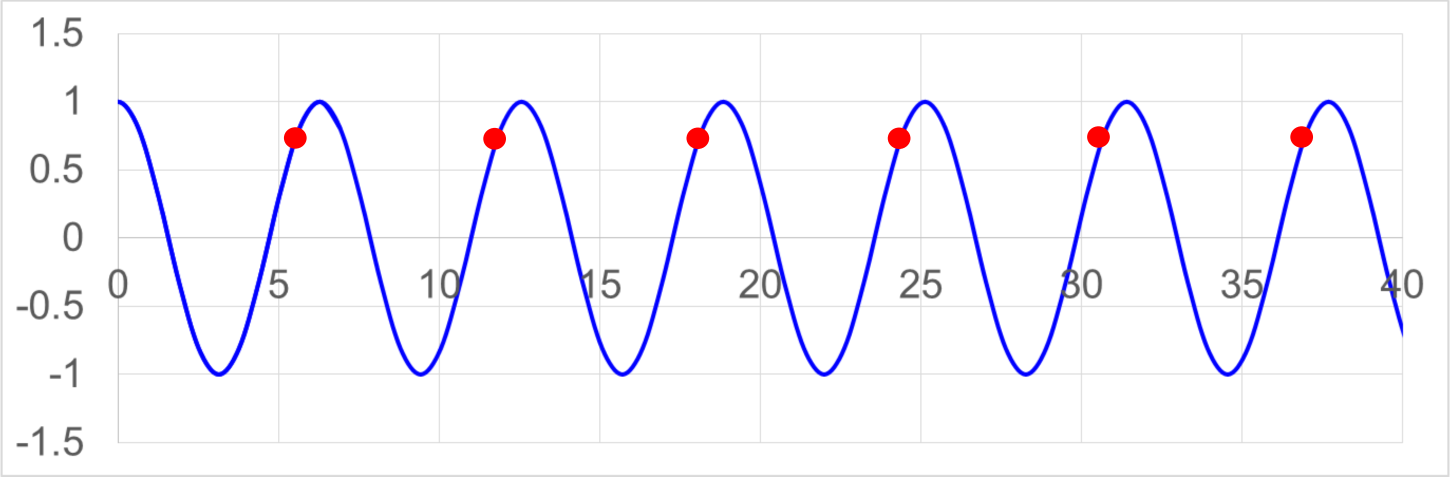
Microtron

- Acceleration of electrons in cyclotron is not efficient due to quick reach of relativistic velocities by electrons
- It is not necessary to have revolution frequency equal to RF frequency, it is sufficient if particles see the same phase in each turn
- This is realized in microtron: after each turn, the length of the orbit increases by exactly the length equal to one RF wavelength, λ
- RF frequency is high, ~3 GHz, λ~ 10 cm

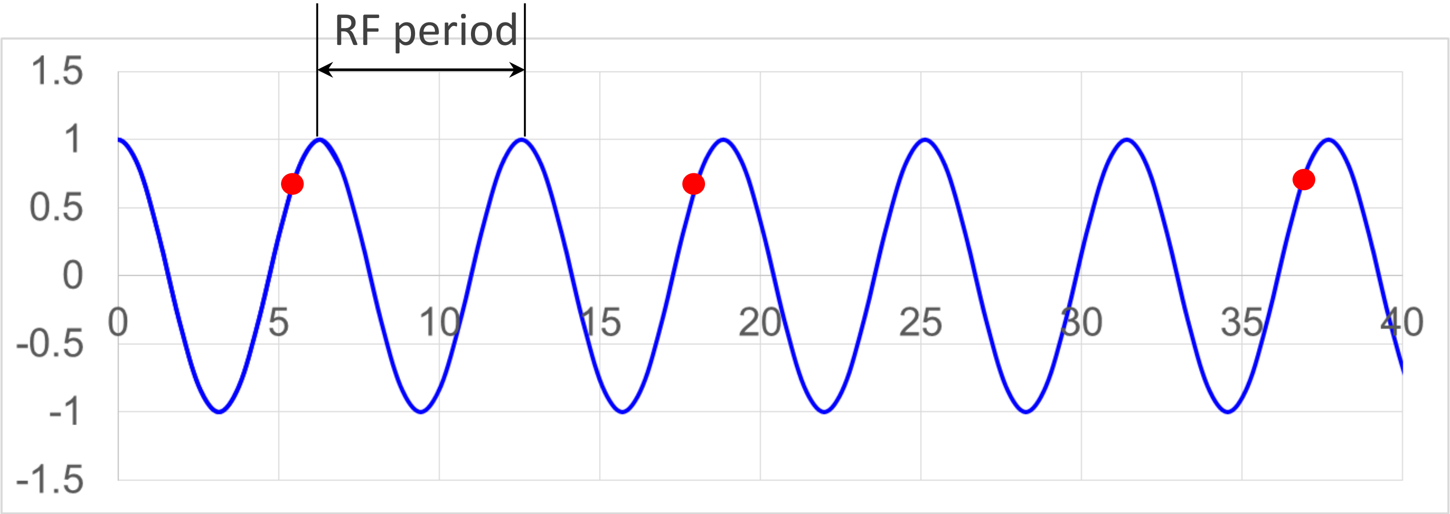


Beam synchronization in RF linac and microtron

- RF linac or cyclotron



- Microtron



Microtron

- Highest energy microtron in University at Mainz (Germany), 1.6 GeV

$$t_i = \frac{2\left(\pi R_i + l\right)}{v_i},$$

- One-turn time of flight in *i*-th turn

$$e v_i B = \frac{m_i v_i^2}{R_i}$$

- Lorentz force = centripetal force

$$R_i = \frac{v_i m_i c^2}{e c^2 B} = \frac{v_i}{e c^2 B} E_i$$

- Bending radius in the magnetic field, E_i is the total beam energy at each turn

$$\Delta t = t_{i+1} - t_i = \frac{2\pi}{e c^2 B}\left(E_{i+1} - E_i\right) = \frac{2\pi}{e c^2 B}\Delta E$$

$$\Delta E = e V_o$$

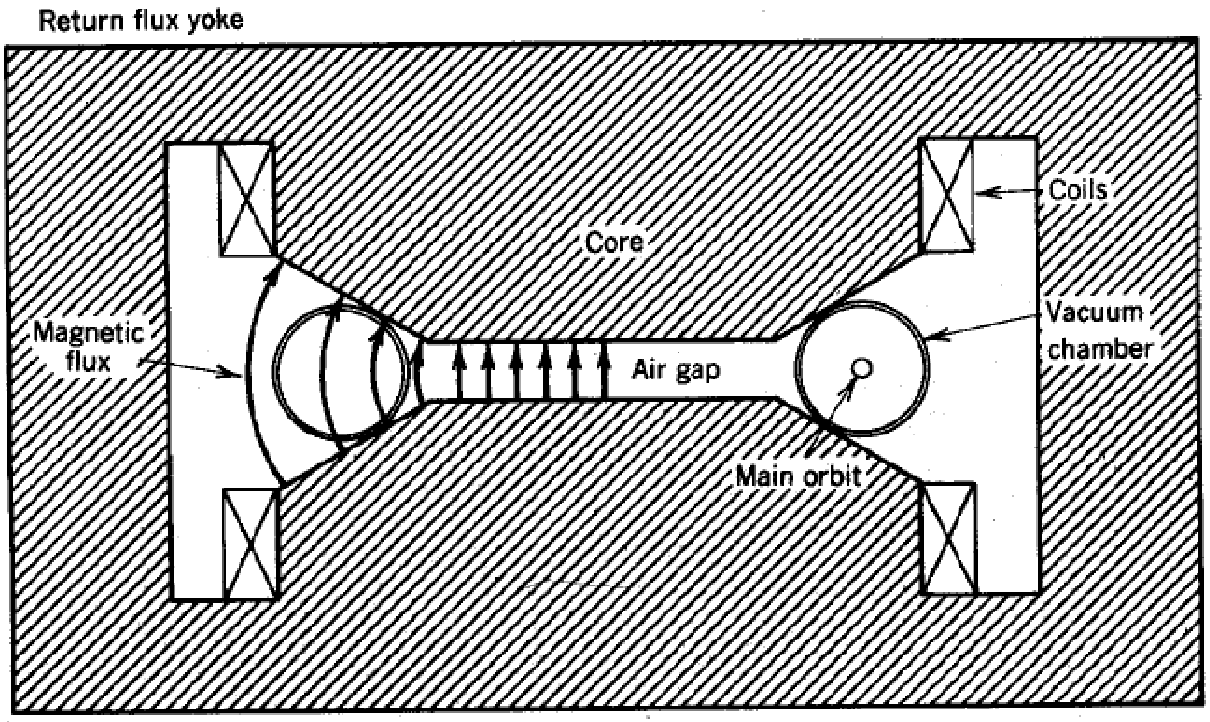
Energy gain per turn

$$\Delta t = \frac{k}{f}, \quad V_0 = k\frac{c^2 B}{2\pi f}$$

The time difference between the turns should be equal to multiple of RF periods, k is an integer number, V_0 is the accelerating voltage per turn

Betatron

- A pulsed electron accelerator
- D.W. Kerst built the first working betatron in 1940
- 20 MeV betatron in 1942
- The acceleration is based on application of the law of induction
- Simple rotational symmetry



Betatron

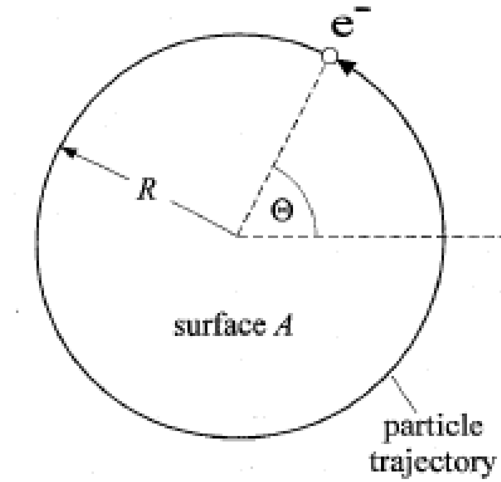
- Faraday's Law of induction, electromotive force

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{r} = -\iint_A \dot{\vec{B}} d\vec{s}, \quad \vec{B}(t) = \vec{B}_0 \sin\omega t$$

$$\frac{d\vec{B}}{d\theta} = 0, \quad rotational\ symmetry,\ B(r) \neq const$$

$$\left\langle \left|\vec{B}\right| \right\rangle = \frac{1}{\pi R^2} \iint_A \vec{B}(r) \cdot d\vec{S},$$

Circular trajectory



Integration over the area of πR^2. This field induces an electric field along the particle trajectory

$$2\pi R\left|\vec{E}\right| = -\pi R^2 \frac{d\left\langle \left|\vec{B}\right| \right\rangle}{dt}, \quad \left|\vec{E}\right| = -\frac{R}{2}\left\langle \left|\dot{\vec{B}}\right| \right\rangle$$

Due to rotational symmetry

$$\frac{d\vec{E}}{d\theta} = 0,$$

$$E_\theta = -\frac{R}{2}\left\langle \dot{B}_z(r) \right\rangle$$

Betatron

- Accelerating force

$$F_\theta = \dot{p}_\theta = -eE_\theta = \frac{eR}{2}\left\langle \dot{B}_z(r) \right\rangle \rightarrow \dot{p}_\theta = \frac{eR}{2}\left\langle \dot{B}_z(r) \right\rangle$$

$$\frac{p_\theta}{e} = B_z(R)\cdot R$$

Centripetal force = Lorentz force on the particle circular trajectory

$$p_\theta = eRB_z(R)$$

$$\dot{p}_\theta = eR\dot{B}_z(R)$$

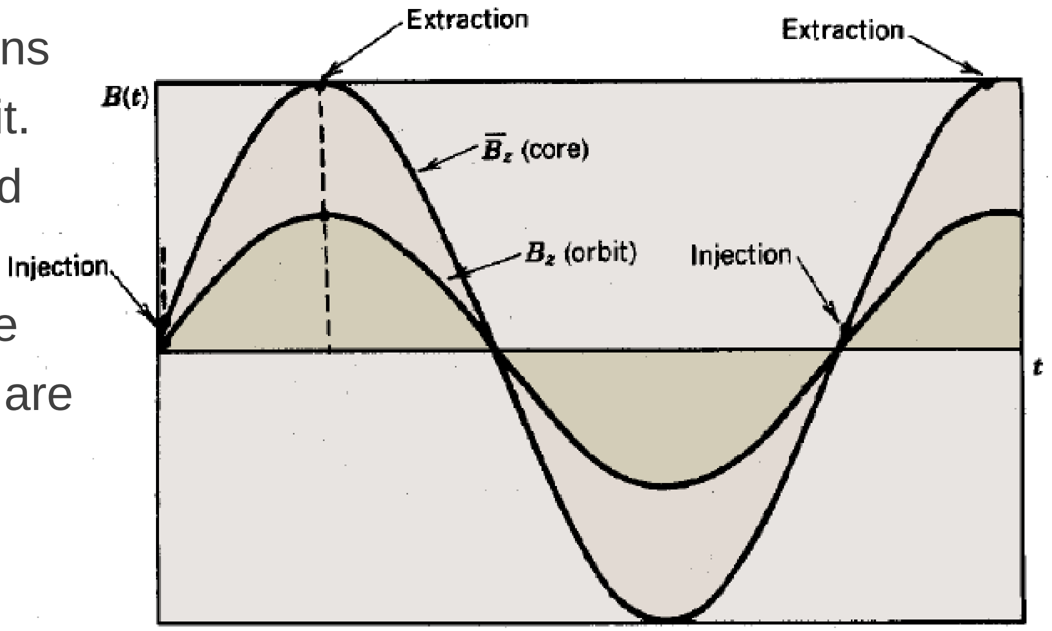
Apply time differentiation to both sides
Of this equation

- Then, we have two equations that should be satisfied during the acceleration of electrons in a betatron

Betatron condition

$$\dot{B}_z(R) = \frac{\langle \dot{B}_z(r) \rangle}{2}, \quad B_z(R,t) = \frac{1}{2}\langle B_z(r,t) \rangle + B_z(R,0)$$

- The stable motion is possible only if this condition is satisfied. The constant field $B_z(R,0)$ can be used to adjust the orbit using correcting coils
- Average magnetic field inside the electron orbit and vertical magnetic field at main orbit as a function of time
- Betatron is a pulsed machine
- Electrons perform oscillations with respect to the central orbit. These oscillations were named "betatron oscillations"
- Nowadays beam transverse oscillations in any accelerator are called "betatron oscillations"

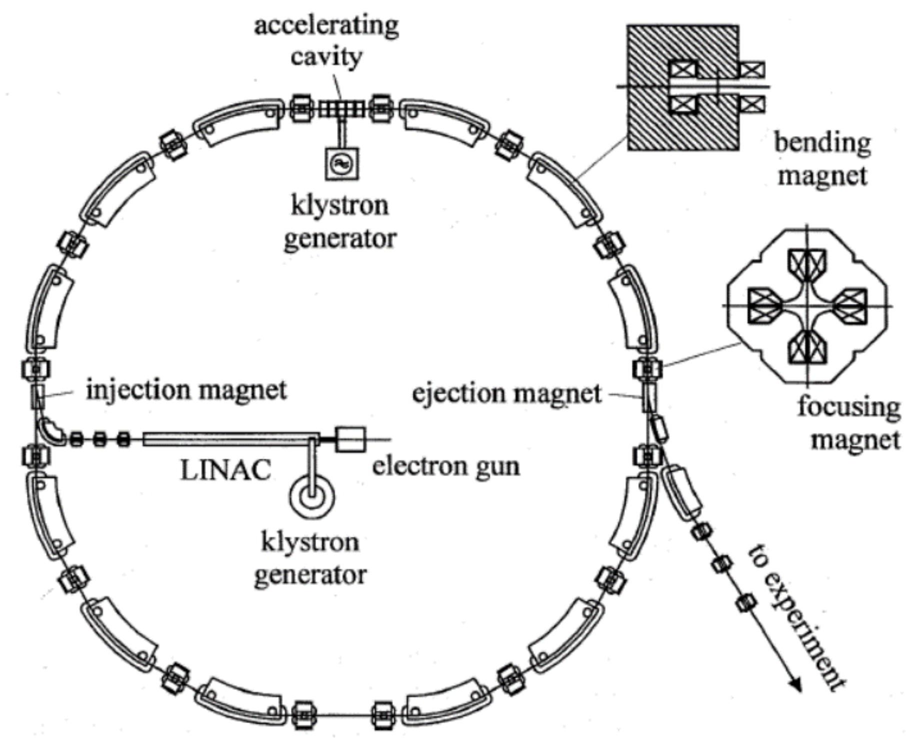


Synchrotron

- For relativistic particles, the orbit radius increase with energy as
- $$R = \frac{E}{qcB}$$ B_{max} ~1.5 T for room temperature magnets, E~ 1. GeV
- B_{max} ~ 6 T for SC magnets
- Very large magnets are required to accelerate particles to 1-2 GeV
- Synchrotron:
  - *R = const*
  - Magnetic field and RF frequency are function of acceleration time
  - Magnetic field must be increased synchronously with beam energy
- Synchrotron can reach any high energy by increasing the perimeter of the ring, LHC perimeter is 28 km
- 3 GeV proton synchrotron (cosmotron) was built at BNL in 1947
- Synchrotron is a pulsed machine
  - Injection
  - Acceleration
  - Extraction

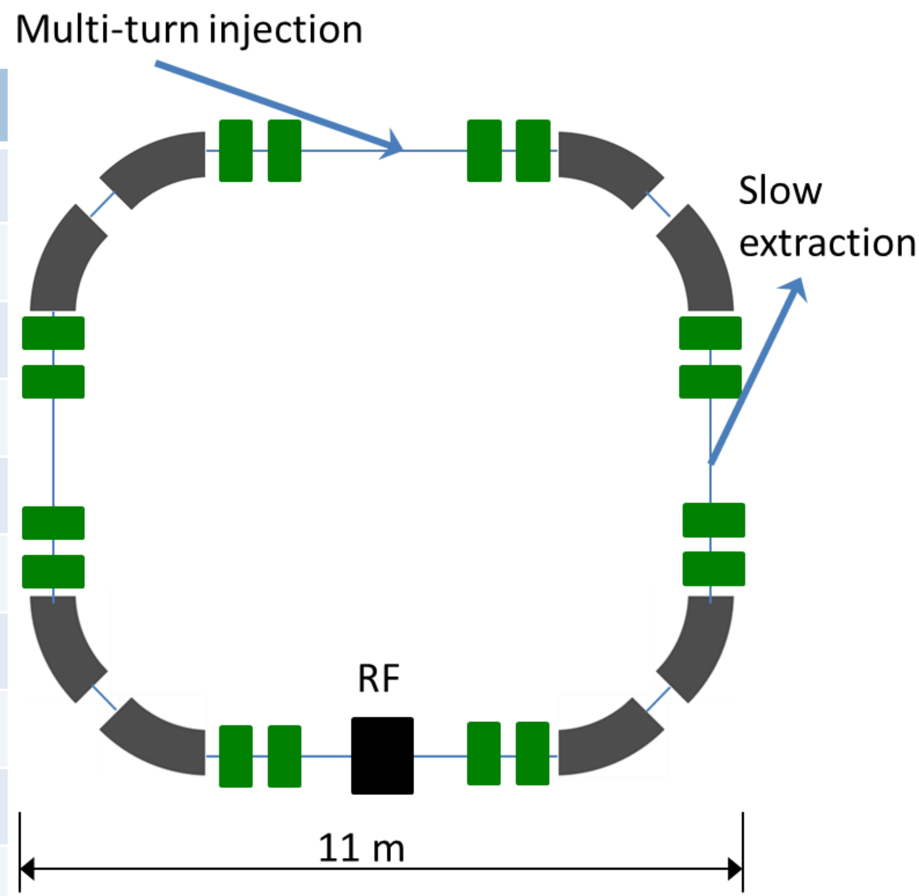
Synchrotron

- During the acceleration cycle B varies from B_{min} to B_{max} . B_{min} can not be zero
  - Main reason: if the field is too low, the errors of the magnetic field can strongly effect the beam quality
- Requires a linac to increase injection energy



Synchrotron for re-acceleration of radioactive beams

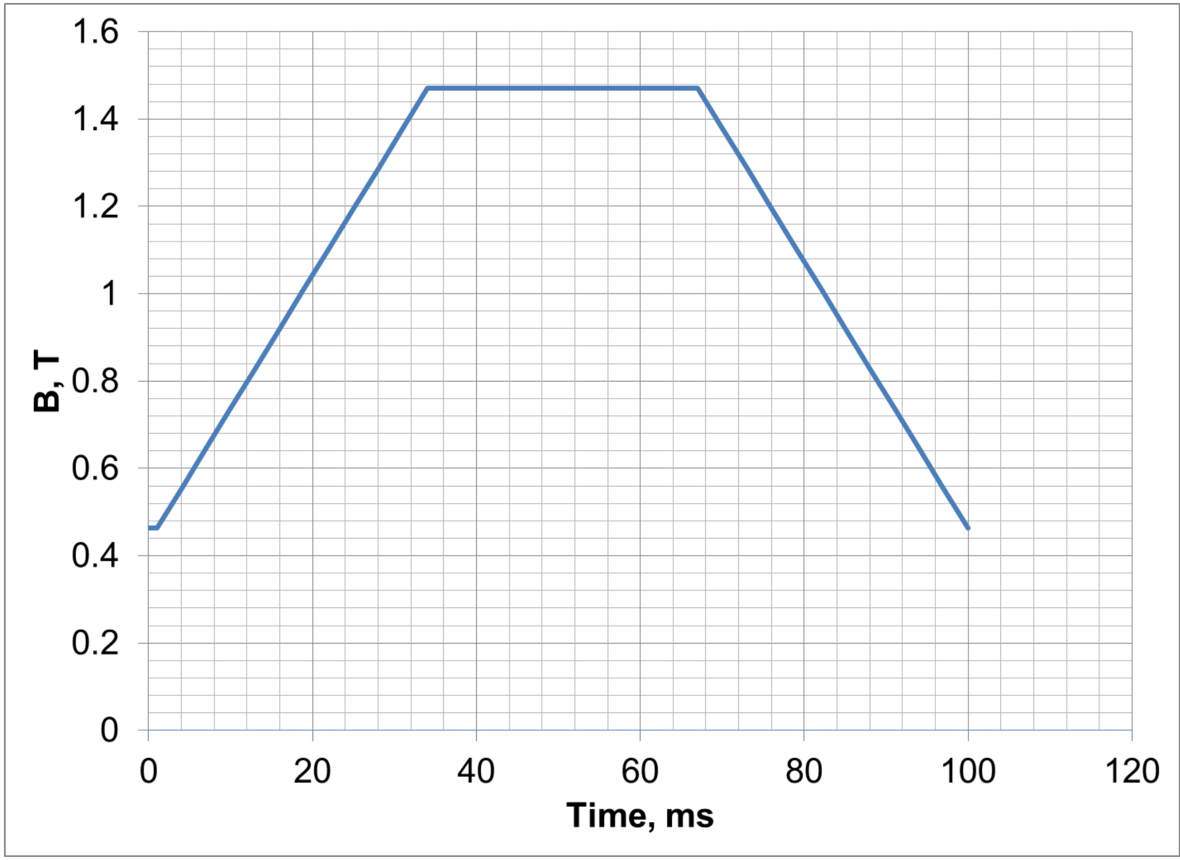
Parameter	Value
Injection energy, MeV/u	2.0
Extraction energy, MeV/u	up to 20.0
Maximum magnetic field, T	1.5
Repetition rate, Hz	10
Q/A	1/5
Revolution frequency, MHz	0.76-2.368
RF harmonic	5
RF frequency range, MHz	3.8-12
Accelerating voltage, kV	4



Acceleration Cycle

- Injection ~10 µs
- Acceleration – 33 ms
- Extraction – up to 33 ms
- Programmable magnet power supply is required
- Repetition rate is 10 Hz
- The beam momentum and magnetic field must satisfy this condition

$$\frac{p}{q} = (B\rho)$$



Synchrotron

- Proton synchrotron
  - The largest proton synchrotron is LHC: 7 TeV

- https://home.cern/about/accelerators

- Electron synchrotron
  - Energy is limited by synchrotron radiation losses

$$\Delta E = \frac{e^2}{3\varepsilon_0 (m_0 c^2)^4} \frac{E^4}{R} \qquad \Delta E[keV] \approx 88.5 \frac{E^4 [GeV^4]}{R[m]}$$

  - Energy loss due to synchrotron radiation $\propto E^4$
    - Mitigation is to increase radius of the ring
  - Majority of electron synchrotrons is up to ~10 GeV
  - The largest electron synchrotron was in CERN, LEP – 80 GeV
    - The same tunnel that houses current LHC

Colliders

- Example: electron-positron collision
- The dynamics of electron positron annihilation is very clear and colliding of electrons and positrons is used for generation of heavy particles
- Collider:

- Total energy available in the c.m. frame which is the same as in laboratory frame in this case: $E+E=2E$
- B-meson mass is 9.47 GeV
- 5 GeV electron and positron beams are sufficient to produce B-meson

Fixed target system

- Electron beam is accelerated and hits fixed target to produce B-mesons via interaction with positrons
- In laboratory frame

	Electron	Positron
Momentum	p_1	0
Relativistic energy	$E_1 = \gamma m_e c^2 = p_1 c$	0

- We assume velocity is *c* $\gamma = \dfrac{E_1}{m_e c^2}$
- In center of mass frame $\sum p_i = 0$

	Electron	Positron
Momentum	p'	$-p'$
Relativistic energy	$E_1' = p'c$	$E_2' = p'c$

$$\gamma' = \frac{E_1'}{m_e c^2}$$

Energy available in the center mass frame

- Total energy available in the center mass system
- $$E^* = E_1' + E_2' = 2p'c = 2\gamma' m_e c^2$$
- We need to find γ'
- Lorentz transformation is applied to energy-momentum 4-vector, only 2 components are not zero:

$$\begin{pmatrix} \frac{E_1}{c} \\ p_1 \end{pmatrix} = \begin{pmatrix} \gamma' & \beta'\gamma' \\ \beta'\gamma' & \gamma' \end{pmatrix} \cdot \begin{pmatrix} \frac{E_1'}{c} \\ p_1' \end{pmatrix}$$

This transformation is similar to the transformation of position-time four vector $\vec{u} = (\vec{x}, ct)$

$$p_1 = \gamma'(\beta' + 1) p_1'$$

- For relativistic particles $\beta \approx 1$ therefore

$$p_1 = 2\gamma' p_1' = 2\gamma'(\gamma' m_e c)$$

$$\gamma m_e c = 2\gamma'^2 m_e c$$

$$\gamma' = \sqrt{\frac{\gamma}{2}}$$

Colliders

- This results in

$$E^* = 2\gamma' m_0 c^2 = 2\sqrt{\frac{\gamma}{2}} m_e c^2 = \sqrt{2\gamma\left(m_e c^2\right)^2} = \sqrt{2E_1 m_e c^2}$$

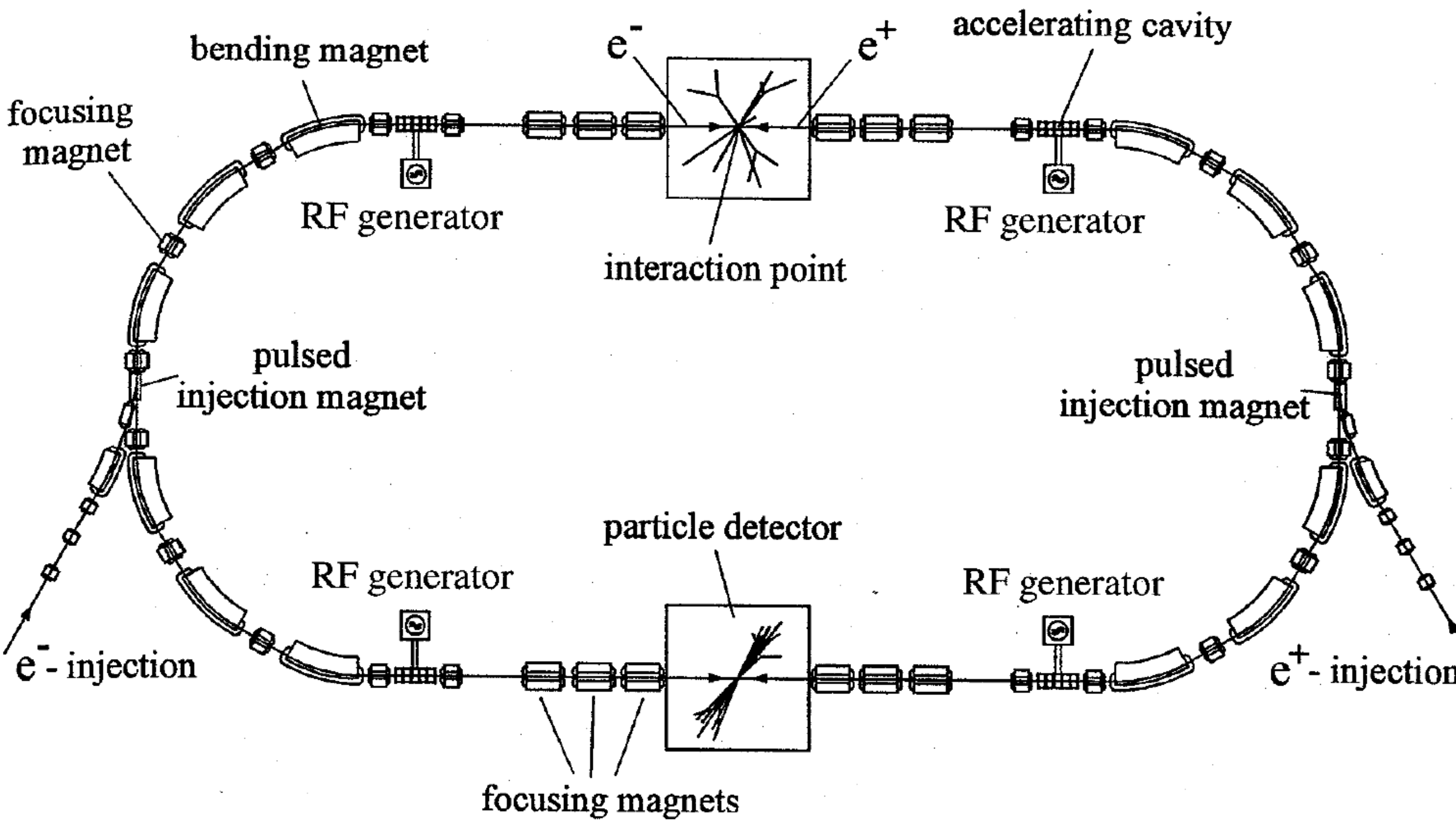
$$E^* = \sqrt{2E_1 m_e c^2}$$

- Example: $m_e c^2 = 0.511 MeV, E_1 = 5 GeV$

$$E_{c.m.} = E^* = 0.071 GeV$$

- Example: to produce 9.47 GeV B-meson we need 87.75 TeV electron with fixed target. Compare to 2×5 GeV in the collider

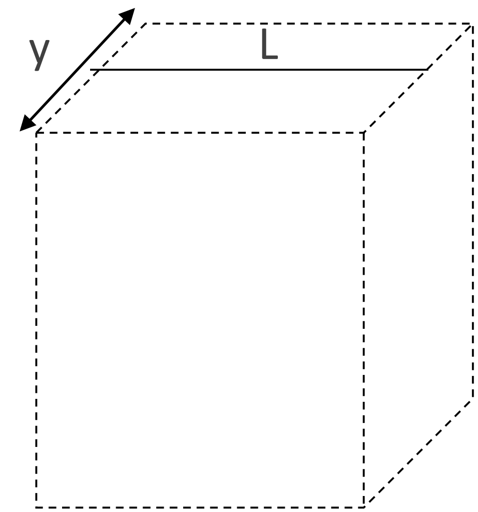
Colliders


bending magnet
e-
e+
accelerating cavity
focusing magnet
RF generator
interaction point
RF generator
pulsed injection magnet
pulsed injection magnet
particle detector
RF generator
RF generator
e- - injection
e+ - injection
focusing magnets

Advanced Acceleration Methods

- How far do accelerating gradients go?
- Superconducting RF acceleration: ~40 MV/m for $\beta \rightarrow 1.0$ particles
- CLIC (Compact Linear Collider, room temperature): ~100 MV/m
  - High frequency accelerating structure
  - Two-beam accelerator: drive beam (high intensity, low energy) couples to main beam
- Dielectric wall accelerator: ~100 MV/m
- Dielectric wakefield acceleration: ~GV/m
- Laser-plasma acceleration ~100 GV/m
  - Electrons to 1 GeV at several cm

Plasma acceleration



- Plasma oscillations
- Imagine a region in plasma where electrons are shifted with respect to ions
- Apply Gauss's law

$$\int_S \vec{E} \cdot d\vec{S} = \frac{1}{\varepsilon_0} \int_V \rho dV$$

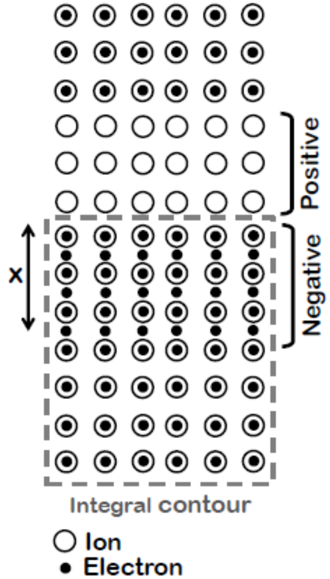
S=y·L
V=y·L·x

- Electric field produced by displaced charges

$$E = \frac{nex}{\varepsilon_0}$$

- Electron motion

$$F = m\frac{d^2x}{dt^2} = -eE = -\frac{ne^2x}{\varepsilon_0}$$



Electric field in displaced plasma

- From equation of motion we can calculate frequency of plasma oscillations

$$\omega_p^2 = \frac{ne^2}{\varepsilon_0 m}$$

$$f_p\left[Hz\right] \approx 9000 \cdot n^{1/2}, \quad n\left[cm^{-3}\right]$$

- Example: n=10^{24} m^{-3} , f=9 THz, λ_p=33 μm

$$E_{\max} \approx \frac{ne}{\varepsilon_0}\lambda_p = \frac{10^{24} \cdot 1.6 \cdot 10^{-19} \cdot 33 \cdot 10^{-6}}{8.85 \cdot 10^{-12}} \sim 600\, GV/m$$

How to excite plasma?

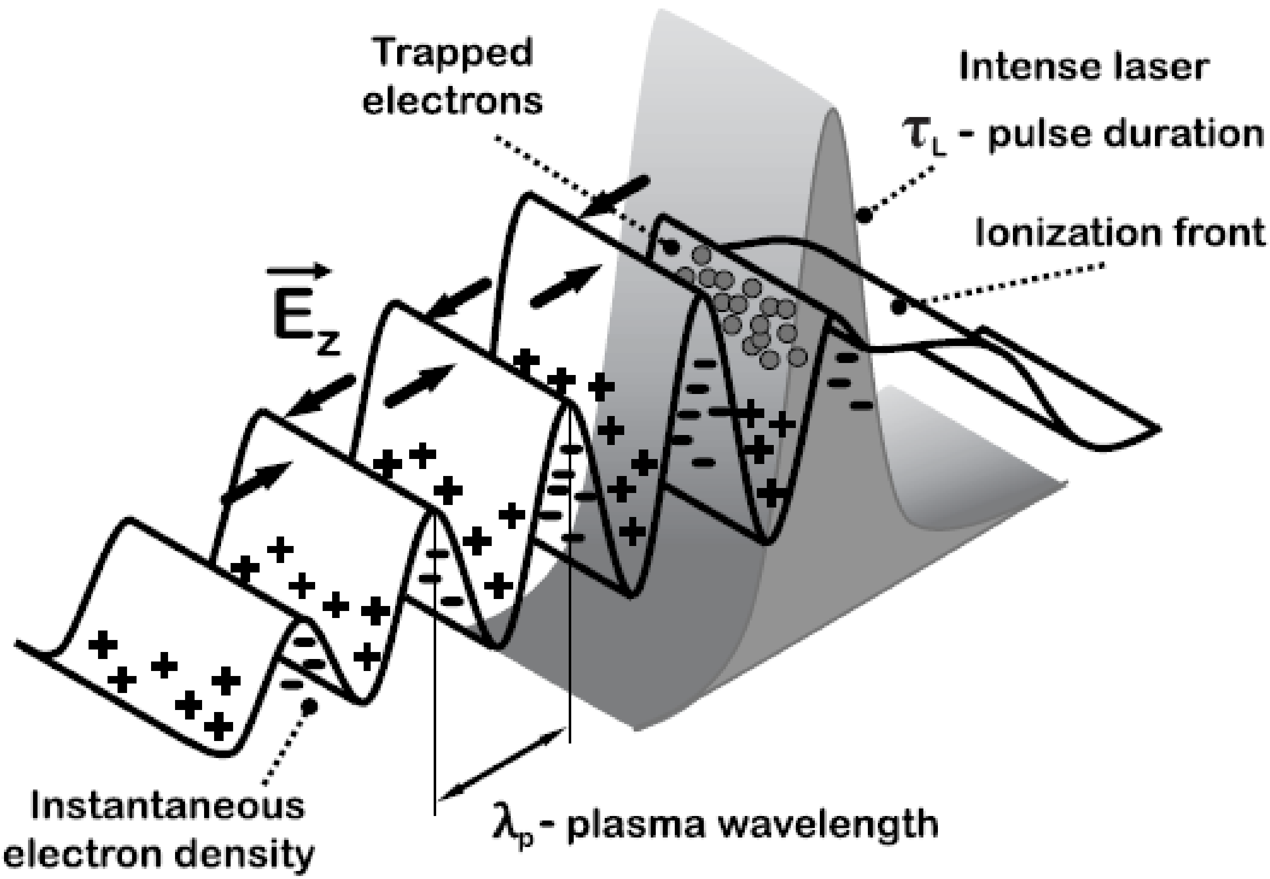
- Use short pulse lasers
- Laser pulse can penetrate plasma if the laser frequency is larger than plasma frequency, otherwise the current of plasma oscillations screens the laser electromagnetic wave

$$\omega_{laser} > \omega_p$$

- Another method to excite the plasma is to send a short bunch of high energy charged particles: plasma Wakefield acceleration.
  - The energy of particles accelerated in plasma will be much higher than the energy of primary bunch

Concept of laser plasma acceleration

- Laser excites plasma EM wave in plasma
  - Can be pre-ionized plasma or just gas
- The electrons of the plasma can be trapped in the EM wave and accelerated



The Berkeley Lab Laser Accelerator (BELLA)

- http://bella.lbl.gov/

Application of accelerators

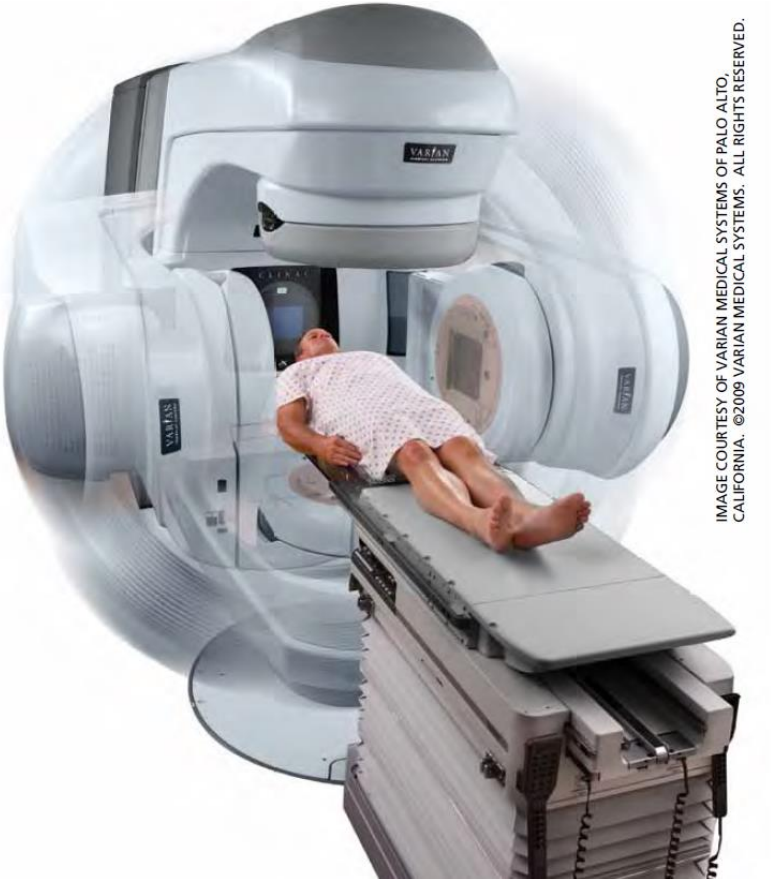
- In high-energy physics (also called particle physics), particles are accelerated to energies far higher than usually found on earth. Collisions then produce particles that did not exist in nature since the Big Bang, yielding data on the properties of these new particles — fundamental information about nature itself.
- Nuclear physics uses the energy of beams to study the internal properties of the atom's nucleus or the dynamics when nuclei interact. Sometimes this results in producing isotopes that do not normally exist in nature.
- Beams for both particle physics and nuclear physics artificially recreate conditions that existed when the universe was much hotter, generating the ambient temperatures from these earlier times.
- Such conditions enable pursuit of big questions, such as the meaning of mass, a concept humans have puzzled over for millennia. Why are particles like electrons nearly massless, and particles like protons or top quarks massive?
- Some 95% of our universe consists of things we don't understand: dark matter and dark energy. Accelerators offer the possibility of answers.

Application of accelerators

- https://science.osti.gov/~/media/hep/pdf/accelerator-rd-stewardship/Report.pdf
- https://science.osti.gov/~/media/hep/pdf/accelerator-rd-stewardship/Workshop_on_Ion_Beam_Therapy_Report_Final_R1.pdf
- https://science.osti.gov/~/media/hep/pdf/accelerator-rd-stewardship/Energy_Environment_Report_Final.pdf

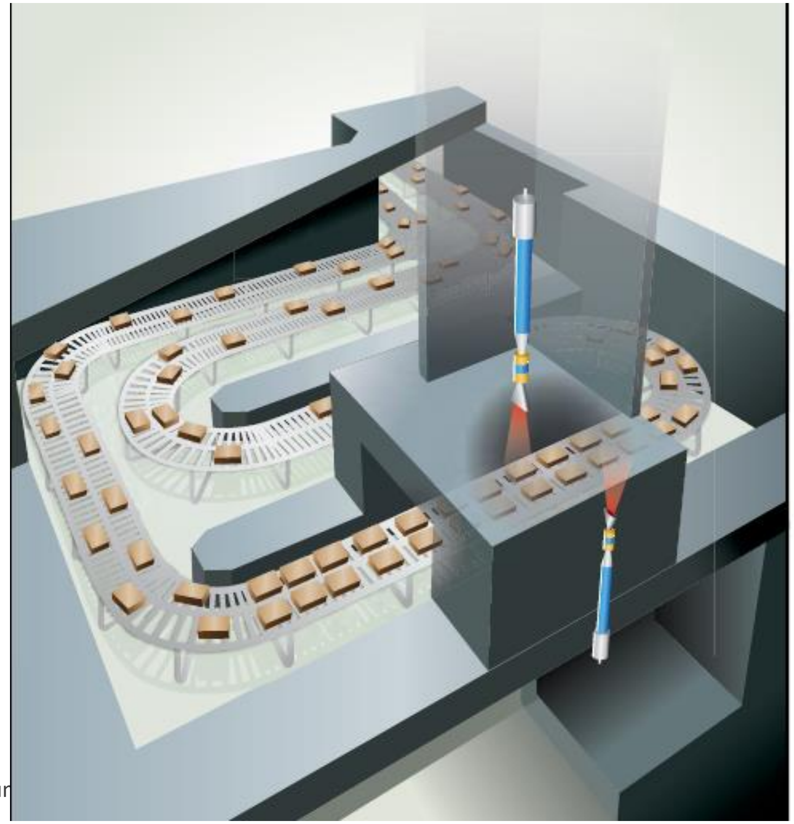
Accelerators for diagnosing illness and fighting cancer

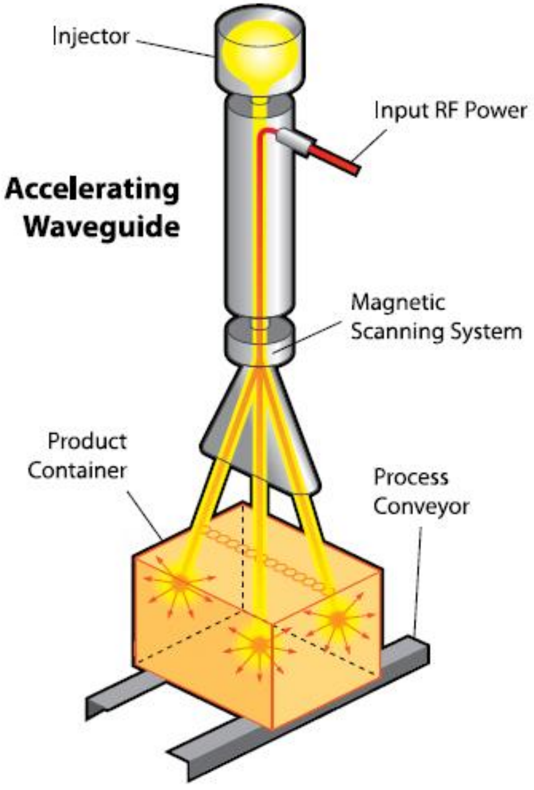
- Loma Linda proton synchrotron



Accelerators to beat food-borne illness

- Electron beams, or X-rays derived from them, can kill dangerous bacteria like E. coli, salmonella and listeria.
- Electron beams can sterilize products effectively, efficiently, and fast. They kill all bacteria. They penetrate packaging, and even the shipping cartons holding the packages, so that there is no danger of contamination during or after sterilization.



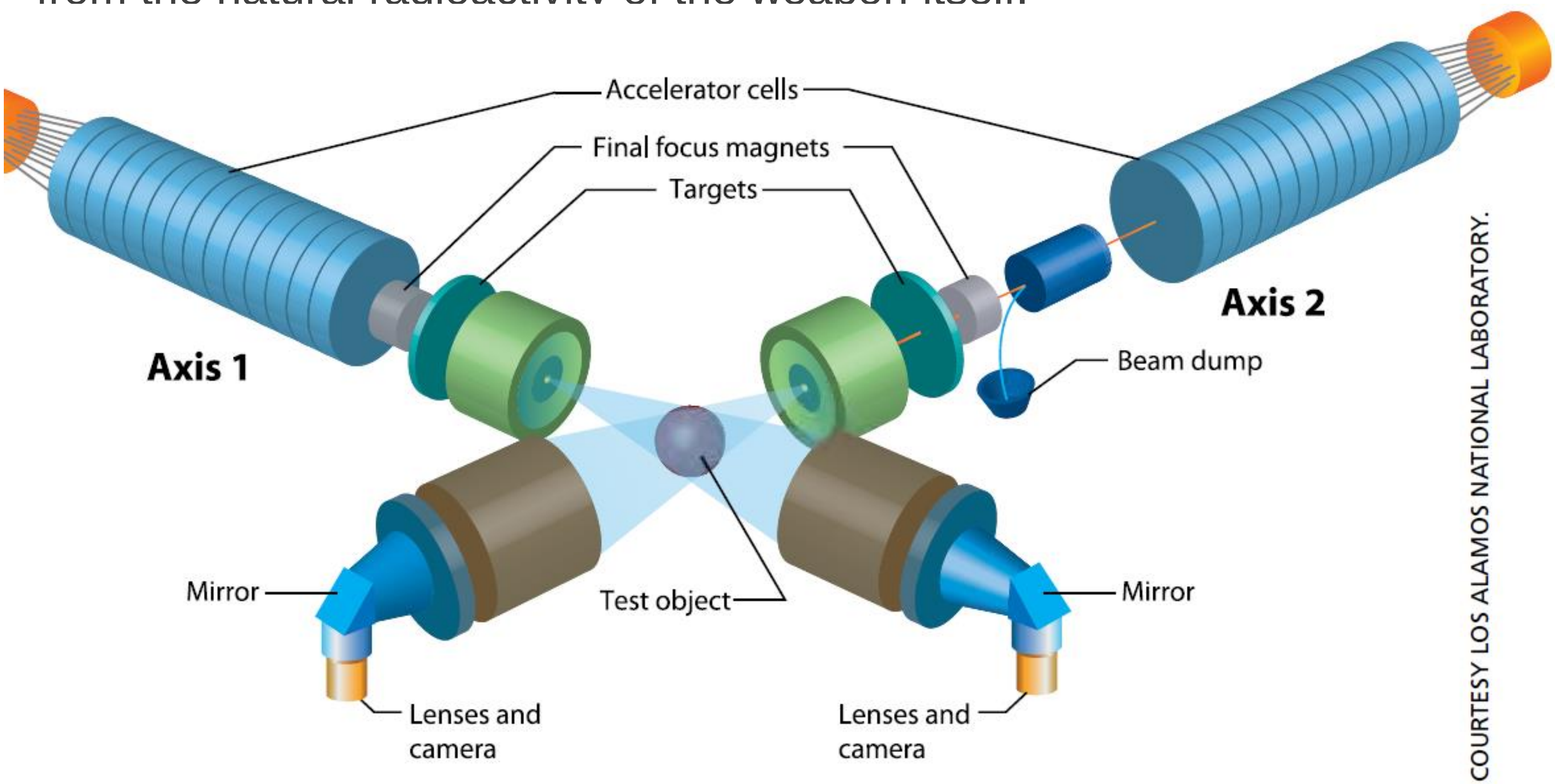


Accelerators and national security

- A single ship can bring up to 8000 tractor-trailer-sized cargo containers into an American port. Seven million containers arrive each year. Before distribution around the country, how can these large steel-walled boxes be inspected for what terrorists might have placed into one or some of them? Accelerators offer answers for scanning various kinds of cargo containers and vehicles effectively and efficiently.
- 6 MV electron accelerator to produce X-rays for cargo inspection

DARHT Facility at LANL

- In a special facility at Los Alamos National Laboratory, two electron accelerators at right angles to each other let scientists monitor realistic but non-nuclear tests of replacement components for the nation's nuclear weapons. "Stockpile stewardship" is necessary because over time some components degrade, perhaps losing functionality, based on interactions from the natural radioactivity of the weapon itself.



Beams of light from beams of particles

Light sources for science and technology

Not all light is visible. In science and technology, the word *light* applies generally to electromagnetic radiation. Most wavelengths of light aren't visible. Light sources generate microwave, infrared, visible, ultraviolet, X-ray and gamma-ray light. An equivalent statement is that light sources generate beams of microwave, infrared, visible, ultraviolet, X-ray and gamma-ray photons. (See illustration A.)

How do physicists generate intense, focused light beams from an accelerator? They use magnets, just as they do to steer particle beams through the accelerator. When a particle beam passes between the north and south poles of an accelerator magnet — N and S in illustration B — the beam not only changes direction, it also emits exceptionally intense, tightly focused light. With magnets, physicists not only can steer a particle beam around a circular accelerator or through a linear one, but — as in illustration C — they can tap the accelerator beam to get light beams. This light can then be directed away from the accelerator, shining down beamlines to scientific experiments or technological uses.

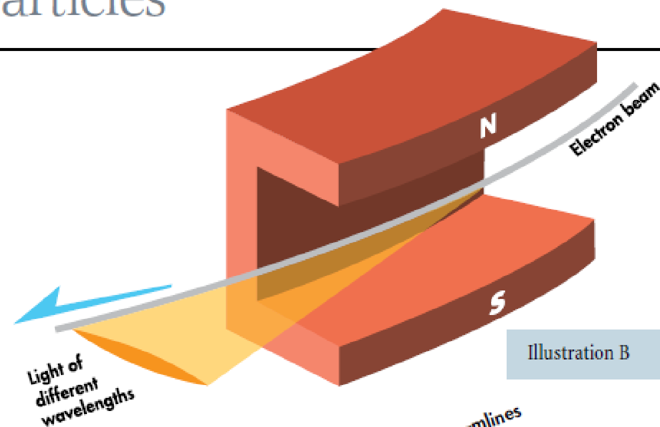


Illustration B

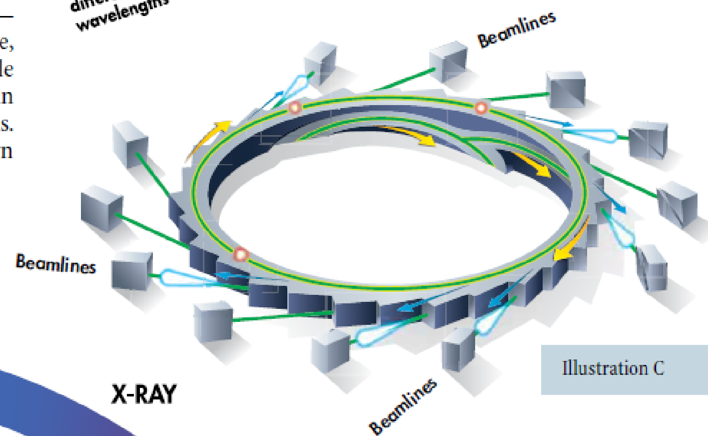
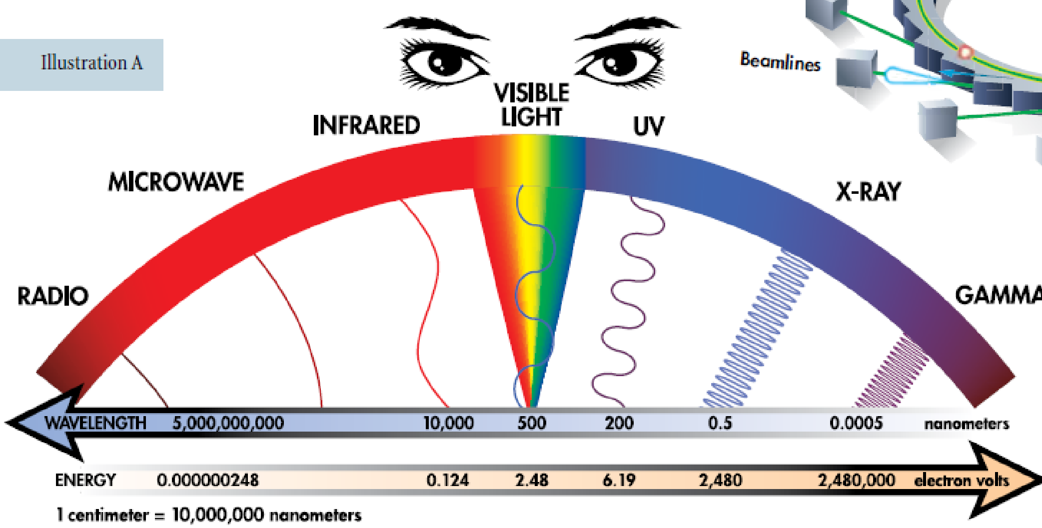
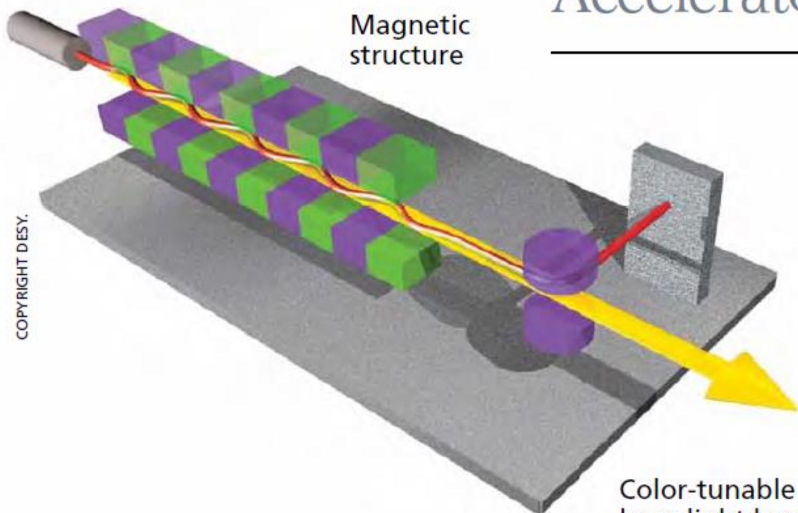


Illustration C

Illustration A





Accelerators energize a new kind of laser

Making laser light from electron beams

Conventional lasers make their extraordinarily useful kind of light by jiggling electrons that are bound in atoms. Accelerator-driven free-electron lasers (FELs, pronounced as three separate letters) make light by using magnets to jiggle electrons that are freed from atoms, as shown at the left. With free electrons, the light's wavelength (color) can be selected. For many applications, that's a very important feature.

Magnets in an FEL jiggle the free electrons in a beam from an accelerator. Operators can select the resulting light's wavelength by varying the beam's energy or the magnetic field strength.

An FEL's beam is delivered in pulses rather than a steady stream. These bursts of light can be timed, with pulse sequences shorter than a trillionth of a second. For many applications, this too is a very important feature.

Powerful X-ray laser light from electron beams

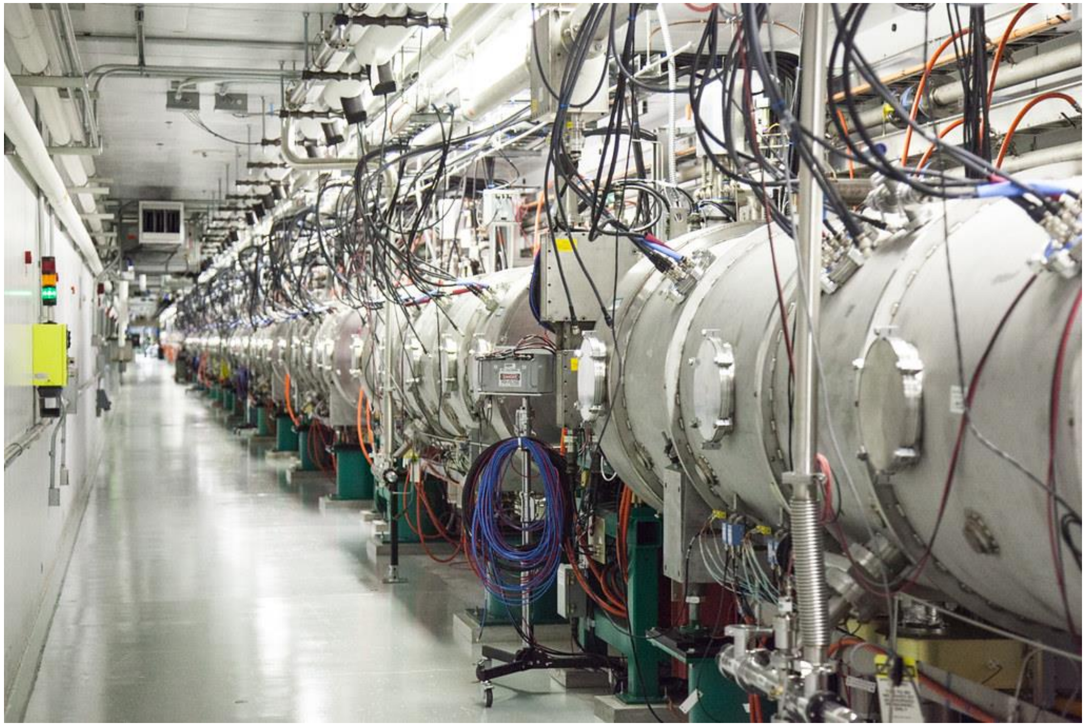
At Stanford University's SLAC National Accelerator Laboratory, the Linac Coherent Light Source (LCLS), shown at the right, is the world's most powerful X-ray laser. The linac (linear accelerator) generates high-energy beams of free electrons for making this special kind of light.

The resulting laser light arrives in staccato bursts one-tenth of a trillionth of a second long. These intense, ultrafast pulses let researchers scrutinize complex, ultrasmall structures by freeze-framing atomic motions. The researchers get to see the fundamental processes of chemistry, drug development and life itself in a new light.

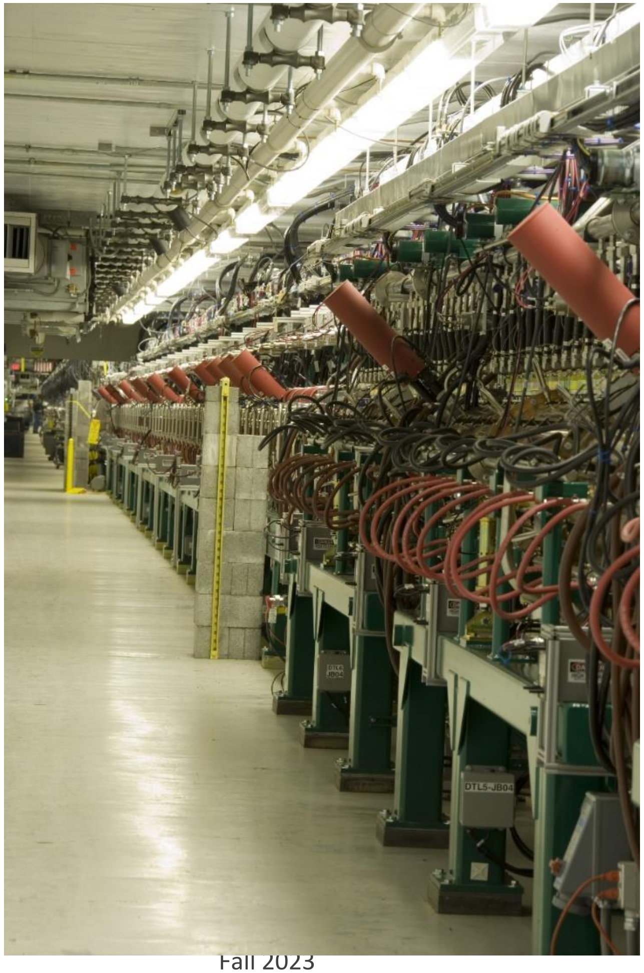
COURTESY SLAC.

Accelerator based neutron science

- The uncharged neutrons can go where charged particles can't, providing detailed snapshots of material structure and "movies" of molecules in motion. This neutron research improves a multitude of products, from medicine and food to electronics, cars, airplanes and bridges.
- Neutron research shows how electromagnetic fields behave inside certain superconductors.

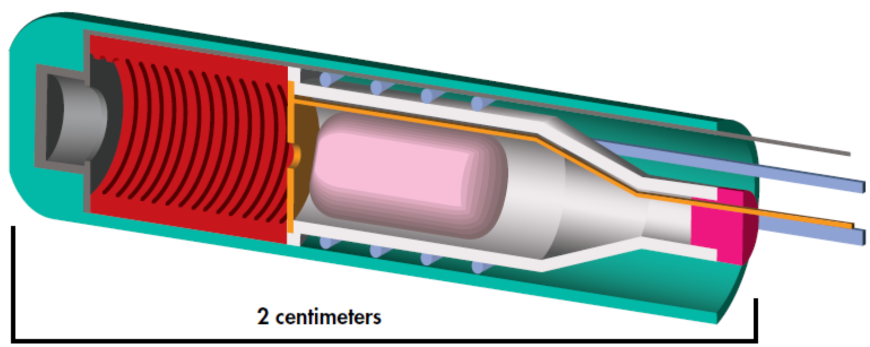


SNS (ORNL) linac provides 1.4 MW proton beam for neutron production



Multiple uses for portable accelerators

- Attach certain instruments to a compact, vacuum-tight accelerator called a portable neutron generator, lower it down a borehole, and you can look for oil.
- A small deuteron-triton accelerator, a "minineutron tube," is used for oil well analysis during drilling, as well as homeland security detection of explosives and fissile materials in luggage or cargo.
- The accelerator generates the neutrons by manipulating isotopes of hydrogen. It can discover oil deposits by detecting porosity, which shows the presence of liquid or gas. Electrical characteristics tell whether a liquid is water or oil.
- Portable neutron generators have other uses too. They can analyze metals and alloys, and they can detect explosives, drugs, or materials for nuclear weapons.



A small deuteron-triton accelerator, a "mini-neutron tube," is used for oil well analysis during drilling, as well as homeland security detection of explosives and fissile materials in luggage or cargo.

Jobs designing, constructing, operating and maintaining accelerators

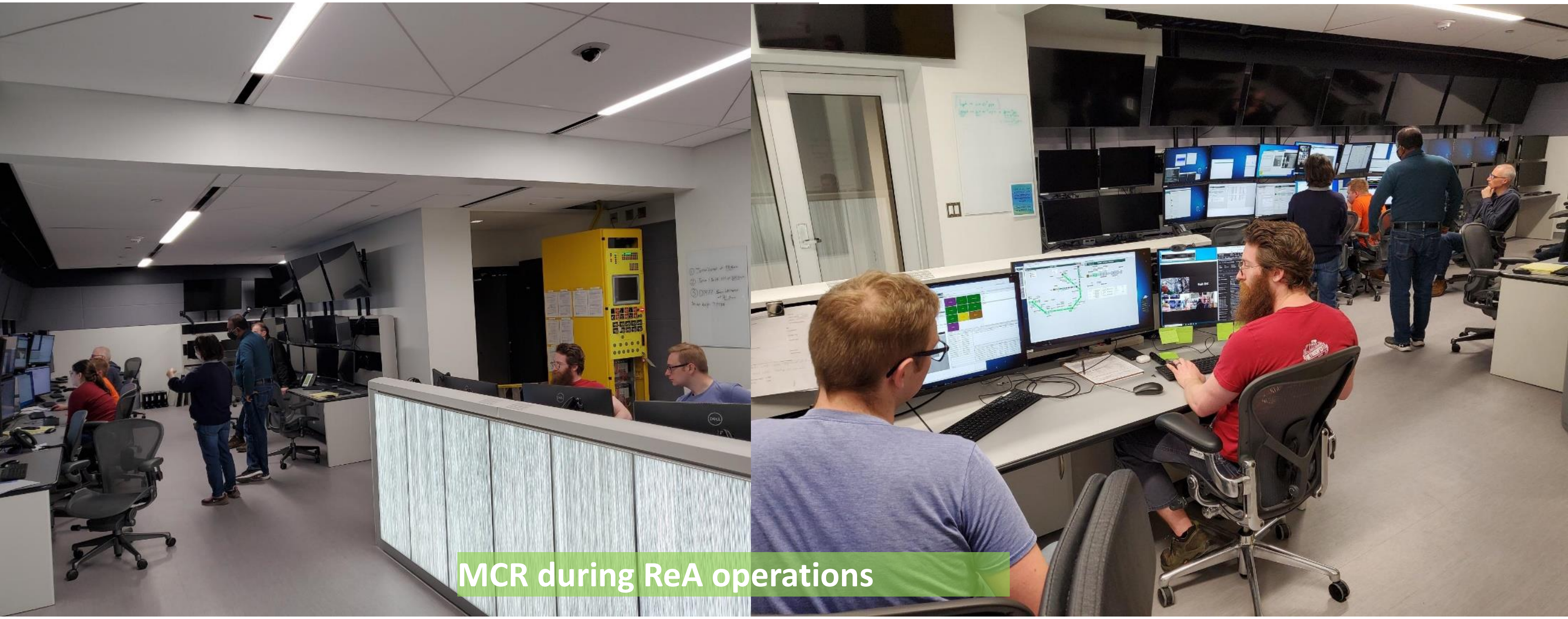
Accelerators bring high-tech jobs. More than 30,000 accelerators worldwide serve an expanding variety of fields — and more than 65 manufacturers are shipping almost a thousand new systems each year. To design, build, operate and maintain these accelerators requires workers with an enormous variety of scientific, engineering, technological, medical and industrial skills. Some market statistics indicate the annual breadth and reach of the high-tech career opportunities involved. The financial size of the accelerator industry is a measure of the many job opportunities there are:

- Industrial electron-beam irradiation generates $90 billion
- Semiconductor components from ion implantation exceed $250 billion
- Goods with materials and parts touched by accelerators yield more than a half-trillion dollars

Accelerator operation

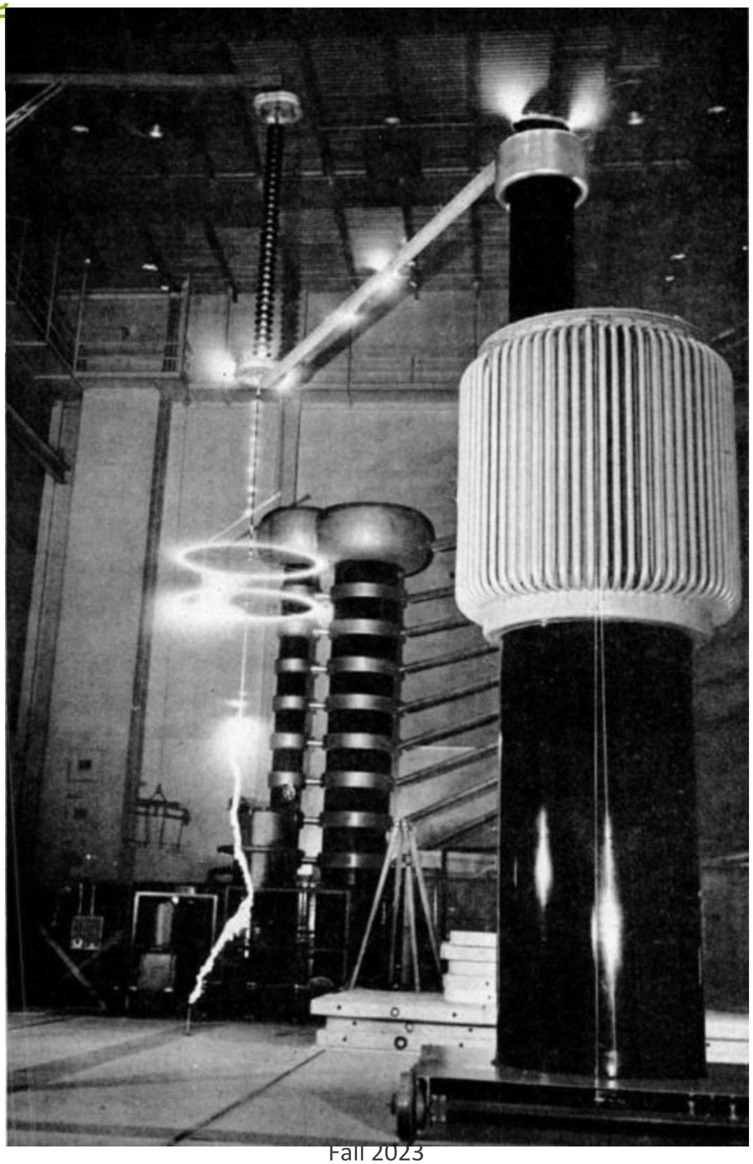
Accelerator construction

FRIB control room


MCR during ReA operations

Corona discharge

- https://en.wikipedia.org/wiki/Corona_discharge



Gauss' law can also be written in terms of the electric field piercing the enclosing Gaussian surface:

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}} \quad \text{(Gauss' law).}$$

5. The electric field *outside a spherical shell of charge* with radius R and total charge q is directed radially and has magnitude

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad \text{(spherical shell, for } r \geq R\text{).} \qquad (23\text{-}15)$$

Here r is the distance from the center of the shell to the point at which E is measured. (The charge behaves, for external points, as if it were all located at the center of the sphere.) The field *inside* a uniform spherical shell of charge is exactly zero:

$$E = 0 \quad \text{(spherical shell, for } r < R\text{).} \qquad (23\text{-}16)$$

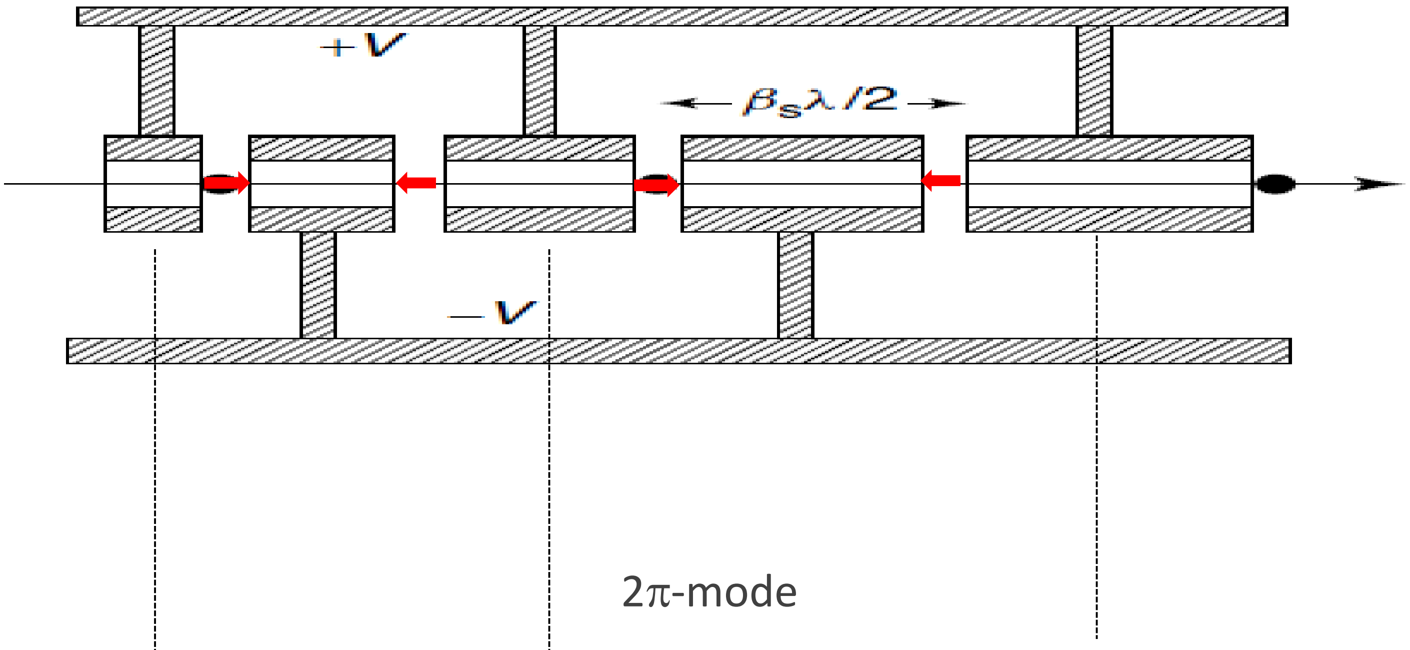
$$V = \frac{q}{4\pi\varepsilon_0 R}$$

Electric potential on the surface of the sphere

π-mode and 2π-mode of accelerating structures

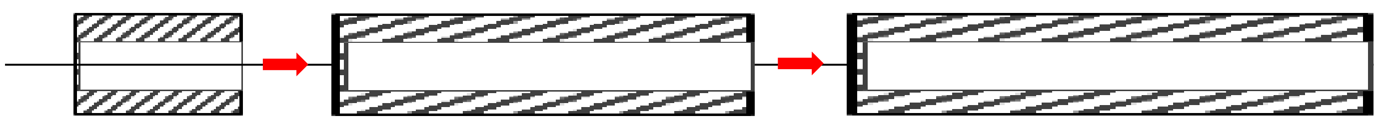
Energy gain

π-mode



$$\Delta E = 4qV\cos\varphi_0$$

2π-mode



$$\Delta E = 2qV\cos\varphi_0$$

+V -V +V -V +V