RF accelerating structures, Lecture 7

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RF accelerating structures

Outline:

1. Introduction;
2. Accelerating, focusing and bunching properties of RF field;
3. RF Cavities for Accelerators

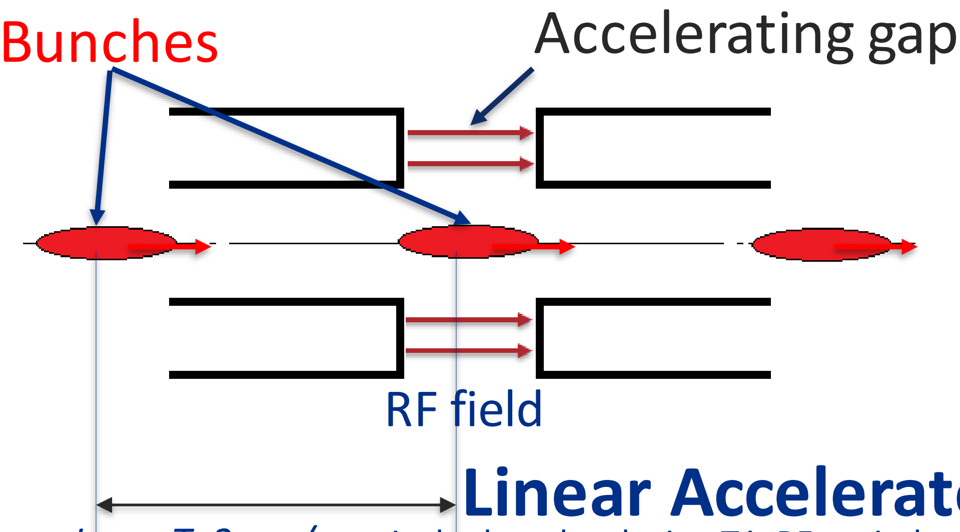
Chapter 1.

Introduction.

Introduction

❖ RF accelerators – acceleration in RF field

- Bunched beam (no particles when the field is decelerating);
- Accelerating RF field is excited in an accelerating gap;

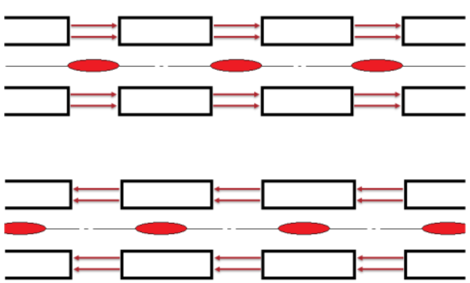


RF field: $E = E_0 \cos(\omega t)$

Particle energy gain $\Delta U \sim E_0$

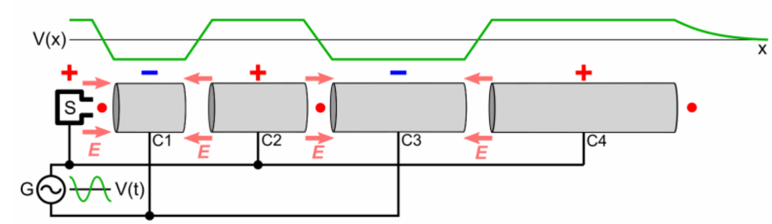
Linear Accelerators:

$L = nvT = 2\pi nv/\omega$, v is the bunch velocity, T is RF period and ω is RF cyclic frequency; n is integer, n=1,2…



$\omega t = 2\pi m$, $m = 0,1,2…$

$\omega t = (2m+1)\pi$, $m = 0,1,2…$



Animation from Wikipedia (https://en.wikipedia.org/wiki/Particle_accelerator)

Linear RF accelerators for scientific applications.

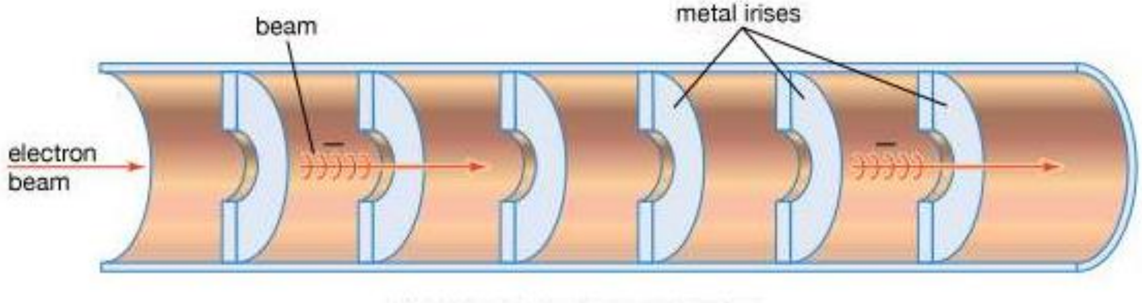
High – Energy Electron accelerators: High Energy Physics, Nuclear Physics, Free-Electron Lasers

High – Energy Proton accelerators: High Energy Physics, Nuclear Physics, source of secondary particles (neutrons, pions, muons, neutrinos), material science, Accelerator-Driven Subcritical reactors (ADS).

Specifics of proton accelerators: protons are non- or weekly relativistic up to high energies: rest mass for protons is 0.938 GeV (compared to 0.511 MeV for electrons).

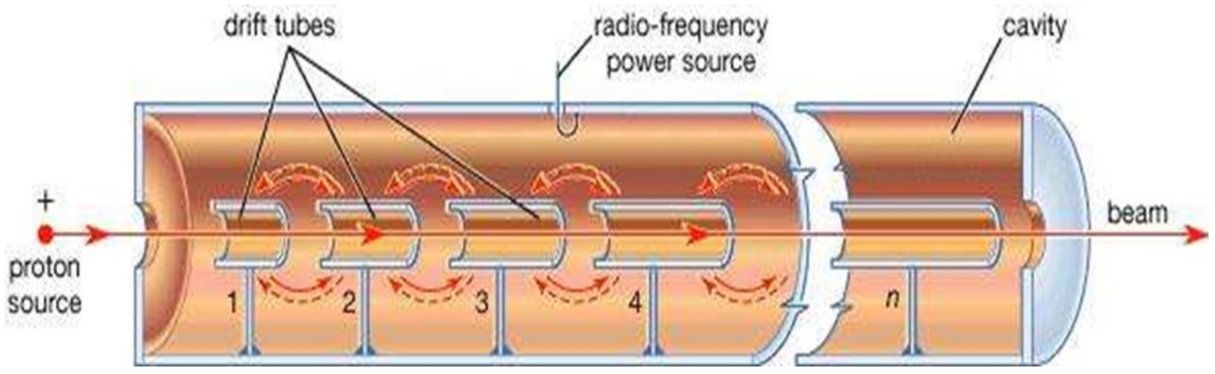
RF linear accelerators

- Travelling wave accelerators.



The wave propagates left to right.

- Standing wave accelerators.

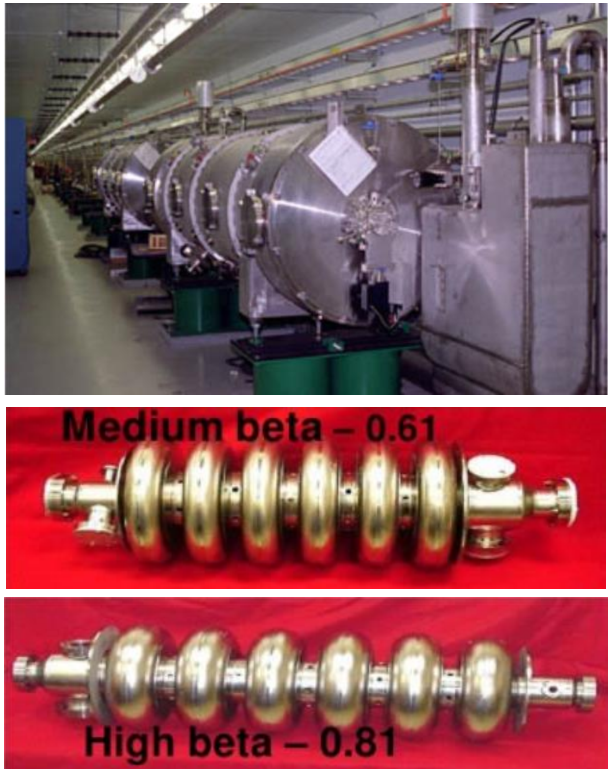


- Room Temperature linear accelerators
- Superconducting linear accelerators

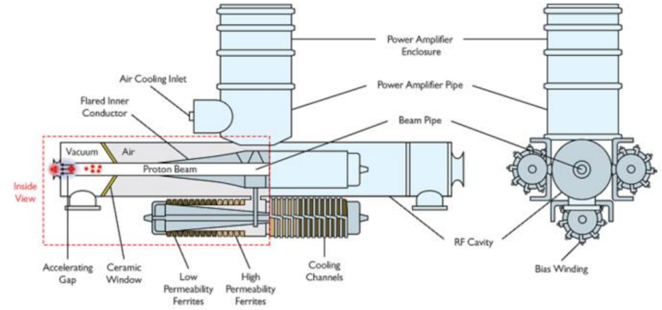
RF cavities for accelerators

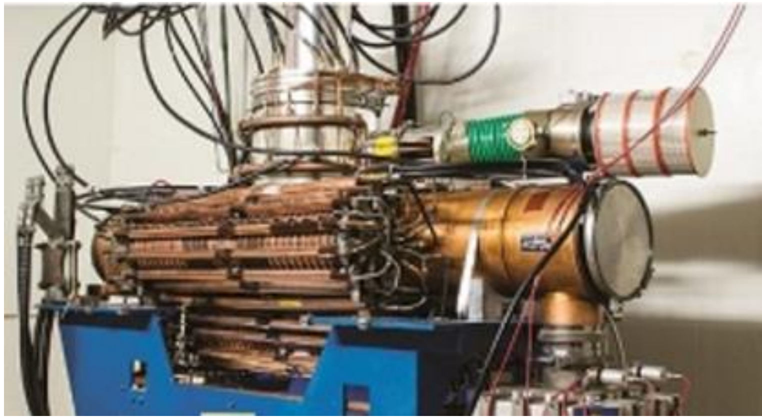
To achieve high accelerating field in the gap resonant RF cavities are used. In all modern RF accelerators, the beam acceleration takes place in a resonance wave (standing or travelling) electromagnetic field excited in an RF cavities.

CW 50.6 MHz cavity of PSI cyclotron. V =1.2 MV



Superconducting 805 MHz multi-cell cavities of SNS linac. V=10-15 MV, DF = 6%.





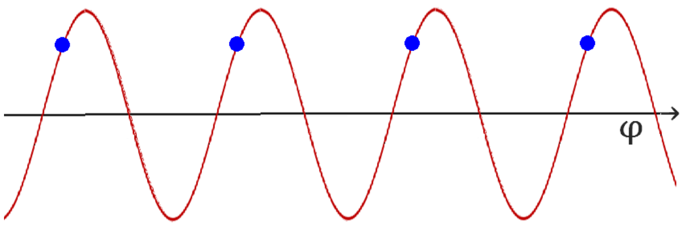
Tunable cavity for FNAL Booster Synchrotron
F=37.8-52.8 MHz. V=60 kV, DF =50%

Acceleration principles:

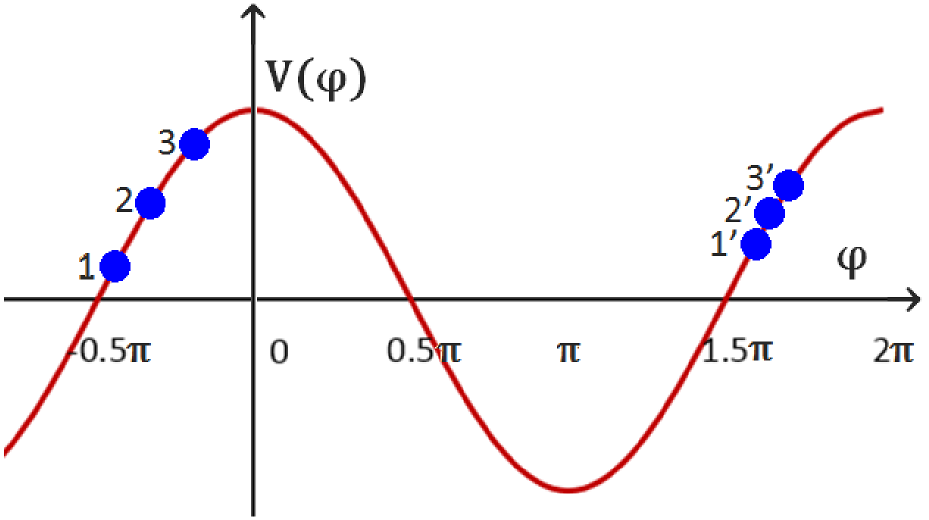
If the charged particle reaches the center of the accelerating gap in arbitrary phase φ, its energy gain is

$$V(\varphi) = V\cos(\varphi)$$

Acceleration: $-\pi/2 < \varphi < \pi/2$



- ***Synchronism:*** the bunches should reach the center of the acceleration gaps in the same accelerating phase.
- ***Autophasing:*** longitudinal dynamics should be stable (no bunch lengthening). For linear accelerator $-\pi/2 < \varphi_s < 0$ (φ_s is the phase of the bunch center)



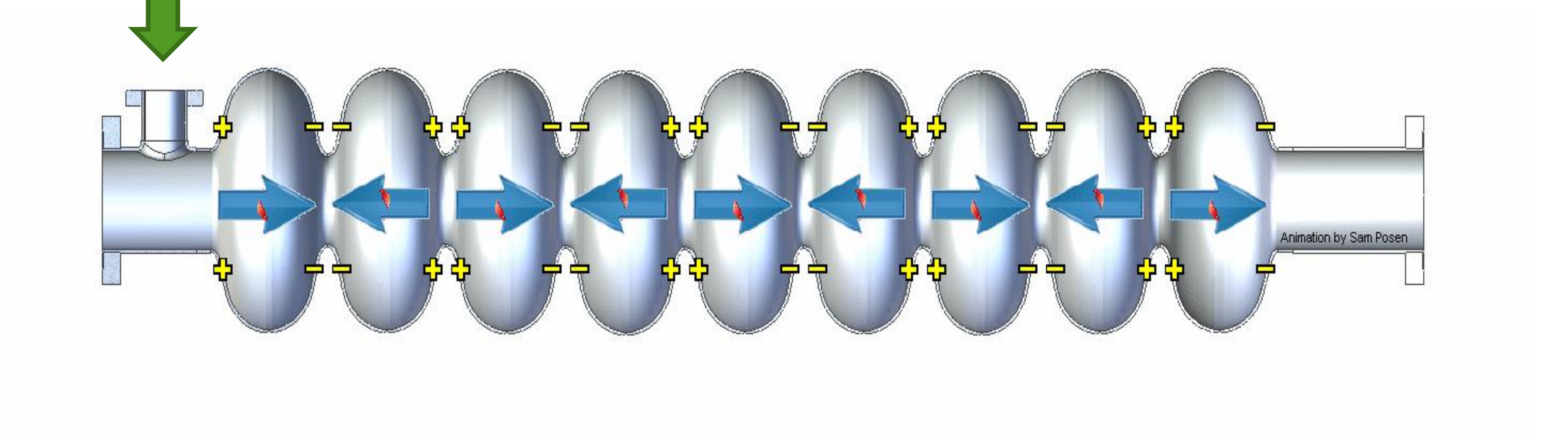
(linear accelerator)

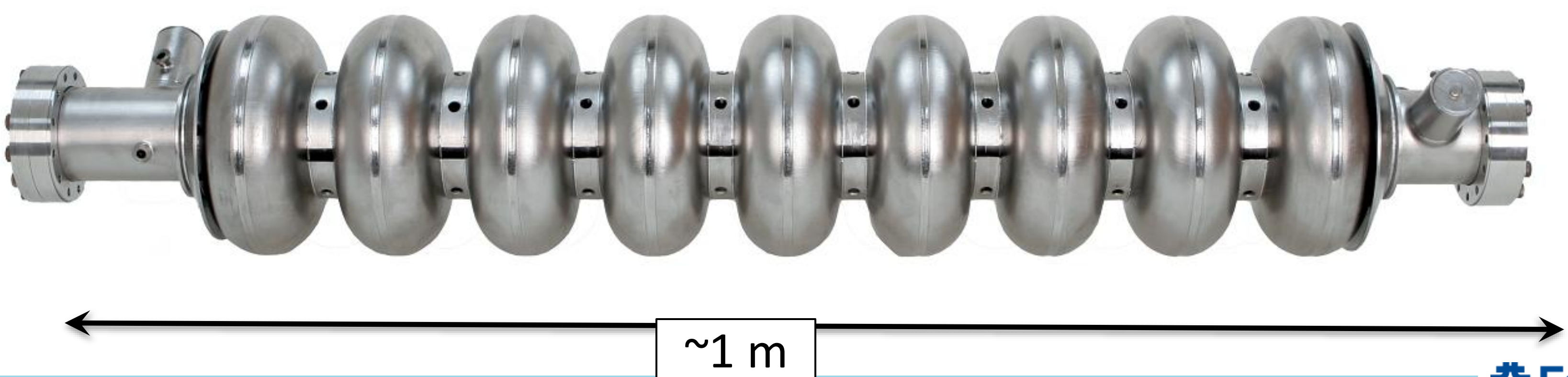
Illustration of synchronism :

1.3 GHz ILC cavity (animation by Sam Posen, FNAL)

Input RF power

Slowed down by factor of approximately 4x10^{9}





Chapter 2.

Accelerating and focusing properties of RF field.

a. Acceleration of charged particles in electromagnetic field;

b. Focusing properties of RF field;

c. Bunching properties of RF field;

d. Summary.

Acceleration and focusing of charged particles in electromagnetic field

Electromagnetic fields in RF cavities are described by Maxwell equations:

$$curl\vec{E} = -\frac{\partial \vec{B}}{\partial t}, \qquad curl\vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}, \qquad \frac{\partial \rho}{\partial t} + div\vec{J} = 0$$

$$div\vec{B} = 0, \qquad div\vec{D} = \rho.$$

Linear media:

$$\vec{D} = \varepsilon\vec{E}, \qquad \vec{B} = \mu\vec{H}, \qquad \vec{J} = \sigma\vec{E}.$$

Harmonic oscillations:

$$\vec{E} = \vec{E}(r)\cdot e^{i\omega t}, \qquad curl\vec{E} = -i\omega\mu\vec{H}, \qquad curl\vec{H} = i\omega\varepsilon\vec{E}.$$

For vacuum:

$$\mu_0 = 4\pi\cdot 10^{-7}\,\frac{H}{m}, \qquad \varepsilon_0 = \frac{10^{-9}}{36\pi} \approx 0.884\cdot 10^{-11}\,\frac{F}{m}.$$

Vacuum impedance Z_0:

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi\ Ohm; \quad \frac{1}{\sqrt{\varepsilon_0\mu_0}} = c$$, c is speed of light.

$$\nabla \equiv \sum_{i=1}^{3} \vec{e}_i \frac{\partial}{\partial x_i}$$

$$grad\ f \equiv \nabla f$$

$$div\ \vec{A} \equiv \nabla\cdot\vec{A}$$

$$curl\ \vec{A} \equiv rot\ \vec{A} \equiv \nabla\times\vec{A}$$

- $\vec{E}$ - electric field strength, V/m
- $\vec{D}$ - electric filed induction, C/m^2
- $\vec{B}$- magnetic field induction, T
- $\vec{H}$ -magnetic field strength, A/m
- ε- permittivity, F/m
- μ – permeability, H/m
- σ –conductivity, S/m
- $\frac{\partial \vec{D}}{\partial t}$ - displacement current density
- $\vec{J}$ - current density, A/m^2
- ρ – charge density, C/m^3

Acceleration and focusing of charged particles in electromagnetic field

From Maxwell equations: $-curlcurl\vec{E} = -\omega^2\varepsilon\mu\vec{E} + i\omega\mu\vec{J}$

For $\vec{J}$=0 $curlcurl\vec{E} = \omega^2\varepsilon\mu\vec{E}$ or

$$\Delta\vec{E} + k^2\vec{E} = 0,$$

Here $k^2 = \omega^2\varepsilon\mu$, $\Delta \equiv grad\ div - curl\ curl$

Same for magnetic field:

$$\Delta\vec{H} + k^2\vec{H} = 0$$

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$ Cartesian (x,y,z)

$$\Delta = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2} + \frac{\partial^2}{\partial z^2}$$ Cylindrical (r,φ,z)

Useful theorems are in Appendix 1

Acceleration and focusing of charged particles in electromagnetic field

❑ Boundary conditions on a conductive wall:

$$\vec{E}_t = Z_s(k)[\vec{H}_t \times \vec{n}],$$

where $Z_s(k)$ is a surface impedance, $\vec{n}$ is directed to the metal.

"Ideal metal": $Z_s = 0$ or $\vec{E}_t = 0$.

- Wall power loss:

$$P = \frac{1}{2} Re \int (\vec{E} \times \vec{H})\, \vec{n}\, dS = \frac{1}{2} \int R_s |H|_t^2 dS\,,$$

R_s is the surface resistance,

$$R_s = Re(Z_s(k))$$

Fermilab

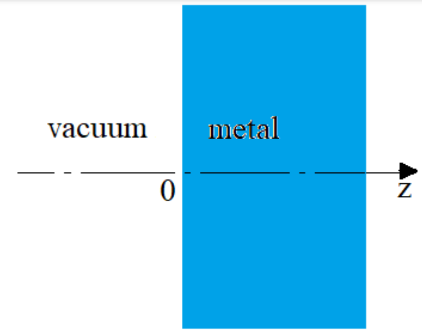
Acceleration and focusing of charged particles in electromagnetic field

1. Normal-conducting metal, classical skin effect (CSE).

$curl\vec{E} = -i\omega\vec{B}, curl\vec{H} = i\omega\vec{D} + \vec{J} \approx \sigma\vec{E}$ (in metal $\omega\vec{D} \ll \vec{J} = \sigma\vec{E}$),

↑ Ohm's law

σ is the wall material conductivity.

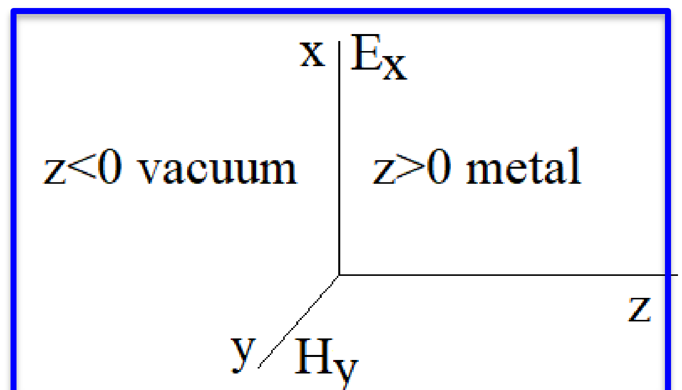


↓

$$curlcurl\vec{H} = -i\omega\mu_0\sigma\vec{H} \rightarrow \frac{d^2 H_y}{dz^2} = -i\omega\mu_0\sigma H_y = -ikZ_0\sigma H_y$$

↓

$$H_y(z) = H_s e^{-(1-i)z/\delta},\ \delta = \sqrt{\frac{2}{kZ_0\sigma}},\ \ H_s = H_y(0)$$



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$$E_x(z) = -\frac{1}{\sigma}\frac{dH_y(z)}{dz} = (1-i)\frac{H_s}{\delta\sigma}e^{-(1-i)z/\delta}$$

$$\left(curl\vec{H} = -\frac{dH_y(z)}{dz} \approx \sigma\vec{E}\right)$$

↓

$$\frac{E_x(0)}{H_y(0)} = Z_s(k) = (1-i)\frac{1}{\delta\sigma}$$

→ $$\boldsymbol{Z_s(k) = (1-i)\sqrt{\frac{kZ_0}{2\sigma}}}$$

Acceleration and focusing of charged particles in electromagnetic field

- Surface impedance

$$Z_s(k) = \sqrt{\frac{kZ_0}{2\sigma}}(1-i)$$

where σ is the wall material conductivity.
For copper at room temperature
(20°C) **σ = 59 MS/m**.

- Surface resistivity:

$$R_s = Re[Z_s(k)] = \sqrt{\frac{kZ_0}{2\sigma}} = \sqrt{\frac{\omega Z_0}{2c\sigma}}.$$

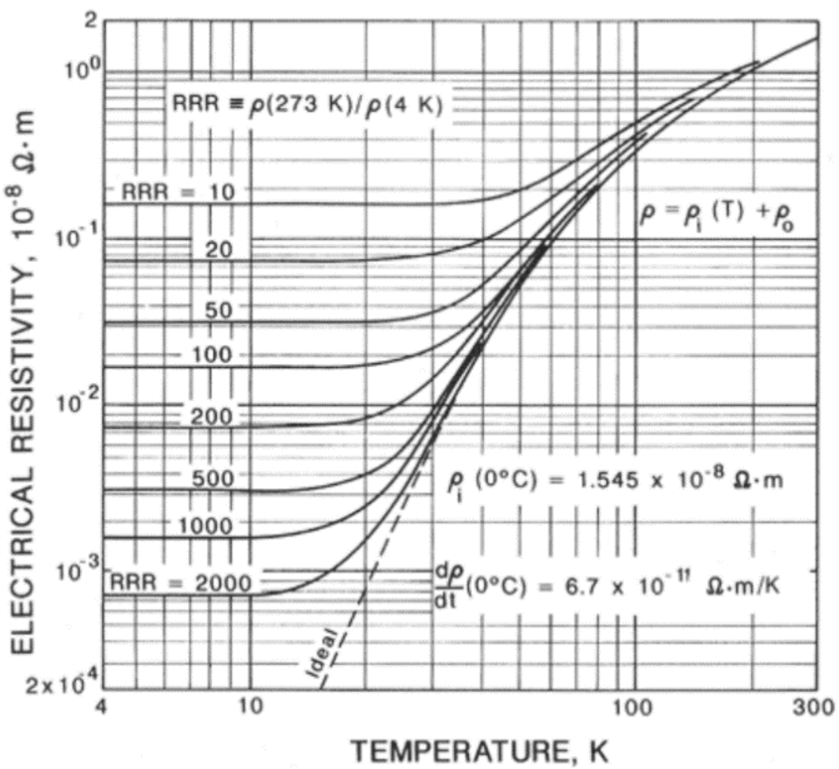
- The RF field $H(z)$ and $E(z)$ decays into the metal exponentially with the distance from the surface z:

$$\frac{H(z)}{H_s} = \frac{E(z)}{E_s} = e^{-(1-i)z/\delta},\ \delta = \sqrt{\frac{2}{kZ_0\sigma}}$$ - classical skin depth.

Acceleration and focusing of charged particles in electromagnetic field

- For pure metals, the conductivity decreases with the temperature.

Copper resistivity $\rho = \sigma^{-1}$ versus temperature for different sample purity:



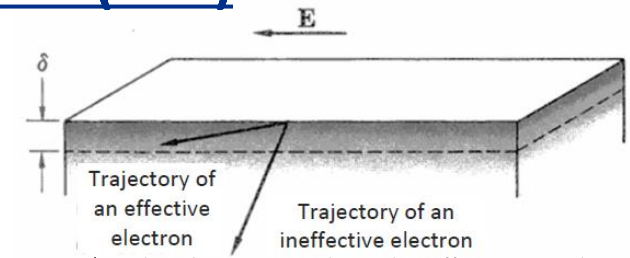
A commonly used measure of purity is the residual resistivity ratio (RRR), defined as the ratio of the resistivity at 273 K or 0°C over the resistivity at 4K:

$$RRR = \frac{\rho(273\ K)}{\rho(4K)}$$

Acceleration and focusing of charged particles in electromagnetic field

2. Normal-conducting metal, anomalous skin effect (ASE)*:

- At low temperature of metal skin depth δ may be smaller than the mean-free path l of conducting electrons, $l = \frac{\sigma(T) Z_0 c v_F}{\omega_p^2}$ (1) **it is anomalous skin effect.**
  Here v_F is the Fermi velocity and ω_p is the plasma frequency of conducting electrons.

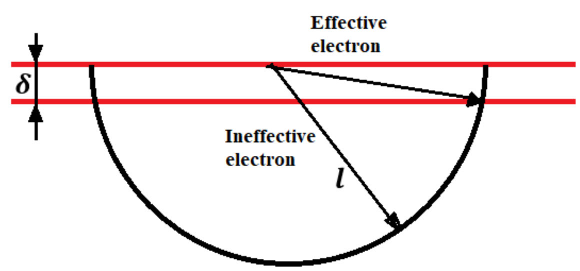


*R. Chambers, Anomalous Skin Effect in Metals. *Nature* **165**, 239–240 (1950).

- Surface impedance frequency dependence can be estimated using simple consideration, so called "ineffectiveness concept" for extreme ASE, when $\delta/l \ll 1$ (A.B. Pippard,1947).
  - Solid angle of all the trajectories of electrons started in a thin layer, it is $\sim 2\pi$ in the case when $\delta/l \ll 1$.
  - Solid angle for the effective electrons is $2\pi \times \delta/l$, and the portion of effective electrons, therefore, is δ/l.

- Effective conductance: $\sigma_{eff} \sim \sigma \cdot \frac{\delta}{l}$ (2) $\longrightarrow \delta = \sqrt{\frac{2}{k Z_0 \sigma_{eff}}}$ (3) , $R_s = \sqrt{\frac{k Z_0}{2\sigma_{eff}}}$ (4)

- From (1-4) it follows that $R_s = Z_0 \cdot \left(\frac{k^2 c v_F}{4\omega_p^2}\right)^{1/3}$ (5).



- Exact formula for extreme ASE based on kinetic equations together with Maxwell equations (G.E. Reuter, E.H. Sondheimer, 1948)

$$R_s = Z_0 \cdot \left(\frac{\sqrt{3} k^2 c v_F}{16\pi\omega_p^2}\right)^{\frac{1}{3}}$$

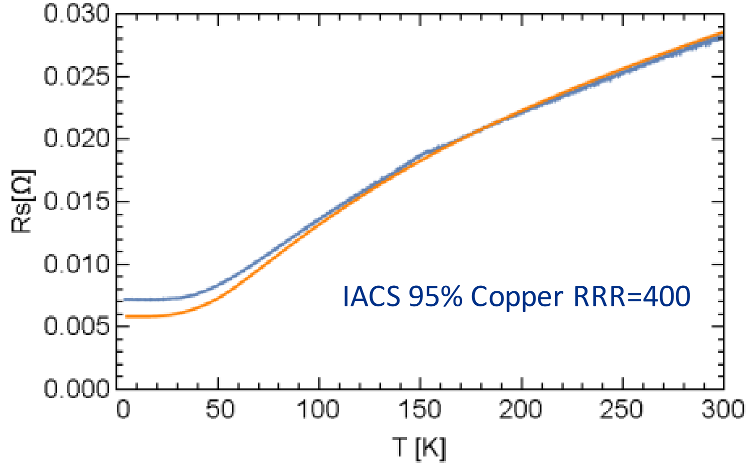
which differs from simple estimation (5) by factor of $\left(\frac{\sqrt{3}}{4\pi}\right)^{-1/3} \approx 2$ only!

Acceleration and focusing of charged particles in electromagnetic field

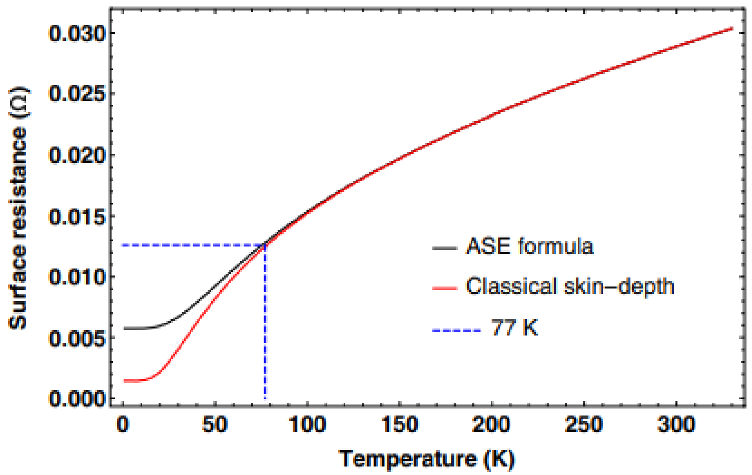
- For <u>extreme</u> anomalous skin effect ($l >> \delta$) the complex surface resistance Z_s may be estimated as

$$Z_s = Z_0 \left(\frac{\sqrt{3}}{16\pi}\frac{cv_F k^2}{\omega_p^2}\right)^{1/3}\left(1-\sqrt{3}i\right), \qquad R_s = Re(Z_s) = Z_0 \left(\frac{\sqrt{3}}{16\pi}\frac{cv_F k^2}{\omega_p^2}\right)^{1/3}.$$

For copper v_F = 1.58e6 m/sec, ω_p = 1.64e16 rad/sec; $\frac{3}{2}\left(\frac{l}{\delta}\right)^2 \gg 1,\ \omega \ll \omega_p$. Note that $\frac{l}{\delta} \sim \sigma(T)^{3/2}$



Calculated pure copper surface resistance (orange) versus measured (blue) for the frequency of 11424 MHz.

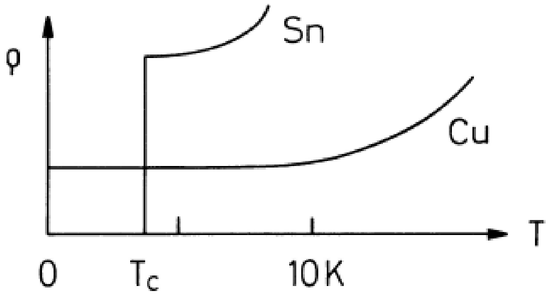


A plot of the surface resistance for copper with RRR = 400 versus temperature at 11424 MHz.

Acceleration and focusing of charged particles in electromagnetic field

2. Superconducting wall:

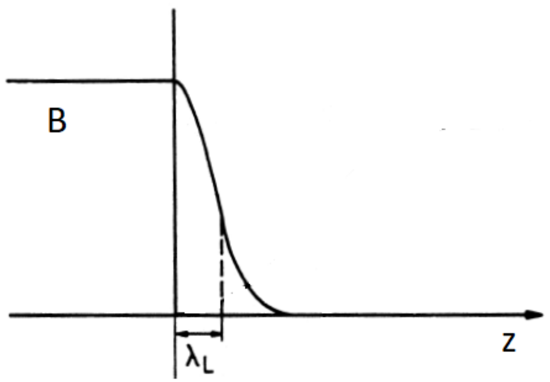
- Superconductivity - the infinitely high conductivity (or zero resistivity) below a ‘critical temperature‘ T_c.



T_c(K):

Al	Hg	Sn	Pb	Nb	Ti	NbTi	Nb_3Sn
1.14	4.15	3.72	7.9	9.2	0.4	9.4	18

- Penetration depth λ_L: $B(z) \sim exp(-z/\lambda_L)$



material	In	Pb	Sn	Nb
λ_L [nm]	24	32	≈ 30	32

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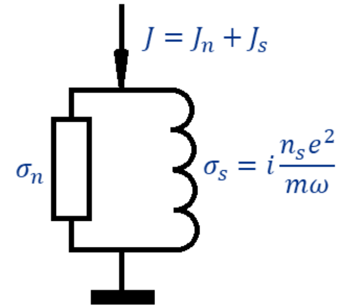
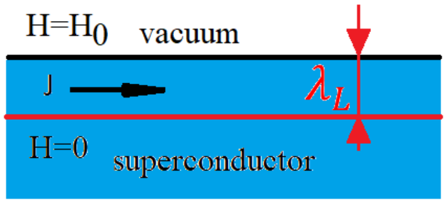
Acceleration and focusing of charged particles in electromagnetic field

- Two-fluid model: current in a superconductor is carried by
  - the superfluid component (Cooper pairs) - J_s;
  - the normal component (unpaired electrons) - J_n.

- At DC – no resistance.
- At AC – resistance caused by electron inertia.





$$Z_s = \frac{1}{\lambda_L(\sigma_n + \sigma_s)}$$

For normal component:

$J_n = \sigma_n E_0 \exp(-i\omega t)$,

For superfluid component:

$$m\dot{v} = -eE_0 \exp(-i\omega t) \rightarrow J_s = -en_s v = i\frac{n_s e^2}{m\omega} E_0 \exp(-i\omega t) =$$

$$= \sigma_s E_0 \exp(-i\omega t) \rightarrow \sigma_s = i\frac{n_s e^2}{m\omega}.$$

$R_s = Re\left(\frac{1}{\lambda_L(\sigma_n+\sigma_s)}\right) \approx \frac{1}{\lambda_L}\cdot\frac{\sigma_n}{|\sigma_s|^2}$, or $R_s \propto \omega^2 \exp\left(-\frac{\Delta}{k_B T}\right)$, because

$\sigma_n \propto \exp\left(-\frac{\Delta}{k_B T}\right)$ and $|\sigma_s|^{-2} \propto \omega^2$. Here $\Delta \sim T_c$ is the energy gap and k_B is the Boltzmann constant.

Phenomenological law for Nb:

$R_{s,BCS} \approx 1.643 \times 10^{-5}\frac{T_c}{T}(f(GHz))^2 e^{-\frac{1.92T_c}{T}}$ (Ω). $\quad R_s = R_{s,BCS} + R_{residual}$

Acceleration and focusing of charged particles in electromagnetic field

Examples:

1. Surface resistance of a copper wall at room temperature for 1.3 GHz.

Mean-free path l is 38 nm compared to classical skin depth δ of 1.9 µm.

$l << \delta \rightarrow$ *classical skin effect (CSE)*. Therefore,

$$R_s = \sqrt{\frac{\omega Z_0}{2c\sigma}} = 9.3 \text{ mOhm};$$

σ=59 MS/m; ω=2π·1.3e9 Hz, Z_0=120π Ohm, c=3e8 m/sec.

2. Surface resistance of a pure copper (RRR=2500) wall at 2 K for 1.3 GHz.

Mean-free path l is 95 µm compared to classical skin depth of 37 nm.

$l >> \delta \rightarrow$ *anomalous skin effect (ASE)*. Therefore,

$$R_s = Z_0 \left(\frac{\sqrt{3}}{16\pi} \frac{c v_F k^2}{\omega_p^2} \right)^{1/3} = 1.3 \text{ mOhm}.$$

v_F = 1.58e6 m/sec, ω_p = 1.64e16 rad/sec, $k = \omega/c = 2\pi$·1.3e9/c.

Classical skin formula gives 0.19 mOhm!

3. BCS resistance of the Nb at 2 K for 1.3 GHz.

$$R_{s,BCS} \approx 1.643 \times 10^{-5} \frac{T_c}{T} (f(GHz))^2 e^{-\frac{1.92 T_c}{T}} = 19 \text{ nOhm}$$

f=1.3 GHz, T_c=9.2 K, T=2K.

Acceleration and focusing of charged particles in electromagnetic field

Dynamics of a charged particle accelerated in a RF field is described by Lorenz equation,

$$\frac{d\vec{p}}{dt} = \vec{F} = e[\vec{E}_0(\vec{r},\mathrm{t}) + \vec{v} \times \vec{B}_0(\vec{r},t)]\,,$$

where $\vec{v}$ is the particle velocity, $\vec{E}_0(\vec{r},\mathrm{t})$ and $\vec{B}_0(\vec{r},t)$ are RF electric and magnetic field oscillating at the cavity resonance frequency ω:

$$\vec{E}_0(\vec{r},\mathrm{t}) = \vec{E}(\vec{r})\, e^{i\omega\mathrm{t}}$$

$$\vec{B}_0(\vec{r},\mathrm{t}) = \vec{B}(\vec{r})\, e^{i\omega\mathrm{t}}$$

Acceleration and focusing of charged particles in electromagnetic field

RF electric field has a longitudinal component next to the beam axis. In cylindrical coordinate it may be expanded over azimuthal harmonics, i.e.,

$$E_z(r,\varphi,z) = \sum_{m=-\infty}^{\infty} e^{im\varphi} E_{m,z}(r,z)$$

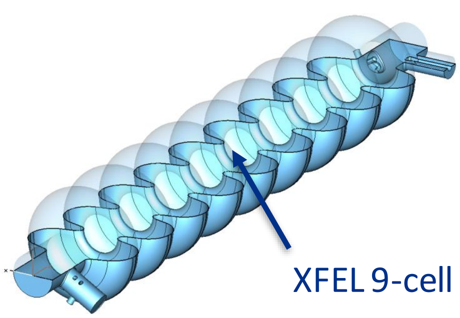
To understand general properties of the acceleration field, the amplitudes may be expanded into Fourier integral for $r < a$, a is the beam aperture :

$$E_{m,z}(r,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} E_{m,z}(k_z,r)\, e^{ik_z z} dk_z \qquad (1)$$

or over the travelling waves existing from z=-∞ to z=∞.

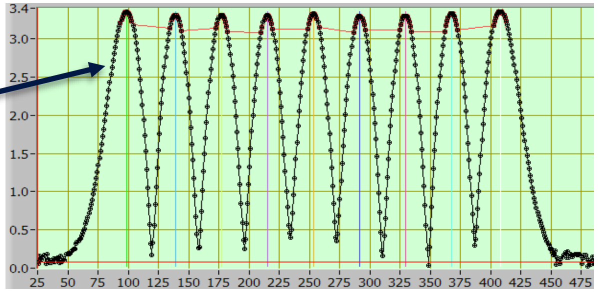
The RF field $\vec{E}_0(\vec{r},\mathrm{t})$ satisfies the wave equation:

$$\Delta\vec{E}_0(\vec{r},\mathrm{t}) + \frac{\partial^2 \vec{E}_0(\vec{r},\mathrm{t})}{\partial t^2} = 0 \qquad (2)$$



XFEL 9-cell 1.3 GHz SW cavity

$|E_{0,z}(0,z)|$



Acceleration of charged particles in electromagnetic field

Substituting expansion (1) to the wave equation (2), we can find, that $E_{m,z}(k_z, r)$ satisfies Bessel equation, and therefore is proportional to the Bessel function $J_m(x)$,

$$E_{m,z}(k_z, r) = E_{m,z}(k_z) J_m(k_\perp r)$$

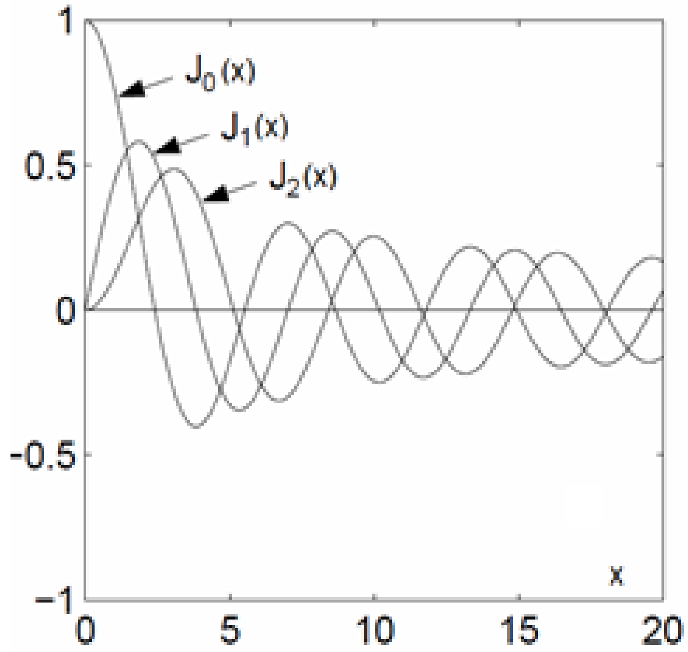
where $k_\perp$ is transverse wavenumber, which it turn satisfies dispersion equation:

$$k_\perp^2 + k_z^2 = \frac{\omega^2}{c^2} \equiv k^2,$$

here c is speed of light.

If the particle velocity $v = \beta c$ and particle transverse coordinates do not change significantly in the cavity, the energy ΔW particle gains in the cavity is equal to

$$\Delta W(r, \varphi) = eRe[\int_{-\infty}^{\infty} dz E_z(r, \varphi, z)\, e^{i\omega t}|_{t=\frac{z}{v}} \ ]$$



Bessel functions $J_m(x)$.

Fermilab

Acceleration of charged particles in electromagnetic field

Performing integration over z one has:

$$\Delta W(r,\varphi) = eRe\left[\sum_{m=-\infty}^{\infty} E_{m,z}\left(\frac{k}{\beta}\right) I_m\left({}^{kr}/_{\beta\gamma}\right) e^{im\varphi}\right],$$

$$I_m(x) = J_m(ix)$$

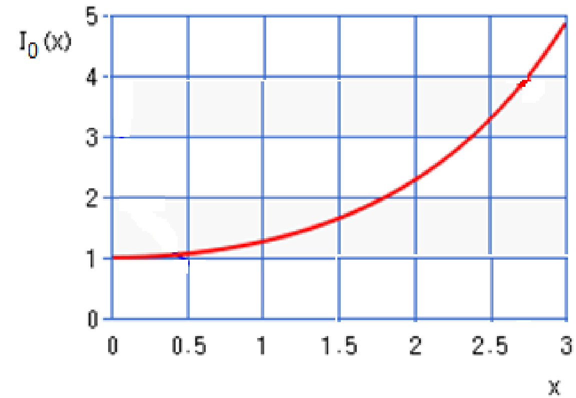
where $I_m(x)$ is modified Bessel function and γ is the particle relativistic factor (note that $k_\perp = ik/\beta\gamma$); i.e., the particle gains the energy interacting with synchronous cylindrical wave having the phase velocity equal to the particle velocity (synchronism: $v_{particle}=\beta c = v_{phase}=\omega/k_z \rightarrow k_z=k/\beta$ and $k_\perp = (k^2-k_z^2)^{1/2}= ik/\beta\gamma$.

Type equation here.

If the cavity and RF field of the operating mode have perfect azimuthal symmetry, one has:

$$\Delta W(r) = eRe\left[E_{0,z}\left(\frac{k}{\beta}\right) I_0\left({}^{kr}/_{\beta\gamma}\right)\right] = e\left|E_{0,z}\left(\frac{k}{\beta}\right)\right| I_0\left({}^{kr}/_{\beta\gamma}\right)\cos\phi$$

where ϕ is the RF phase.

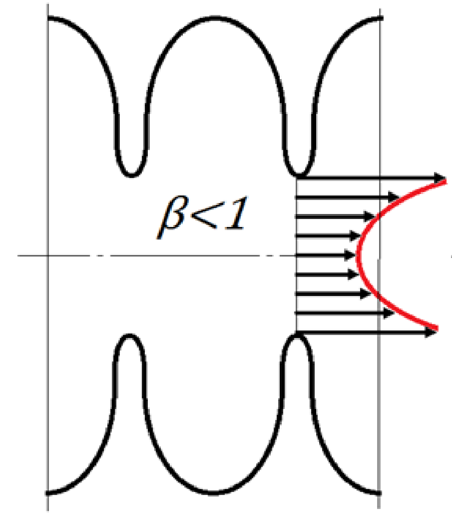


Acceleration of charged particles in electromagnetic field

- For a very slow particle, i.e., when $\beta << 1$, if $kr/\beta\gamma >> 1$ one has

$$I_0\left({}^{kr}/_{\beta\gamma}\right) \sim \frac{1}{\sqrt{kr}} e^{{}^{kr}/_{\beta\gamma}}.$$

It means that for low-beta particle the energy gain increases with the radius r.



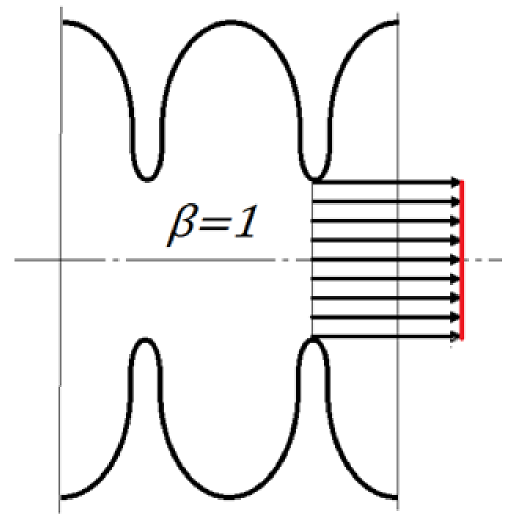
- For the ultra-relativistic particle $k_\perp \to 0$ and one has

$$\mathrm{V}(r,\varphi) = eRe\left[\sum_{m=-\infty}^{\infty} E_{m,z}(k)\, r^m e^{im\varphi}\right],$$

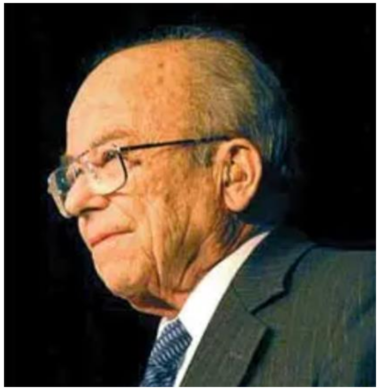
For the RF field having perfect azimuthal symmetry

$$\mathrm{V}(r) = e|E_{0,z}(k)|cos\phi$$

and the particle energy gain does not depend on the transverse coordinate.



Focusing properties of RF field

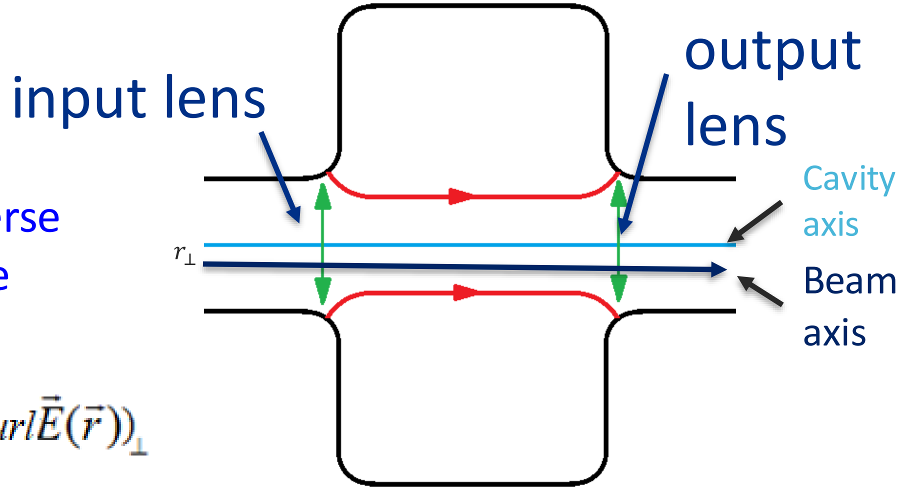


W. Panofsky

In addition to acceleration, the RF field provides deflection of the beam. Let's consider the particle transverse momentum change causes by the cavity RF field. The particle moves on the trajectory $z=vt$ parallel to the axis but has off-set $\vec{r}_\perp$.

According to **Panofsky – Wenzel theorem (Appendix 2)**, change of transverse momentum caused by RF field is related to change of the longitudinal momentum:

$$\Delta\vec{p}_\perp = \frac{iv}{\omega}\vec{\nabla}_\perp(\Delta p_\parallel).$$



The differential operator $\vec{\nabla}_\perp$ acts on the transverse coordinates $\vec{r}_\perp$ only; longitudinal and transverse momentum changes are (Appendix 2):

$$\vec{F}_\perp(\vec{r}) = \frac{d\vec{p}_\perp}{dt} = \vec{E}_\perp(\vec{r}) + (\vec{v}\times\vec{H}(\vec{r}))_\perp = \vec{E}_\perp(\vec{r}) + (\vec{v}\times\frac{i}{\omega}curl\vec{E}(\vec{r}))_\perp$$

$$\Delta p_\parallel = e\int_{-\infty}^{\infty} E_z(\vec{r})e^{i\omega t}dt\,|_{t=z/v} = \frac{e}{v}\int_{-\infty}^{\infty} E_z(\vec{r})e^{i\omega z/v}dz;$$

$$\Delta\vec{p}_\perp = e\int_{-\infty}^{\infty}\left[\vec{E}_\perp(\vec{r}) + (\vec{v}\times\frac{i}{\omega}curl\vec{E}(\vec{r}))_\perp\right]e^{i\omega t}dt\,|_{t=z/v} = e\int_{-\infty}^{\infty}\left[\vec{E}_\perp(\vec{r}) + \frac{iv}{\omega}\vec{\nabla}_\perp E_z(\vec{r}) - \frac{iv}{\omega}\frac{\partial\vec{E}_\perp(\vec{r})}{\partial z}\right]e^{i\omega t}dt\,|_{t=z/v} =$$

$$= e\int_{-\infty}^{\infty}\frac{iv}{\omega}\vec{\nabla}_\perp E_z(\vec{r})\,e^{i\omega t}dt\,|_{t=z/v} = \frac{iv}{\omega}\vec{\nabla}_\perp(\Delta p_\parallel).$$

No acceleration → no deflection!

Focusing properties of RF field

Therefore,

$$Re\Delta p_\perp = -\frac{e}{\omega}\sum_{m=-\infty}^{\infty} \left|E_{m,z}\left(\frac{k}{\beta}\right)\right| \cdot \vec{\nabla}_\perp \left[I_m\left({}^{kr}/_{\beta\gamma}\right) \cdot \cos\left(m(\varphi - \varphi_m)\right)\right] \cdot \sin\phi$$

where φ_m is polarization of the azimuthal harmonics. The maximum of transverse momentum change is shifted in RF phase versus the maximum the energy gain by -90°: for the particle accelerated on crest of the RF field, transverse momentum change is zero. In order to get longitudinal stability in low-energy accelerator one needs to accelerate the particle at $\phi < 0$. One can see that for the field having perfect azimuthal symmetry

$$Re\Delta p_\perp = -\frac{e}{\omega}\left|E_{0,z}\left(\frac{k}{\beta}\right)\right| \cdot \vec{\nabla}_\perp \left[I_0\left({}^{kr}/_{\beta\gamma}\right)\right] \cdot sin\phi = -\frac{e}{\beta\gamma c} V_{max}(0) \cdot I_1\left({}^{kr}/_{\beta\gamma}\right) \cdot sin\phi$$

Near the axis, where ${}^{kr}/_{\beta\gamma} \ll 1$ one has for the trajectory angle r'in the end of acceleration $r' = \frac{\Delta p_\perp(r)}{p_{||}} \approx -\frac{kr}{2\beta^3\gamma^3}\frac{V_{max}(0)}{m_o c^2/e} \cdot sin\phi$,

where m_o is the particle rest mass.

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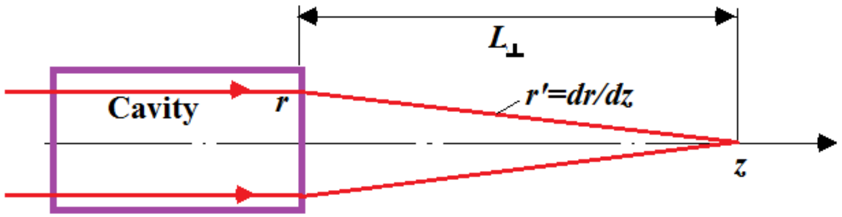
Focusing properties of RF field

❖ The focusing distance $L_\perp$ is

$$L_\perp = -\frac{r}{r\prime} = \frac{2\beta^3\gamma^3}{\omega/c}\frac{m_o c^2/e}{V_{max}(0)\cdot sin\phi}$$



Thin lens

- Focusing distance $L_\perp$ is inversed proportional to the RF frequency and proportional to β^3 . Because of this, at low energies the cavity provides strong defocusing ($\phi < 0!$), and this defocusing should be compensated by external magnetic focusing system.
- To mitigate this defocusing, one should use lower RF frequency ω in low energy parts of the linac ($L_\perp \sim 1/\omega$).

❖ For an ultra-relativistic particle in this case one has:

$$Re\Delta p_\perp = -\frac{e}{\omega}\sum_{m=-\infty}^{\infty}|E_{m,z}(k)| \cdot mr^{m-1} \cdot sin\phi$$

and in the case of perfect azimuthal symmetry of the field $\Delta p_\perp = 0.$ However, the RF field provides transfer momentum change for ultra-relativistic particle, i.e., focusing. The reason is that the particle transverse coordinate and energy change during acceleration because of the initial trajectory angle and influence of the RF field.

In this case the transverse momentum change is proportional to the RF amplitude squared.

Focusing properties of RF field

The transport matrix (thick lens) which determines relationship between the input and output transverse coordinates and angles (x and x' respectively) of the relativistic particle is calculated, for example, in [*]. For the RF cavity operating at π-mode (slide) for the particle accelerated on crest, the transport matrix is the following:

$$\begin{bmatrix} x \\ x' \end{bmatrix}_f = \begin{bmatrix} cos(\alpha) - \sqrt{2}\, sin(\alpha) & \sqrt{8}\, \frac{\gamma_i}{\gamma'} sin(\alpha) \\ -\frac{3\gamma'}{\sqrt{8}\gamma_f} sin(\alpha) & \frac{\gamma_i}{\gamma_f}\left[cos(\alpha) + \sqrt{2}\, sin(\alpha)\right] \end{bmatrix} \begin{bmatrix} x \\ x' \end{bmatrix}_{in},$$

where $(x,x')_i$ initial coordinate and angle, $(x,x')_f$ are final parameters, γ_i and γ_f are initial and final relativistic factors, γ' is the acceleration gradient over the rest mass in electron-Volts ($\gamma' = \Delta W_{max}/L_c m_o c^2$ (L_c is the cavity length) and $\alpha = \frac{1}{\sqrt{8}} ln \frac{\gamma_f}{\gamma_i}$.

- Note, that the angle x'_f at the cavity output for $x'_i = 0$ is proportional to the gain over the particle energy squared:

$$x'_f = \frac{\Delta p_\perp}{p_{||}} \approx \frac{3}{8}\left(\frac{V_{max}}{\gamma m_0 c^2}\right)^2 \frac{x_i}{L_c} \quad \text{and} \quad L_\perp = \frac{x_i}{x'_f} \approx \frac{8}{3} L_c \left(\frac{\gamma m_0 c^2}{V_{max}}\right)^{\mathbf{2}}$$

*****J. Rosenzweig and L. Serafini, "Transverse Particle Motion in Radio-Frequency Linear Accelerators," *Phys. Rev. E*, vol. 49, Number 2 (1994).

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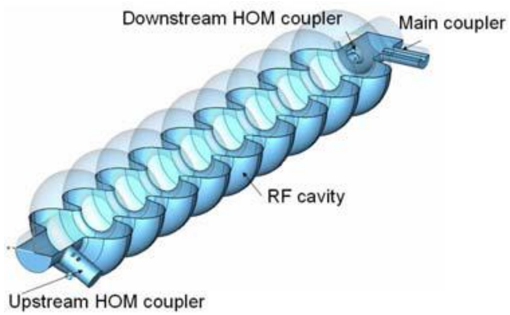
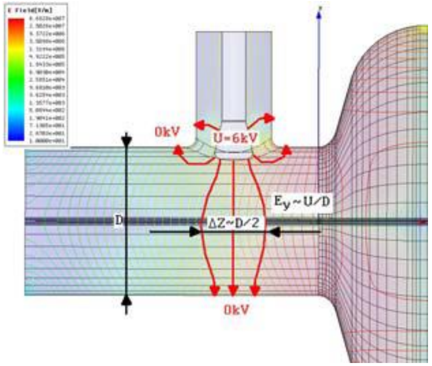
Focusing properties of RF field

RF acceleration elements (cavities, acceleration structures) typically have no perfect axial symmetry because of design features, coupling elements or manufacturing errors.

- Elliptical SRF cavities have the input couplers, which introduce dipole field components.
- Low-beta cavities like Half-Wave Resonators or Spoke Resonators have quadrupole RF field perturbations cause by spokes, which influence the beam and should be compensated by correctors.





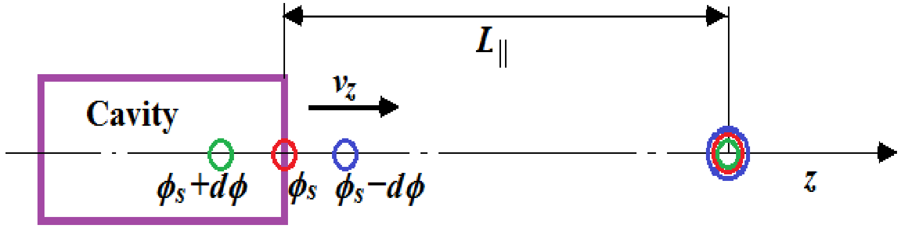
The angle x'_f at the cavity output for x'_i =0 caused by multipole perturbation of m^{th} order (m>0) is linear with respect to the ratio of the gain over the particle energy (ultra-relativistic case):

$$x'_f = \frac{\Delta p_\perp}{p_{||}} \approx \frac{m}{ka}\left(\frac{V_{max}(a)}{\gamma m_0 c^2}\right)\left(\frac{x_i}{a}\right)^{m-1}$$

- It may strongly influence the beam dynamics leading to the beam emittance dilution or result in strong quadrupole beam defocusing .
- On the other hand, the octupole perturbations may be used for the cavity alignment.

Bunching of charged particles in electromagnetic field

Because the particle velocity depends on the energy, the cavity RF field provides the beam bunching (Appendix 2):



Longitudinal "focusing" distance $L_{||}$:

$$L_{||} = -\frac{\beta^3 \gamma^3}{\omega/c} \frac{m_o c^2/e}{V_{max}(0) \cdot sin\phi_s} = -\frac{1}{2} L_{\perp}$$

For the bunch longitudinal stability $L_{||}$ should be >0, or ϕ_s <0 . In this case, one has transverse defocusing.

Note that for small energy (and therefore small β) the bunching may be too strong, and low RF frequency is to be used for acceleration.

Example:

SSR1 cavity (PIP II H^- accelerator):

f=325 MHz; V_{max} = 1 MV; φ_s = -34°;

$m_0c^2=E_0$=938 MeV; E= 10 MeV →$\beta\approx(2E/E_0)^{1/2}$=0.146, $\gamma\approx 1$.

❖ The focusing distance $L_\perp$ is

$$L_\perp = \frac{2\beta^3\gamma^3}{\omega/c}\frac{m_oc^2/e}{V_{max}(0)\cdot sin\phi} = \mathbf{-1.55\ m}$$

❖ Longitudinal "focusing" distance $L_{||}$:

$$L_{||} = -\frac{2\beta^3\gamma^3}{\omega/c}\frac{m_oc^2/e}{V_{max}(0)\cdot sin\phi_s} = -\frac{1}{2}L_\perp = \mathbf{78\ cm}$$

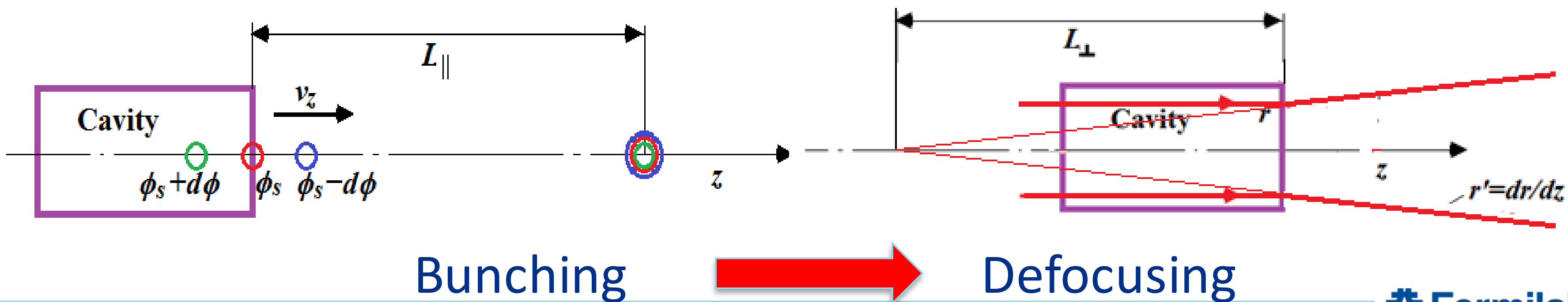
Summary:

- Acceleration of a charged particle moving in axisymmetric RF field parallel to the axis at the radius r is proportional to $I_0(kr/\beta\gamma)$;
  - for non-relativistic particle it increases with the radius → for low-energy particles one should use <u>low</u> frequency;
  - for ultra-relativistic particle it does not depend on the radius.
- Focusing of the accelerating particle is related to acceleration;
  - the maximum of transverse momentum change of the non-relativistic particle is shifted in RF phase versus the maximum the energy gain by -90°
  - The focusing distance of the non-relativistic particle is propositional to $\beta^3\gamma^3\lambda/V_{max}$ → for low-energy particles one should use <u>low</u> frequency.

Summary (cont):

- The focusing distance for ultra-relativistic particles is <u>quadratic</u> versus the ratio of particle energy over the voltage.
- The focusing distance for ultra-relativistic particles in multipole fields is <u>linear</u> versus the ratio of particle energy over the voltage; multipole perturbations may strongly affect the beam dynamics.

- The bunching "focusing" distance of the non-relativistic particle is propositional to $\beta^3\gamma^3\lambda/V_{max} \rightarrow$ **for low-energy particles one should use <u>low</u> frequency.**
- The sign is opposite to the focusing: **<u>if the bunch is bunched, it is defocused</u>**, and *vice versa*.
- **<u>In low-energy accelerators external focusing is necessary!</u>**



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Chapter 3.

RF Cavities for Accelerators.

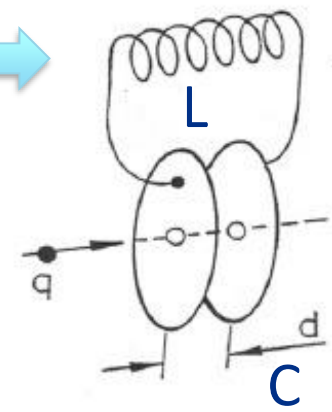
a. Resonance modes – operation mode, High-Order Modes;
b. Pillbox cavity
c. Cavity parameters:
- **Acceleration gradient;**
- **R/Q;**
- **Q_0 and G-factor;**
- **Shunt Impedance;**
- **Field enhancement factors (electric and magnetic);**

d. Cavity excitation by the input port;
e. Cavity excitation by the beam;
f. High-Order Modes (HOMs);
g. Types of the cavities and their application;
h. Tools for cavity simulations.

RF cavity:

$\omega_0 = (LC)^{-1/2}$

❑ An LC circuit, the simplest form of RF resonator:



This circuit and a resonant cavity share common aspects:

- Energy is stored in the electric and magnetic fields
- Energy is periodically exchanged between electric and magnetic field
- Without any external input, the stored power will turn into heat.

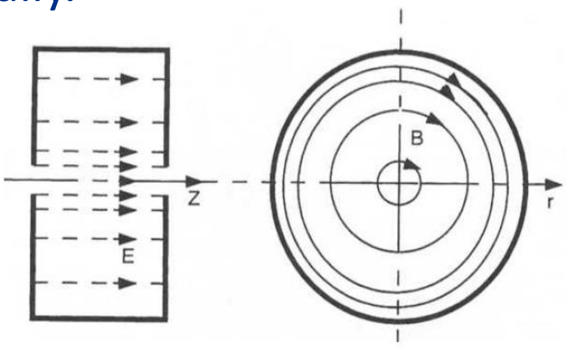
❑ To use such a circuit for particle acceleration, it must have opening for beam passage in the area of high electric field (capacitor).

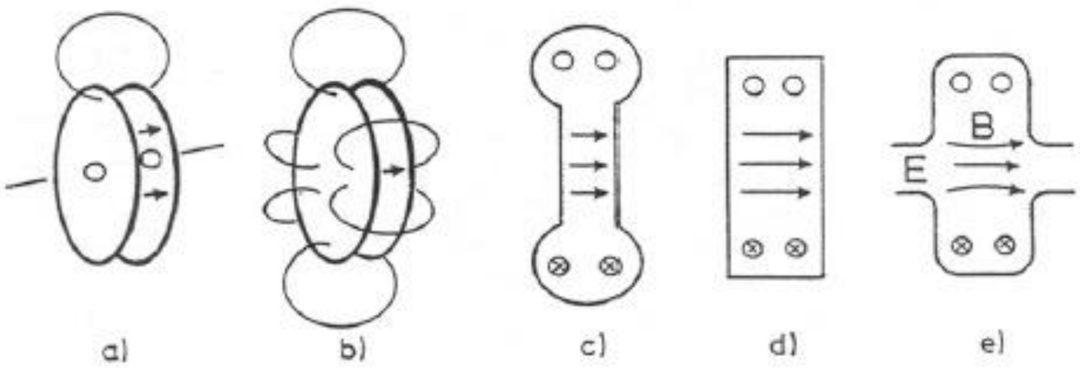
❑ As particles are accelerated in vacuum, the structure must provide vacuum space. A ceramic vacuum break (between the two electrode of the capacitor) can be used to separate the beam line vacuum from the rest of the resonator. Or the resonant structure can be enclosed in a vacuum vessel.

From LC circuit to an accelerating cavity:

- Alternatively, we can use “cavity resonators”.
- Metamorphosis of the ***LC*** circuit into an accelerating cavity:

1. Increase resonant frequency by lowering ***L***, eventually have a solid wall.
2. Further frequency increase by lowering ***C*** → arriving at cylindrical, or “pillbox” cavity geometry, which can be solved analytically.
3. Add beam tubes to let particle pass through.





Pillbox geometry:

- Electric field used for acceleration is concentrated near the axis
- Magnetic field is concentrated near the cavity outer wall

Cavity resonators:

A cavity resonator is a closed metal structure that confines electromagnetic fields in the RF or microwave region of the spectrum.

- Such cavities act as resonant circuits with extremely low losses. The RF loss for cavities made of copper is typically 3-4 orders lower than for resonant circuits made with inductors and capacitors at the same frequency.

- Resonant cavities can be made from closed (or short-circuited) sections of a waveguide or coaxial line. Ferrite-loaded cavities are used at low frequencies to make cavities compact and allow very wide frequency tuning range.

- The cavity wall structure can be made stiff to allow its evacuation.

- Electromagnetic energy is stored in the cavity and the only losses are due to finite conductivity of cavity walls and dielectric/ferromagnetic losses of material filling the cavity.

Modes in an RF cavity:

$$\Delta\vec{E} + k^2\vec{E} = 0, \quad \Delta\vec{H} + k^2\vec{H} = 0.$$

where $k = \omega\sqrt{\mu\varepsilon}$

Boundary conditions

$$\vec{n} \times \vec{E} = 0$$

$$\vec{n} \cdot \vec{H} = 0$$

- There are an infinite number of orthogonal solutions (eigen modes) with different field structure and resonant frequencies (eigen frequencies).
- For acceleration in longitudinal direction the lowest frequency mode having longitudinal electric field component is used.

Properties of resonance modes:

- Relation between eigenvalue k_m and eigenfunction $\vec{H}_m$:

$$k_m^2 = \frac{\int_V |curl\vec{H}_m|^2 dV}{\int_V |\vec{H}_m|^2 dV}; \quad \omega_m = ck_m = \frac{k_m}{\sqrt{\varepsilon\mu}}, \quad \lambda_m = \frac{2\pi}{k_m}.$$

- The eigen functions are orthogonal

$$\int_V \vec{E}_m \cdot \vec{E}_n \, dV = 0, \qquad \int_V \vec{H}_m \cdot \vec{H}_n \, dV = 0, \qquad \text{if } \; k_m^2 \neq \; k_n^2$$

- The <u>average</u> energies stored in electric and magnetic fields are equal:

$$\frac{1}{4}\int_V \mu|\vec{H}_m|^2 dV = \frac{1}{4}\int_V \varepsilon|\vec{E}_m|^2 dV = \frac{W_0}{2} \rightarrow W_0 = \frac{1}{2}\int_V \mu|\vec{H}_m|^2 dV = \frac{1}{2}\int_V \varepsilon|\vec{E}_m|^2 dV$$

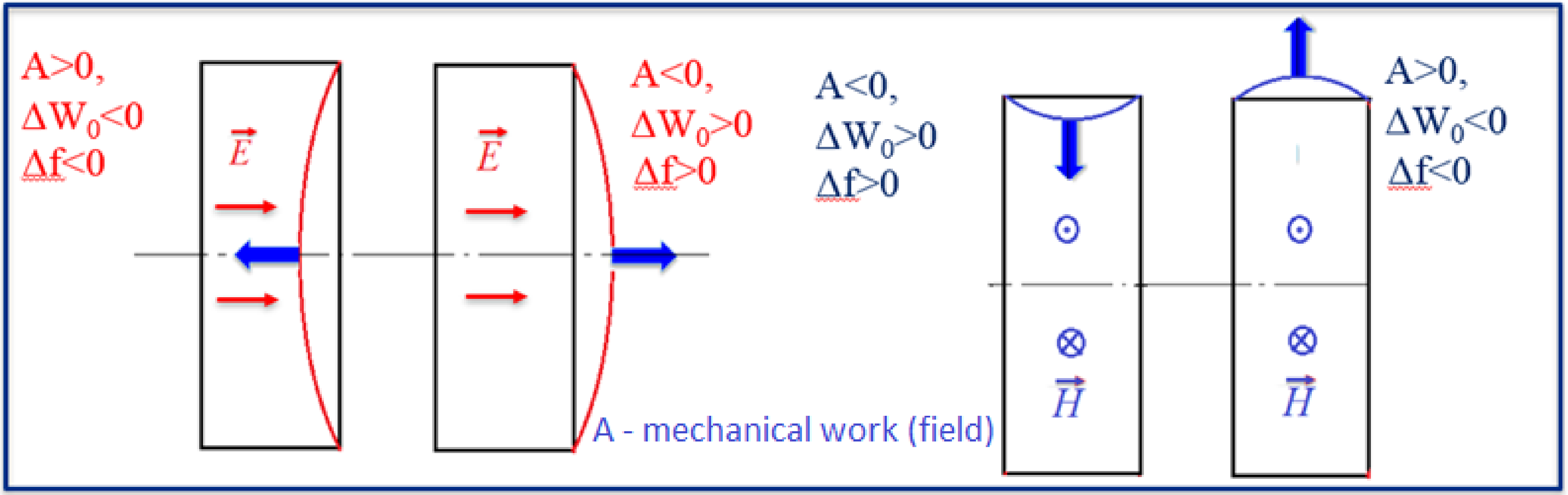
- The eigenmode variation property: for small cavity deformation one has: $\frac{W_0}{\omega_0} = const$

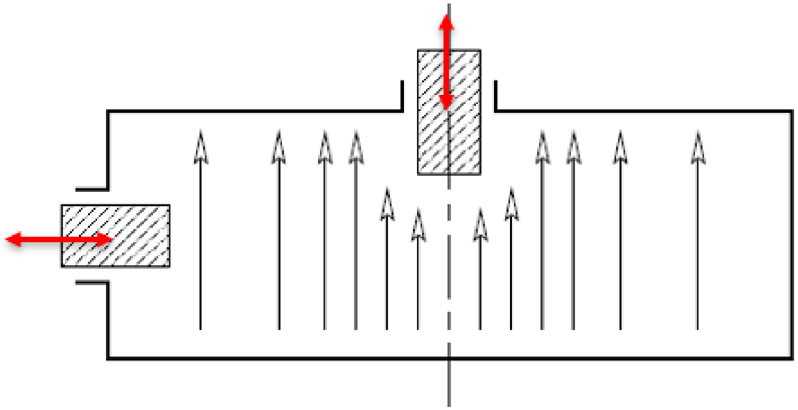
W_0 us stored energy, ω_0 is the mode circular resonance frequency.

See for details <u>Appendix 3</u>

Properties of resonance modes :

Cavity mechanical tuning is based on the eigenmode variation property:





Resonance modes and pillbox cavity:

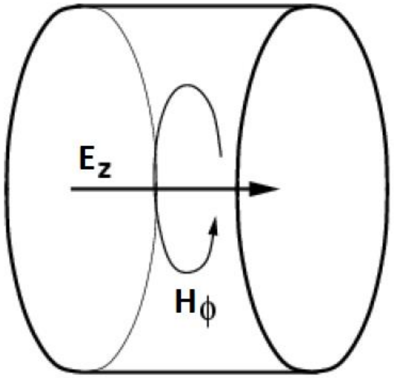
- For pillbox cavities there are two families of the eigen modes:
  - TM -modes, which have no longitudinal magnetic fields;
  - TE-modes, which have no longitudinal electric fields.
- The modes are classified as TM_{mnp} (TE_{mnp}), where integer indices *m, n,* and *p* correspond to the number of variations E_z (H_z) has in φ, r, and z directions respectively (see Appendix 5).

- For "monopole" modes in the axisymmetric cavity of arbitrary shape
  - TM-modes have only azimuthal magnetic field component;
  - TE -modes have only azimuthal electric field component.

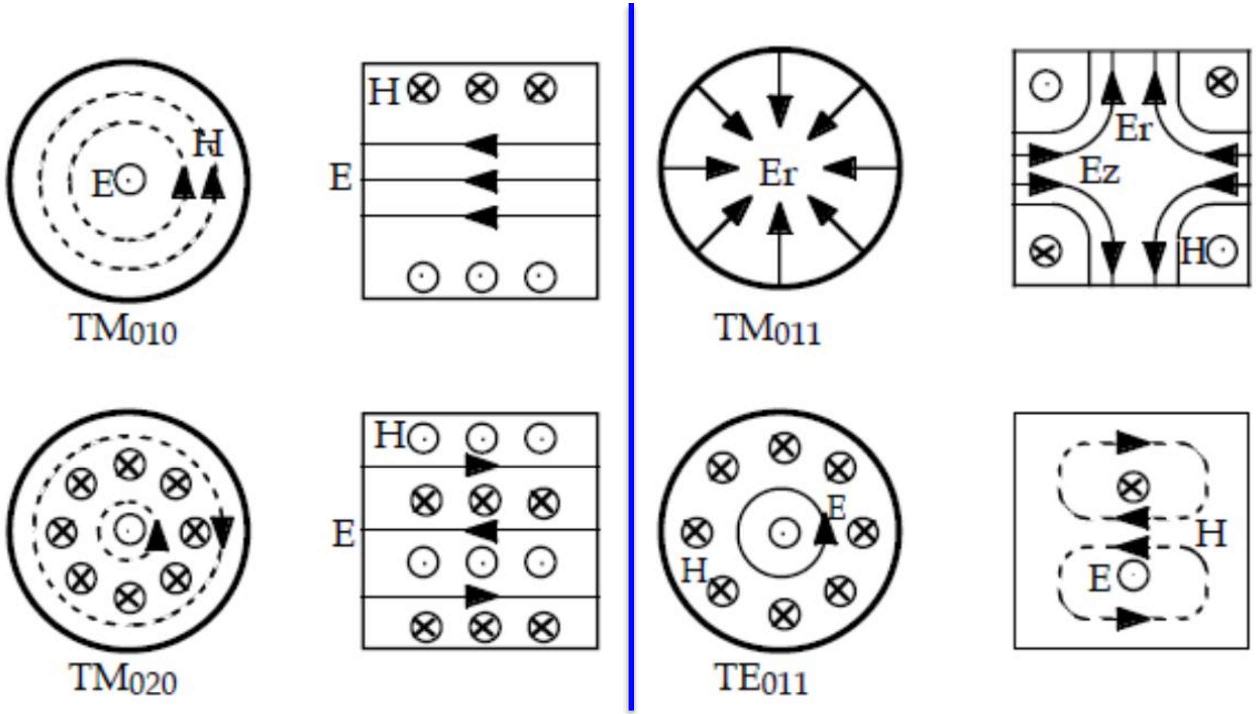
  For acceleration, lowest TM-mode is used, which has longitudinal electric filed on the axis.

Resonance modes and pillbox cavity:

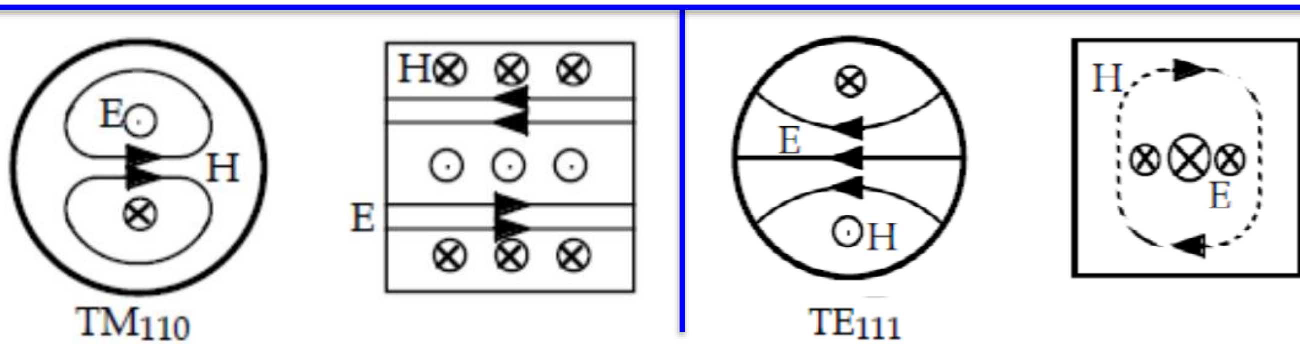
- Most of acceleration cavities have axial symmetry (slightly violated by perturbations – coupling units, manufacturing errors, etc).
- The modes in the axisymmetric cavity of arbitrary shape have azimuthal variations, $\vec{E}, \vec{H} \sim \exp(im\varphi)$:
  -For acceleration TM-modes with m=0 (“monopole”) are used;
  -Dipole (m=1) TM-modes are used for the beam deflection.
- The simplest cavity is a pillbox one:



Resonance modes and pillbox cavity:



Modes with m=0 (“monopole modes”)



Modes with m=1 (dipole modes)

Modes in a pillbox RF cavity:

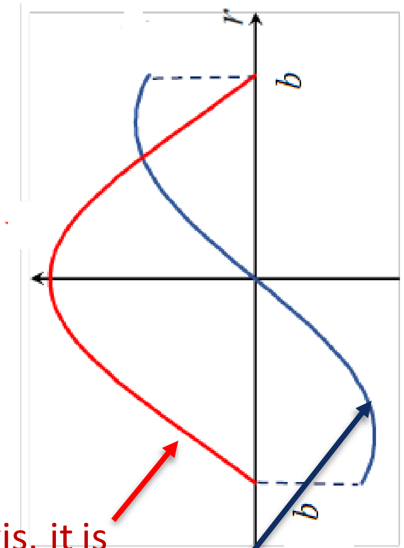
- While TM_{010} mode is used for acceleration and usually is the lowest frequency mode, all other modes are “parasitic” as they may cause various unwanted effects. Those modes are referred to as High-Order Modes (HOMs). Modes with m=0 – “monopole”, with m=1 – “dipole”, etc.

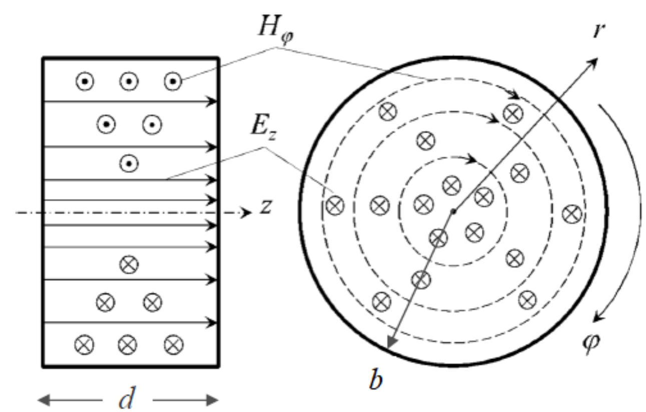
$$E_z = E_0 J_0\left(\frac{2.405r}{b}\right)e^{i\omega t}$$

$$H_\phi = -i\frac{E_0}{Z_0}J_1\left(\frac{2.405r}{b}\right)e^{i\omega t}$$

$$\omega_{010} = \frac{2.405c}{b},\ Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$\lambda_{010} = 2.61b$$





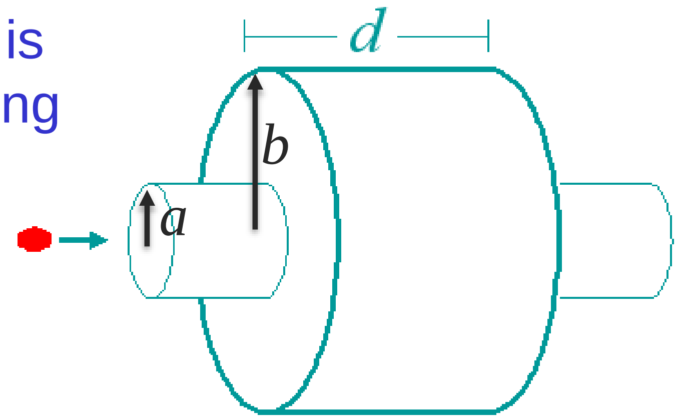
- Electric field is concentrated near the axis, it is responsible for acceleration.
- Magnetic field is concentrated near the cylindrical wall, it is responsible for RF losses.

Next to the cavity axis
$E_z(r) \sim const;$
$H_\varphi(r) \sim r$

Note that electric and magnetic fields are shifted in phase by 90 deg. For vacuum Z_0=120π Ohms; b is the pillbox radius, d is its length.

Accelerating voltage and transit time factor*:

Assuming charged particles moving along the cavity axis, and the particle velocity change is small, one can calculate *maximal* accelerating voltage V as



$$V = \left| \int_{-\infty}^{\infty} E_z(r=0,z) e^{i\omega_0 z/\beta c} dz \right|$$

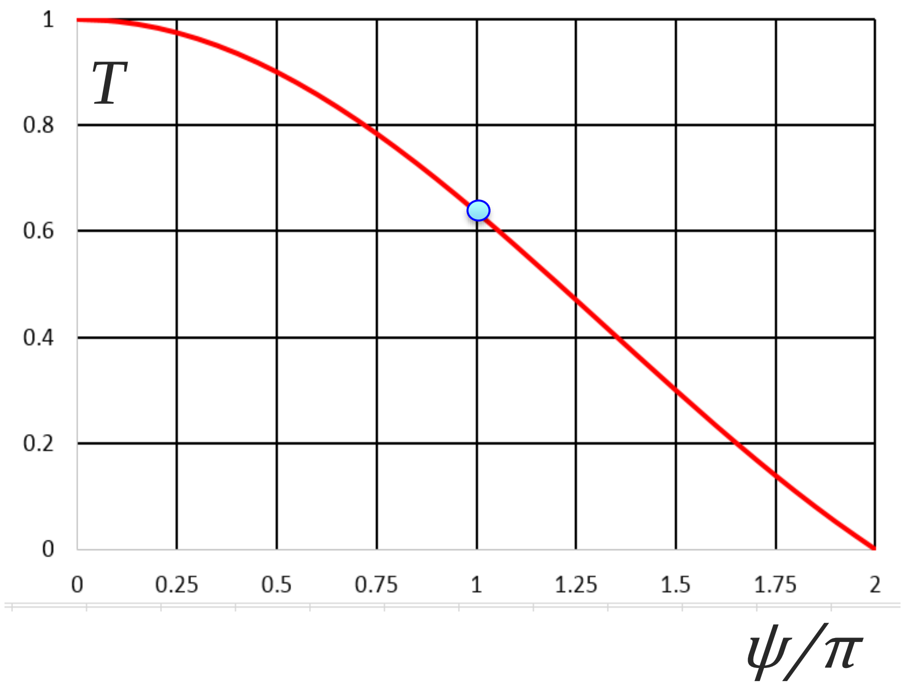
For the pillbox cavity one can integrate this analytically:

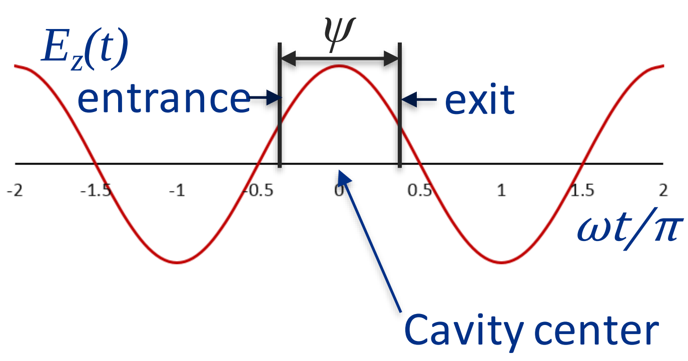
$$V = E_0 \left| \int_0^d e^{i\omega_0 z/\beta c} dz \right| = E_0 d \frac{\sin\left(\frac{\omega_0 d}{2\beta c}\right)}{\frac{\omega_0 d}{2\beta c}} = E_0 d \cdot T$$

where T is the transit time factor, $T(\psi) = \frac{\sin(\psi/2)}{\psi/2}$, $\psi = \frac{\omega_0 d}{\beta c}$

***Details are in Appendix 4**

Acceleration gradient





Note that maximal acceleration takes place when the RF field reaches maximum when the particle is the cavity center.
In order to "use" all the field for acceleration , $\psi=\pi$ (or $d = \beta\lambda/2$) and $T = 2/\pi$ for the pill box cavity. $\lambda = 2\pi c/\omega_0$ – wavelength.

- ***Acceleration gradient E is defined as $E=V/d=E_0T$***

*Unfortunately, the cavity length is not easy to specify for shapes other than pillbox so usually it is assumed to be **d = βλ/2**. This works OK for multi-cell cavities, but poorly for single-cell cavities or cavities for slow particle acceleration.*

RF cavity parameters:

- Stored energy U:

$$U = \frac{1}{2}\mu_0 \int_V |\mathbf{H}|^2 dv = \frac{1}{2}\varepsilon_0 \int_V |\mathbf{E}|^2 dv$$

- Losses in the cavity. There are the losses P_c in a cavity caused by finite surface resistance R_s:

$$P_c = \frac{1}{2} R_s \int_S |\mathbf{H}|^2 ds$$

For normal conducting metal at room temperature (no anomalous skin effect)

$R_s = \frac{1}{\sigma\delta}$, where σ is conductivity and δ is skin depth, $\delta = \frac{1}{\sqrt{\pi f \mu_0 \sigma}}$

Example:
For copper at room temperature σ=59.6 MS/m; R_s= 9.3 mOhm@1.3 GHz

Simple formula for estimation: $\delta = 0.38 \cdot (30/f(\mathrm{GHz}))^{1/2}$ [mkm]

Fermilab

RF cavity parameters:

Unloaded quality factor Q_0:

$$Q_0 \equiv \frac{\omega_0 \cdot (\text{stored energy})}{\text{average power loss}} = \frac{\omega_0 U}{P_c} = \frac{\omega_0 \mu_0 \int_V |\mathbf{H}|^2 dv}{R_s \int_S |\mathbf{H}|^2 ds}$$

Quality factor Q_0 roughly equals to the number or RF cycles times 2π necessary for the stored energy dissipation.

One can see that

$$Q_0 = \frac{G}{R_s}$$

where G is so-called geometrical factor (same for geometrically similar cavities),

$$G = \frac{\omega_0 \mu_0 \int_V |\mathbf{H}|^2 dv}{\int_S |\mathbf{H}|^2 ds}$$

RF cavity parameters:

$$G = \frac{\omega_0 \mu_0 \int_V |\mathbf{H}|^2 dv}{\int_S |\mathbf{H}|^2 ds}$$

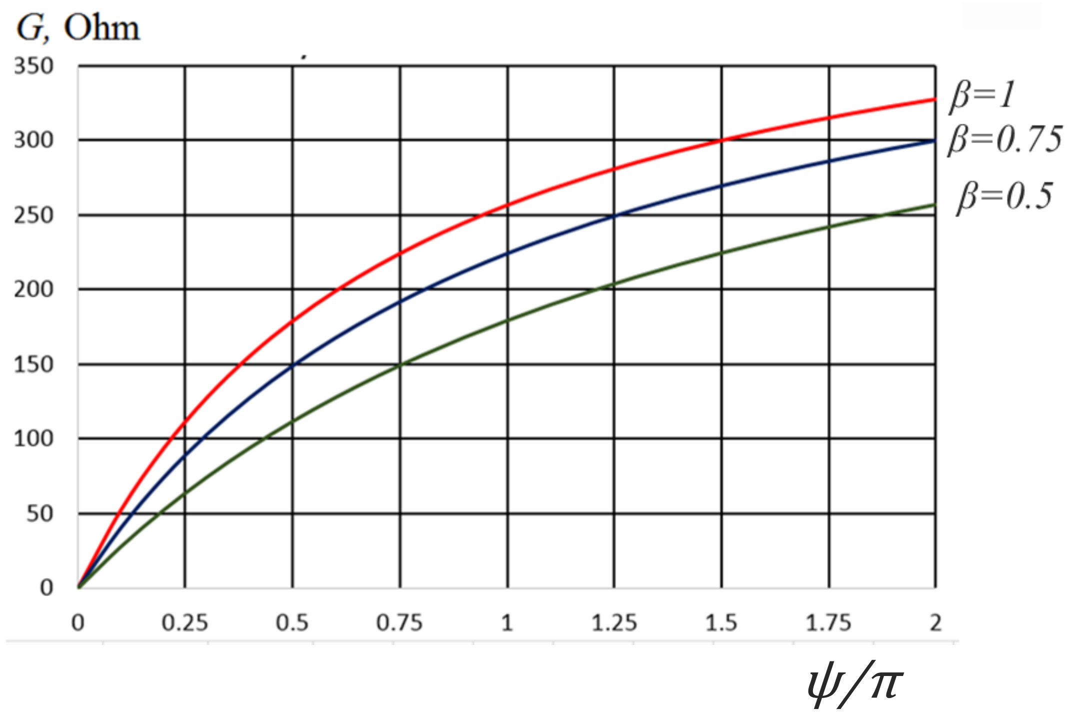
For a pillbox cavity:

$$H_\varphi = J_1(kr),\ k = \frac{\omega_{010}}{c} = \frac{2.405}{b}$$

$$\int_V |\vec{H}_m|^2 \, dV = \pi d b^2 J_1^2(kb), \quad \oint_S |\vec{H}_m|^2 \, dS = 2\pi b(b+d) J_1^2(kb)$$

and

$$G = 1.2 Z_0 \frac{1}{1+b/d}.$$



RF cavity parameters:

For a room-temperature pillbox cavity

$$Q_0 = \frac{1}{\delta}\frac{bd}{b+d}.$$

$$Q_0 = \frac{\omega_0 \mu_0 \int_V |\mathbf{H}|^2 dv}{R_s \int_S |\mathbf{H}|^2 ds}$$

- For pillbox having $\psi=\pi$ and $\beta=1$ $(d = \lambda/2 = \pi b/2.405)$, G =257 Ohms.
- Therefore, 1.3 GHz RT copper pillbox cavity has Q_0 = 2.6e4.
- For 1.3 GHz SRF Nb cavity at 2K one has Q_0 = 3e10 (R_s= 8.5 nOhm).

For geometrically similar RT cavities Q_0 scales as $f^{-1/2}$ *or* $\lambda^{1/2}$!

RF cavity parameters:

Estimation of the unloaded Q_0 for an arbitrary room-temperature cavity:

$$Q_0 = \frac{\omega_0 \mu_0 \int_V |\vec{H}|^2 \, dV}{R_s \int_S |\vec{H}|^2 \, dS}, \quad R_s = \frac{1}{\sigma\delta},$$

Taking into account that $\omega_0 \sigma = \frac{2}{\delta^2 \mu_0}$ one has:

$$Q_0 = \frac{2}{\delta} \frac{\int_V |\vec{H}|^2 \, dV}{\oint_S |\vec{H}|^2 \, dS}.$$

One may introduce the average surface and volume fields:

$$Q_0 = \frac{2}{\delta} \frac{V |H|_V^2}{S |H|_S^2}, \quad 2|H|_V^2 = A|H|_S^2.$$

for accelerating mode $A \sim 1$

For convex figures $V/S \sim a_{av}/6$ (cube: $V/S = a/6$. sphere: $V/S = 2R/6$) and

$$Q_0 \approx \frac{1}{6} \frac{a_{av}}{\delta} A.$$

Note, that $a_{av} \sim \lambda$, $\delta \sim \sqrt{\lambda}$ and $Q_0 \sim \lambda^{1/2}$,

RF cavity parameters:

- An important parameter is the cavity shunt impedance R, which determines relation between the cavity accelerating voltage V and power dissipation:

$$R = \frac{V^2}{P_c}$$

- Another important parameter is (R/Q), which determines relation between the cavity voltage V and stored energy U. Itis necessary to estimate the mode excitation by the accelerated beam. It does not depend on the surface resistance and is the same for geometrically similar cavities:

$$\frac{R}{Q} = \frac{V^2}{\omega_0 U} = 2\frac{\left|\int_{-\infty}^{\infty} E_z(\rho = 0, z) e^{i\omega_0 z/\beta c} dz\right|^2}{\omega_0 \mu_0 \int_V |\mathbf{H}|^2 dv}$$

Note that $R = \frac{R}{Q} \cdot Q_0$ and power dissipation $P_c = \frac{V^2}{\frac{R}{Q} \cdot Q_0} = \frac{V^2 \cdot R_S}{\frac{R}{Q} \cdot G}$

*Sometimes they use a “circuit” definition: $\frac{R}{Q} = \frac{V^2}{2\omega_0 U}$

Fermilab

RF cavity parameters:

For a pillbox cavity:

$$V = E_0 d \cdot T(\psi),$$

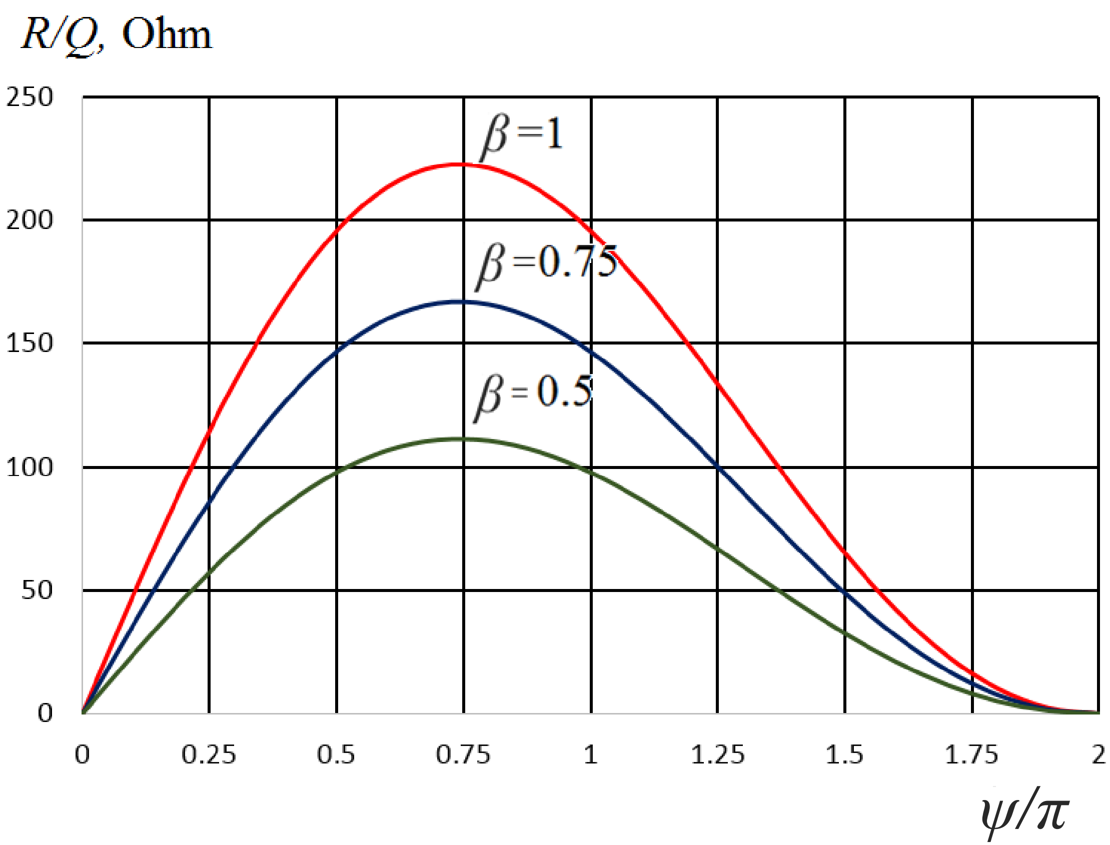
$$\int_V |\vec{H}_m|^2 \, dV = \frac{E_0^2}{Z_0^2} \pi d b^2 J_1^2(kb),$$

$$J_1(kb) = J_1(2.405) = -0.519$$

$$\omega_0 \mu_0 = Z_0 2.405/b$$

and

$$\frac{R}{Q} = 0.98 Z_0 d/b \cdot T^2(\psi)$$



- For pillbox having $\psi=\pi$ and $\beta=1$ $(d=\lambda/2=\pi b/2.405)$, R/Q =196 Ohms.
- R/Q is maximal for for $\psi \approx 3\pi/4$.

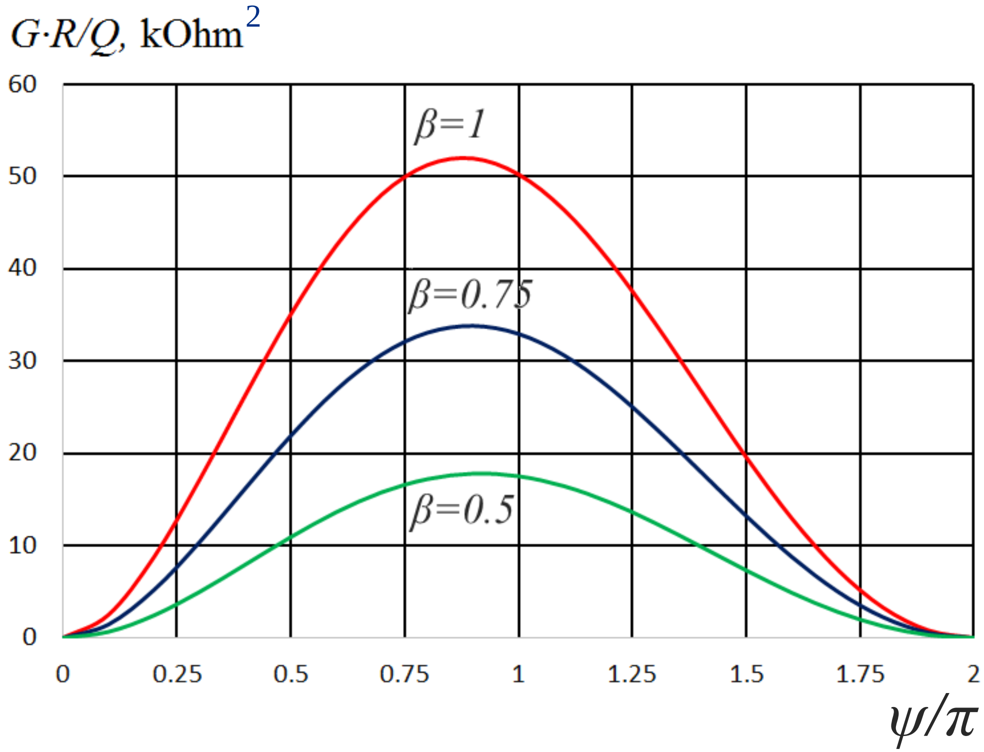
RF cavity parameters:

The power loss in the cavity walls is

$$P_c = \frac{V^2 \cdot R_s}{G \cdot (R/Q)}$$

Therefore, the losses are determined by $G{\cdot}R/Q$.

For pillbox:



$G{\cdot}R/Q$ is maximal for $\psi \approx 0.9\pi$

RF cavity parameters:

Gradient limitations are determined by surface fields:

❖ RT cavities:

-breakdown (determined mainly by E_{peak})

-metal fatigue caused by pulsed heating (determined by B_{peak})

❖ SRF cavities:

-quench (determined by B_{peak})

Field enhancement factors:

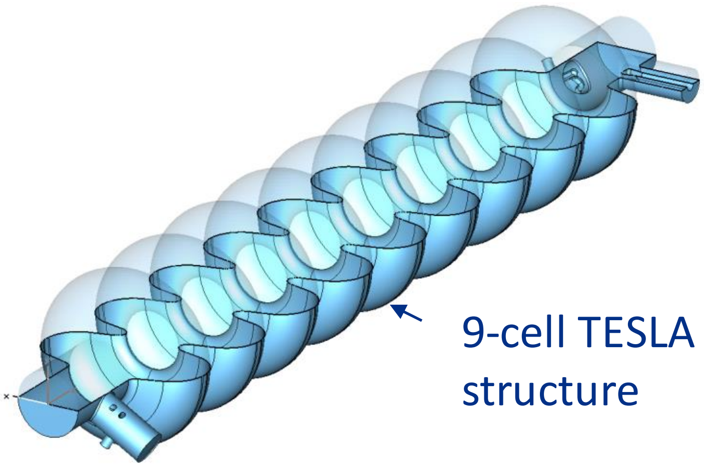
- Surface electric field enhancement: $K_e = E_{peak}/E$, E_{peak} is maximal surface electric field. K_e *i*s dimensionless parameter.

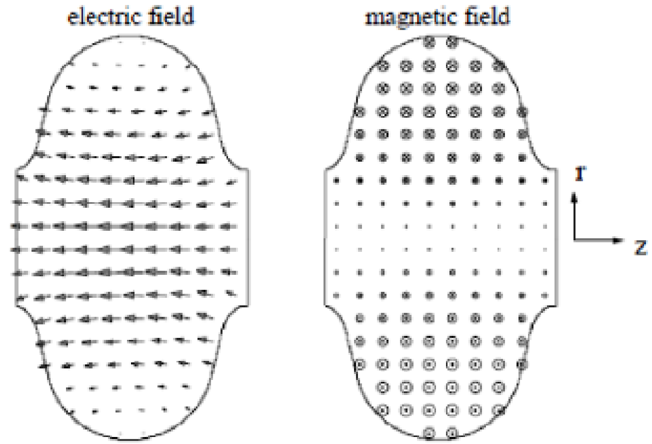
- Surface magnetic field enhancement: $K_m = B_{peak}/E$, B_{peak} is maximal surface magnetic field. K_m is in mT/(MV/m)

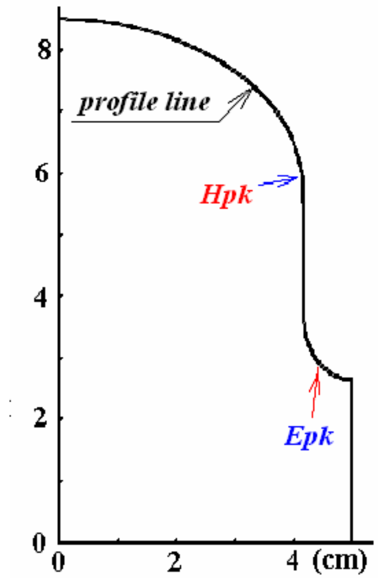
Here E is acceleration gradient.

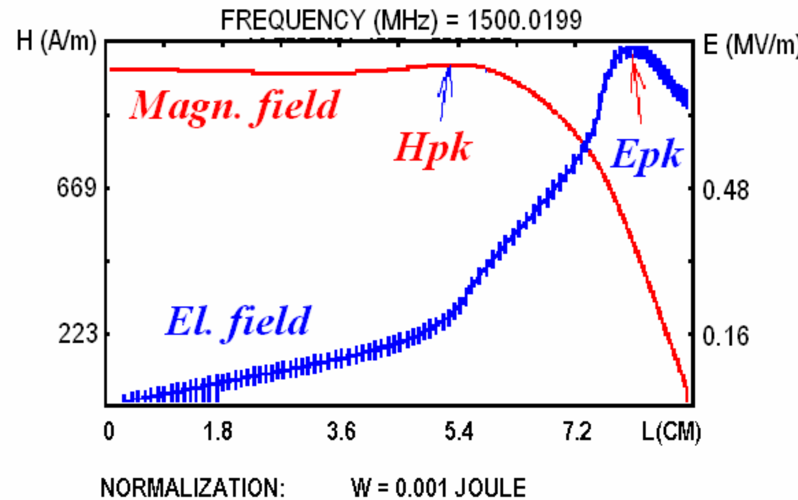
RF cavity parameters:

Field enhancement factors – example:









9-cell TESLA structure

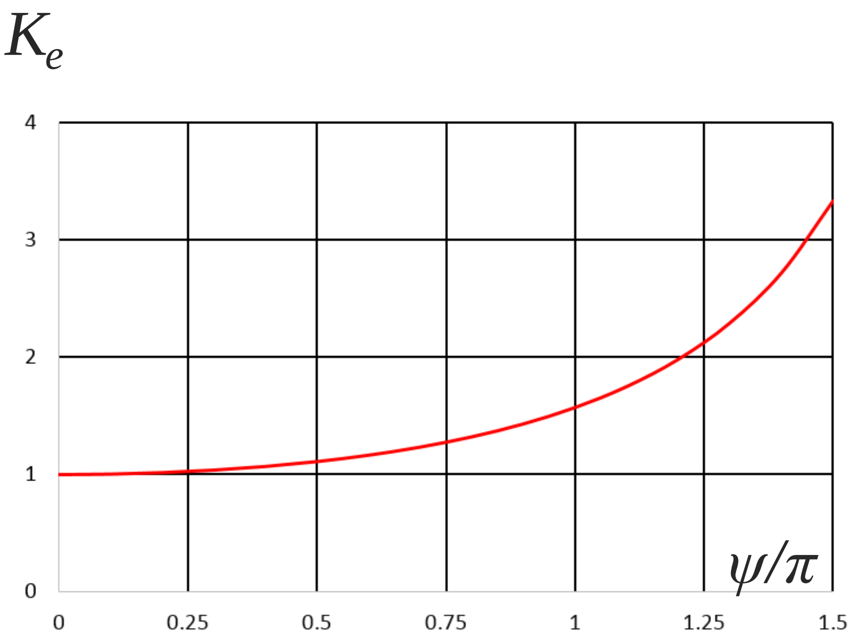
$$K_e = E_{peak}/E = 2; \quad K_m = B_{peak}/E = 4.16\, mT/MV/m$$

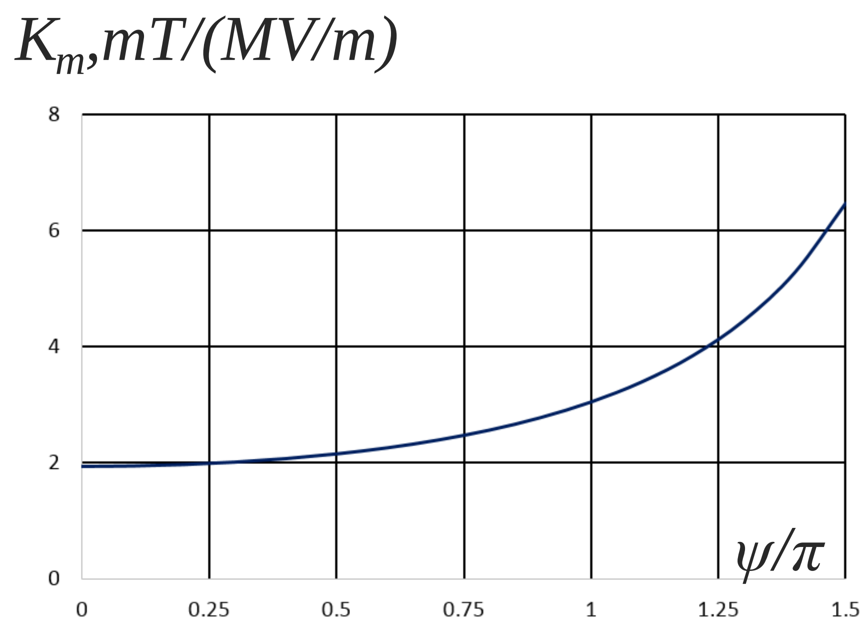
Geometry of an inner half-cell of a multi-cell cavity and field distribution along the profile line.

RF cavity parameters:

For a pillbox cavity:

- Surface electric field enhancement: $K_e = E_{peak}/E = 1/T(\psi)$
  ($E_{peak}=E_0$, $E= E_0 T(\psi)$, *see slide 47*)
- Surface magnetic field enhancement: $K_m = B_{peak}/E =$
  $= 1.94/T(\psi)$ [mT/(MV/m)]
  ($B_{peak} = E_0 \cdot J_1(2.405r/b)_{max}/c = 0.582E_0/c$, *see slide 46*)





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Example:

Let's consider a pillbox cavity for high-energy electrons ($\beta \approx 1$), f=500 MHz, or wavelength $\lambda = c/f$=0.6 m. The mode is TM_{010}. The cavity voltage V is 3 MV.

1. The cavity radius b (Slide 46):

$$b = 2.405c/(2\pi f) = 230 \text{ mm}.$$

2. The cavity transit time factor for $\psi = \pi$ (Slide 47):

$$T = \sin(\pi/2)/(\pi/2) = 2/\pi = 0.64.$$

3. The cavity length d (Slide 48):

$$d = \beta\lambda/2 = 300 \text{ mm}.$$

4. The cavity G- factor (Slide 51):

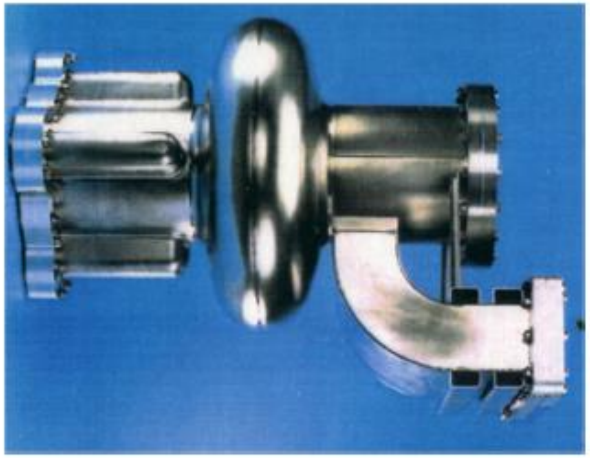
$$G = 1.2Z_0/(1+b/d) = 256 \text{ Ohm}$$

5. The cavity R/Q (Slide 55):

$$R/Q = 0.98Z_0(d/b)T^2 = 196 \text{ Ohm}$$

Pillbox *vs.* "real life" SC cavity

Quantity	Cornell SC 500 MHz	Pillbox
G	270 Ω	257 Ω
R/Q	88 Ω/cell	196 Ω/cell
$E_{\text{pk}}/E_{\text{acc}}$	2.5	1.6
$B_{\text{pk}}/E_{\text{acc}}$	$5.2\,mT/(\text{MV/m})$	$3.05\,mT/(\text{MV}/\text{m})$



Cornell SC 500 MHz

- In "real life" cavities, sometimes it is necessary to damp higher-order modes (HOMs) to avoid beam instabilities.
- The beam pipes are made large to allow HOMs propagation toward microwave absorbers.
- This enhances B_{pk} and E_{pk} and reduces R/Q.

Example:

6. Surface resistance R_s for room-temperature copper (Slide15):

$$R_s = \sqrt{\frac{\omega Z_0}{2c\sigma}} = 5.8\ mOhm$$

7. Surface resistance R_s for superconducting Nb at 2 K (Slide 20):

$$R_{s,BCS} \approx 1.643 \times 10^{-5} \frac{T_c}{T} (f(GHz))^2 e^{-\frac{1.92 T_c}{T}} = 2.8\ nOhm$$

8. Copper cavity unloaded quality factor Q_0 (Slide 50):

$Q_0 = G/R_s = 44e3$

9. Nb cavity unloaded quality factor Q_0 at 2 K (Slide 50):

$Q_0 = G/R_s = 9e10$ *(compared to 44e3 for a copper cavity!)*

Example:

10. Copper cavity wall power dissipation (Slide 56):

$$P_{diss} = \frac{V^2}{\frac{R}{Q} \cdot Q_0} = 1\ MW$$ – *unacceptable!*

11. Nb cavity wall power dissipation (Slide 56):

$$P_{diss} = \frac{V^2}{\frac{R}{Q} \cdot Q_0} = 0.5\ W!$$

12. Acceleration gradient (Slide 48):

$$E = V/d = 3\ MeV/0.3m = 9\ MV/m$$

13. Peak surface electric and magnetic fields (Slide 60):

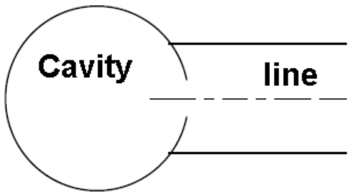
$$E_{peak} = K_e \cdot E = E/T = 14.1\ MV/m = 141\ kV/cm$$ – *OK for SC*

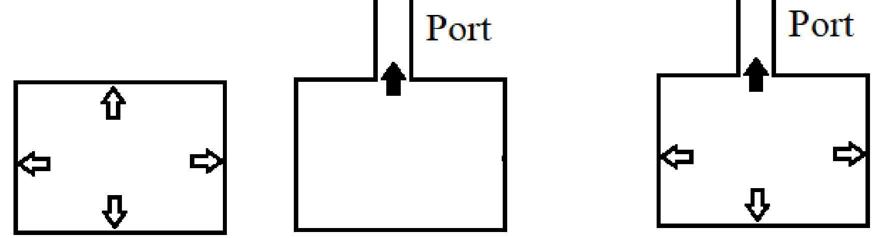
$$B_{peak} = K_m \cdot E = 1.94 \cdot E/T\ \ mT = 27\ mT$$ – *OK for SC*

SC cavities allow much higher acceleration gradient at CW!

The cavity coupling to the line:

Let's consider the cavity coupled to the feeding line.





Wall loss:	Port radiation:	Wall loss and port radiation:
$Q_0=\omega_0 U/P_0$	$Q_{ext}=\omega_0 U/P_{ext}$	$Q_L=\omega_0 U/(P_0+P_{ext})$

$$\frac{1}{Q_L}=\frac{1}{Q_0}+\frac{1}{Q_{ext}}$$

If the incident wave is zero (i.e., if the RF source is off), the loss in the cavity is a sum of the wall P_0 loss and the loss coasted by the radiation to the line P_{ext}:

$$P_{tot}=P_0+P_{ext}$$

$$P_0=\frac{V^2}{R/Q\cdot Q_0}, \quad P_{ext}=\frac{V^2}{R/Q\cdot Q_{ext}}$$

where we have defined an external quality factor associated with an input coupler. Such *Q* factors can be identified with all external ports on the cavity: input coupler, RF probe, HOM couplers, beam pipes, etc. The total power loss can be associated with the loaded *Q* factor, which is

$$\frac{1}{Q_L}=\frac{1}{Q_0}+\frac{1}{Q_{ext1}}+\frac{1}{Q_{ext2}}+\dots$$

because $P_{tot}=P_0+P_{ext1}+P_{ext2}\dots=\frac{V^2}{R/Q\cdot Q_L}$

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*Details are in Appendix 7

Coupling parameter:

For each port a coupling parameter β can be defined as

$\beta \equiv \frac{Q_0}{Q_{ext}}$ and, therefore, $\frac{1}{Q_L} = \frac{1+\beta}{Q_0}$

It tells us how strongly the couplers interact with the cavity. Large implies that the power leaking out of the coupler is large compared to the power dissipated in the cavity walls:

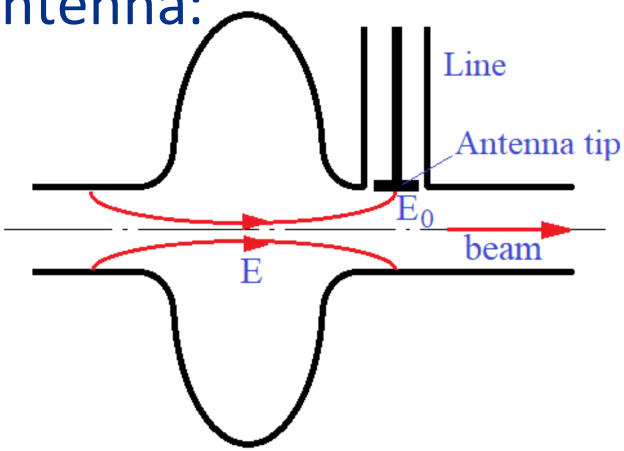
$$P_{ext} = \frac{V^2}{R/Q \cdot Q_{ext}} = \frac{V^2}{R/Q \cdot Q_0} \cdot \beta = \beta P_0$$

In order to maintain the cavity voltage, the RF source should compensate both wall loss and radiation to the line. Therefore, the RF source should deliver the power to the cavity which is

$$P_{tot} = P_{forw} + P_0 = (\beta + 1) P_0$$

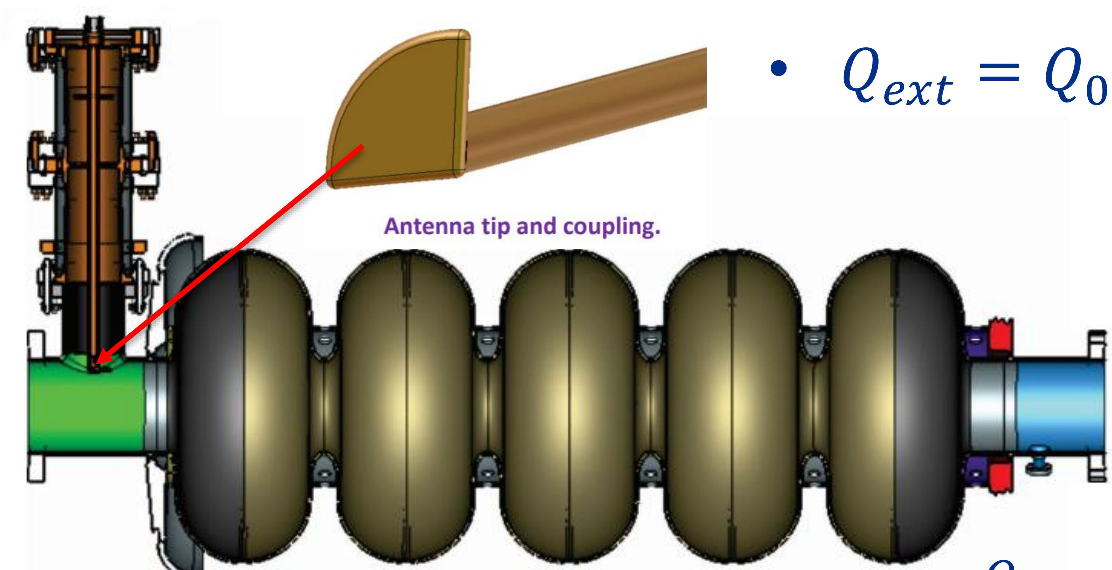
The cavity coupling to the line

Antenna:



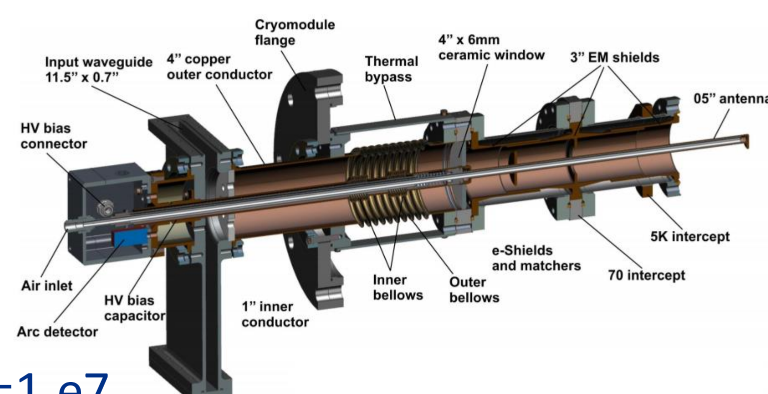
- Antenna tip square is S;
- The line has impedance Z;
- Electric field on the tip is E_0
- Antenna tip has a charge q:

  $q = E_0\varepsilon_0 S \rightarrow I = \omega q = \omega\, E_0\varepsilon_0 S = kSE_0/Z_0;$
- Radiated power $P_{ext} = \frac{1}{2} Z I^2$;
- Loss in the cavity $P_0 = \dfrac{V^2}{R/Q \cdot Q_0}$
- $Q_{ext} = Q_0 \dfrac{P_0}{P_{ext}} = 2\dfrac{Z_0^2}{Z \cdot R/Q} \cdot \left(\dfrac{V}{kSE_0}\right)^2$



Antenna tip and coupling.

Structure of 650 MHz coupler, new design



Q_{ext} =1.e7

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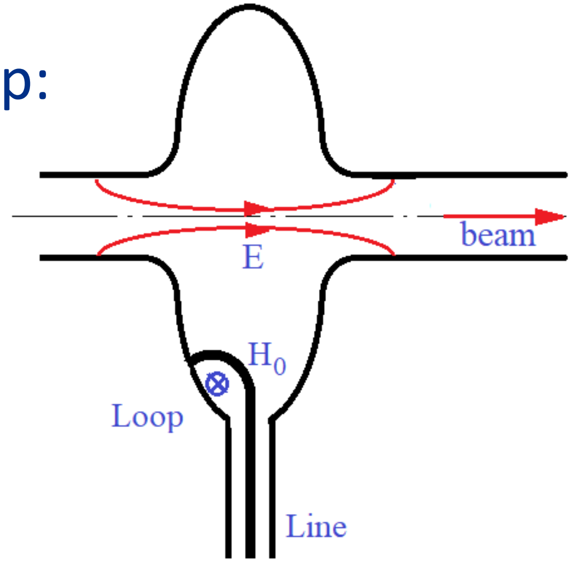
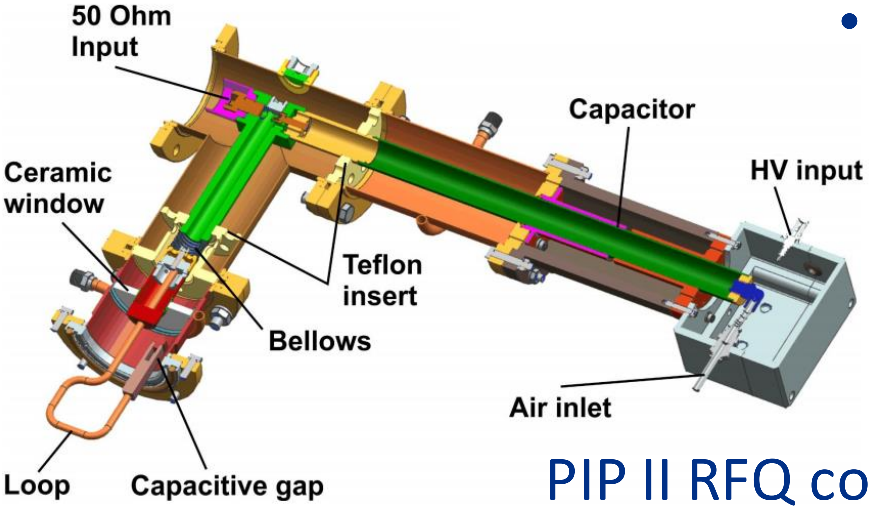
The cavity coupling to the line

Loop:



- Loop square is S;
- The line has impedance Z;
- Magnetic field on the loop is H_0
- Voltage induced on the loop U:

  $U = \omega H_0 \mu_0 S$; ⬅ $curl\vec{E} = -i\omega\mu_0\vec{H}$
- Radiated power $P_{ext} = \frac{U^2}{2Z}$;
- Loss in the cavity $P_0 = \frac{V^2}{R/Q \cdot Q_0}$
- $Q_{ext} = Q_0 \frac{P_0}{P_{ext}} = 2\frac{Z}{R/Q} \cdot \left(\frac{V}{kSH_0Z_0}\right)^2$



PIP II RFQ coupler

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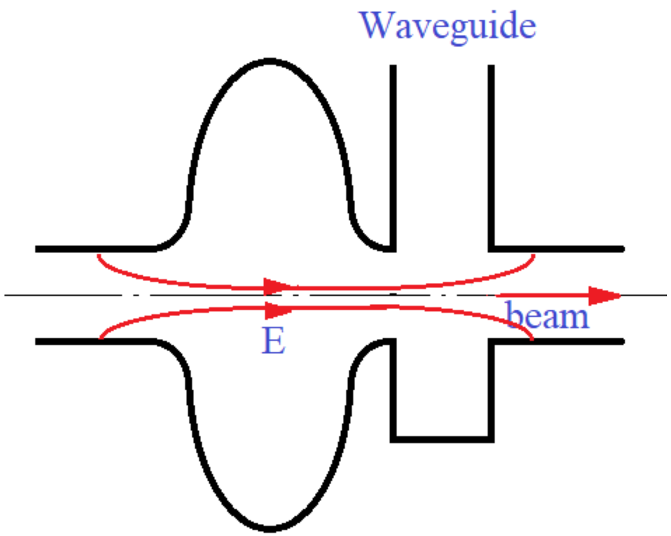
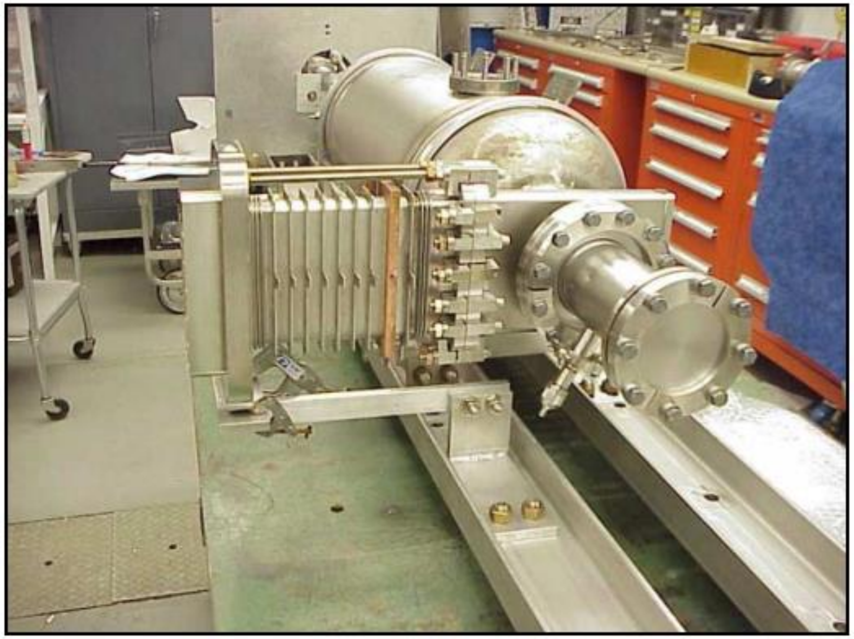
The cavity coupling to the line

Waveguide:





Waveguide on Cavity String

CEBAF couplers

Cavity excited by the beam (Appendix 6):

- If the cavity is excited by the beam with the *average* current I having the bunches separated by the length equal to integer number of RF periods, i.e., in resonance, the excited cavity voltage provides maximal deceleration. The beam power loss is equal to the cavity loss, i.e., radiation and wall loss:

$$-VI = \frac{V^2}{\left(\frac{R}{Q}\right)Q_L} \qquad (1)$$

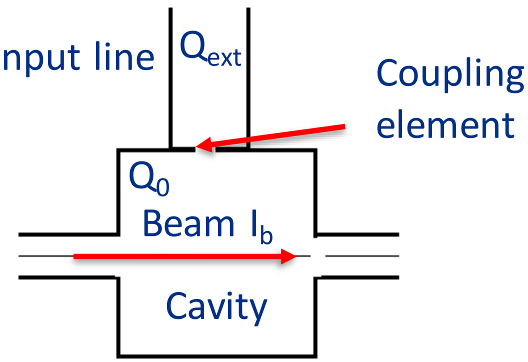
or

$$V = -I\left(\frac{R}{Q}\right)Q_L$$

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}} = \frac{1+\beta}{Q_0}$$

$\beta = \frac{Q_0}{Q_{ext}}$ -coupling parameter

(See Slide 65)

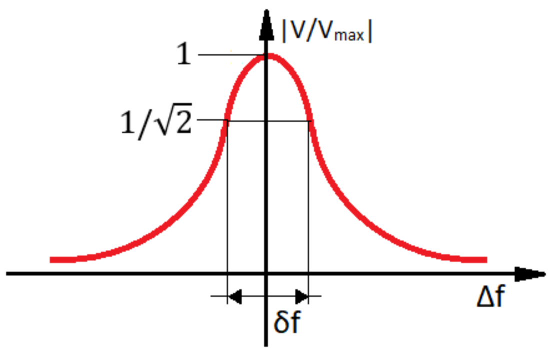


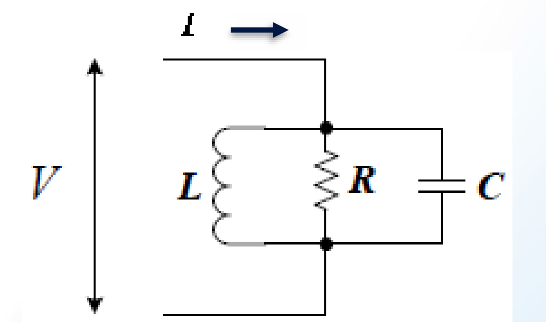
- The cavity excited by the beam off the resonance, the voltage is $V \approx -\frac{I\left(\frac{R}{Q}\right)Q_L}{1+iQ_L\frac{2\Delta f}{f}}$

where Δf is the distance between the beam spectrum line and the cavity resonance frequency f.

- Cavity bandwidth:

$\delta f = f/Q_L;$





$L=(R/Q)/2\omega;$
$C=2/\omega(R/Q);$
$R=(R/Q)Q_L/2;$
$\omega=2\pi f$

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Acceleration cavity operating in CW regime:

Energy conservation law:

$$P_0 = P_{backward} + P_{diss} + P_{beam}$$

$$P_0 = E_0^2/(2Z),$$

Here Z is the transmission line impedance; E_0 is the incident wave amplitude in the transmission line.

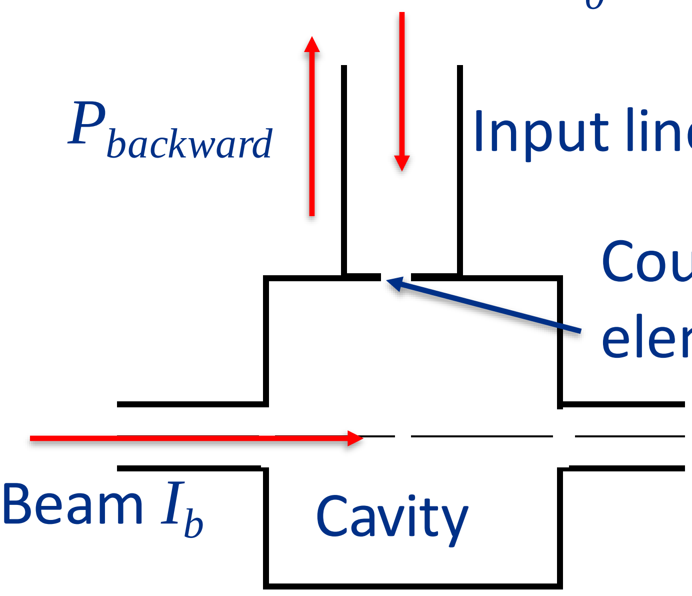
- $P_{backward} = (E_0 - E_{rad})^2/(2Z),$

E_{rad} is the amplitude of wave radiated from the cavity to the transmission line.

$$\beta = P_{rad}/P_{diss},$$

$$P_{rad} = E_{rad}^2/(2Z) = \beta P_{diss} = \beta V^2/(Q_0 \cdot R/Q) = V^2/(Q_{ext} \cdot R/Q);$$

$$P_{beam} = V \cdot I_b.$$



Details are in Appendix 8

Acceleration cavity operating in CW regime:

- If the line is matched to the transmission line,
  - coupling is optimal, $\beta = \beta_{opt}$ then

$P_{backward} = 0, \;\; E_0 = E_{rad},$

and therefore,

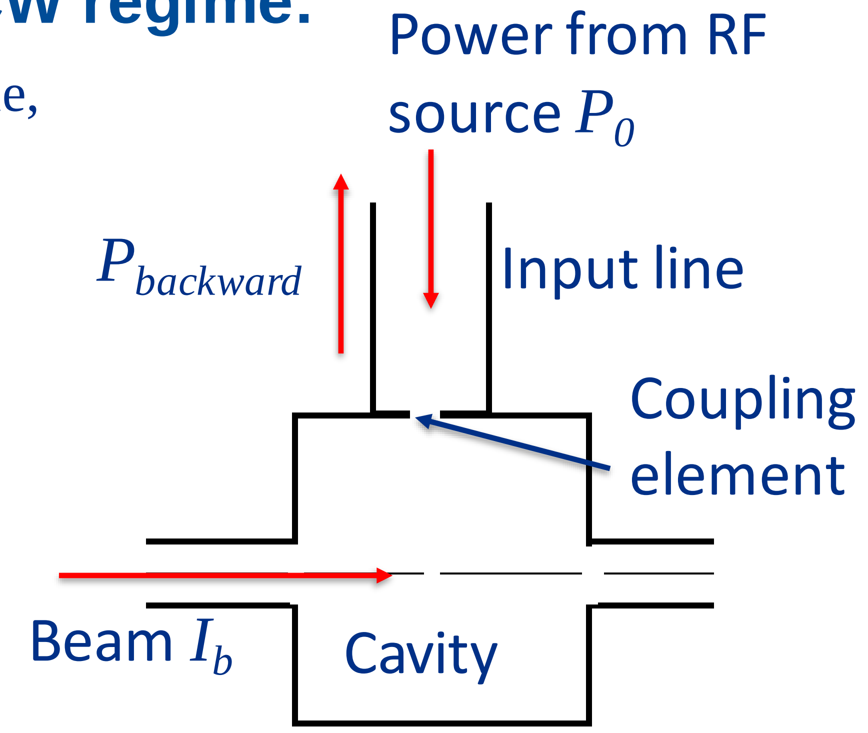
$P_{rad} = P_0 = P_{diss} + P_{beam}$, or

$\beta_{opt} V^2/(Q_0 \cdot R/Q) = V^2/(Q_0 \cdot R/Q) + VI$

and $\beta_{opt} = I \cdot Q_0 \cdot R/Q/V + 1.$

$For \; \beta_{opt} >> 1, \;\; \beta_{opt} \approx I \cdot Q_0 \cdot R/Q/V$ and

$$Q_L = Q_0/(1+\beta_{opt}) \approx V/(R/Q \cdot I)$$



(see Slide 65)

Details are in Appendix 8

Acceleration cavity operating in pulsed regime:

Energy conservation law:

$$dW/dt = P_0 - P_{backward} - P_{diss} - P_{beam} \quad (1)$$

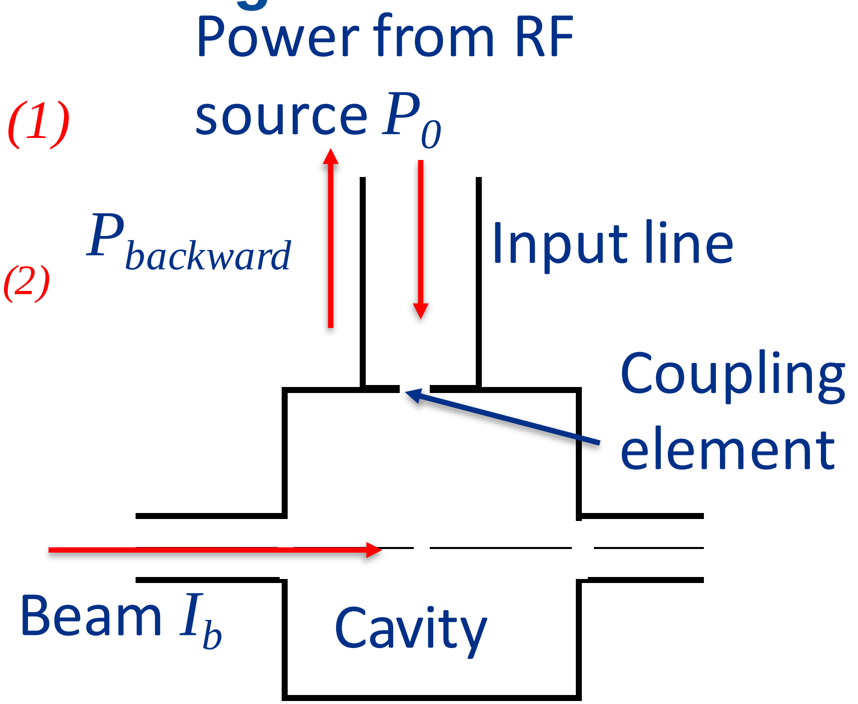
$$P_0 = E_0^2/2Z;\ P_{backward} = (E_0 - E_{rad})^2/2Z,$$
$$\beta = P_{rad}/P_{diss},\ P_{rad} = E_{rad}^2/2Z = W = V(t)^2/[(R/Q)Q_{ext}] \quad (2)$$
$$P_{beam} = V(t)I$$
$$W = V(t)^2/(R/Q)\omega;$$
$\tau = 2Q_L/\omega$ – time constant.

V_0 –operating voltage,

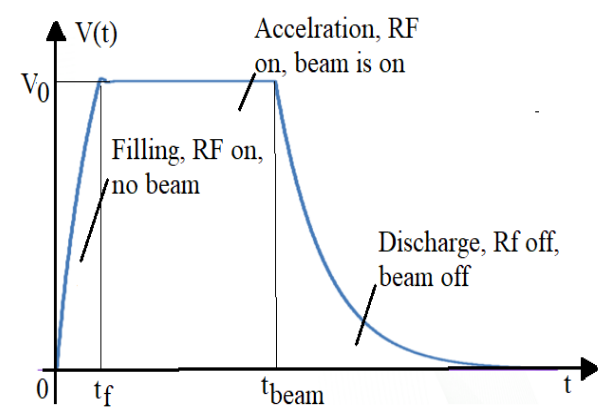
If $\beta >> 1$, $Q_L \approx V_0/(R/Q \cdot I)$.

Substituting (2) to (1) we have:

$$dV/dt = (2V_0 - V(t) - I \cdot (R/Q) \cdot Q_L)/\tau$$



- RF on:
  - Cavity filling, no beam: $I=0$, $V(t) = 2V_0(1 - exp(-t/\tau))$;
    If the filling time $t_f = \tau ln2$, $V(t_f) = V_0$,
  - Acceleration, the beam is on, $V_0 = I \cdot (R/Q) \cdot Q_L$,
    $dV/dt = (V_0 - V(t))/\tau = 0$ and $V(t) = V_0$;
- RF is off:
  $dV/dt = -V(t)/\tau$, the cavity discharge, $V(t) = V_0 \cdot exp(-t/\tau)$.



Example:

Let's consider a SC Nb pillbox cavity for high-energy electrons (β≈1), f=500 MHz, or wavelength $\lambda=c/f$=0.6 m. The mode is TM_{010}. The cavity voltage V is 3 MV. The beam current I is 1 A.

1. The cavity R/Q (Slide 60):

$$R/Q = 0.98 Z_0 (d/b) T^2 = 196\, Ohm$$

2. Nb cavity unloaded quality factor Q_0 at 2 K (Slides 60-63):

$$Q_0 = G/R_s = 9e10$$

3. The cavity loaded quality factor (Slide 71):

$$Q_L \approx V/(R/Q)I = 1.5e3$$

3. The optimal coupling (Slide 71):

$$\beta = Q_0/Q_L - 1 \approx Q_0/Q_L = 6e7$$

4. The power necessary for acceleration (Slide 71):

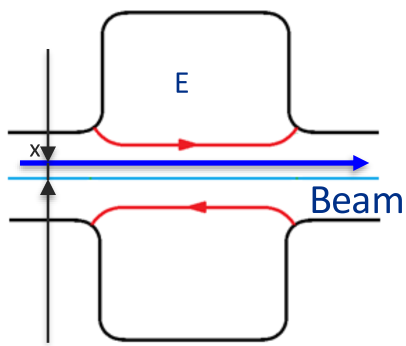
$$P_0 = P_c + P_{beam} \approx P_{beam} = VI = 3\ MW\ (compared\ to\ P_c = 0.5\ W!)$$

High-Order Modes in cavities:

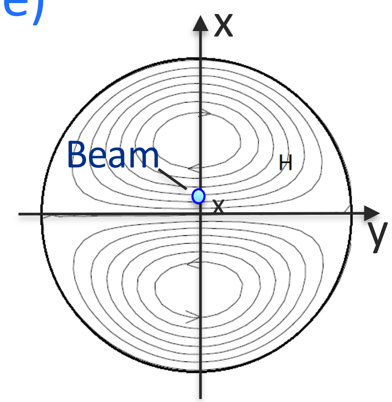
- ❑ Possible issues:
- Trapped modes;
- Resonance excitation of HOMs;
- Collective effects:
  - Transverse (BBU) and longitudinal (klystron-type instability) in linear accelerators;

- Additional losses;
- Emittance dilution (longitudinal and transverse)
- Beam current limitation.

- ❑ Longitudinal modes;
- ❑ Transverse modes.
- ❑ HOM dampers;



Dipole (transverse) mode



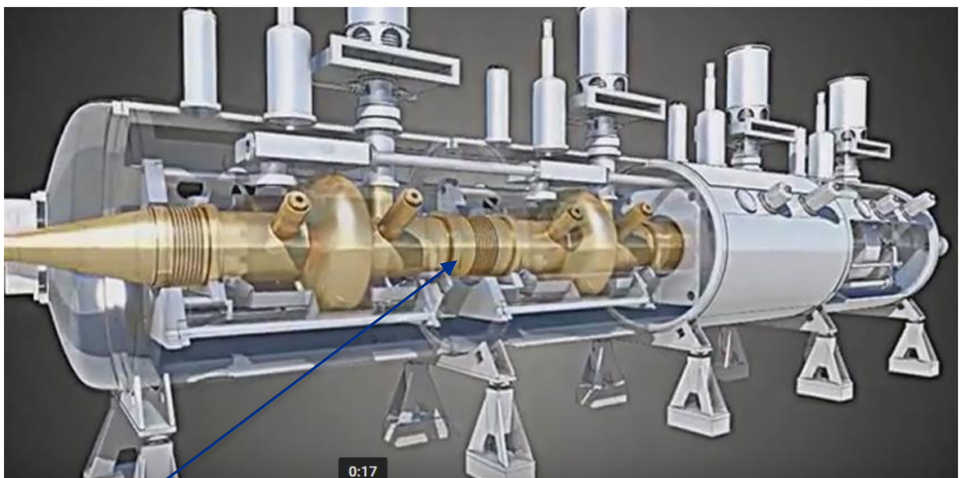
Near axis: E_z~x, E_x ~ const, H_y ~ const

High-Order Modes in cavities:

- ❑ Longitudinal modes:

$V_{HOM} = I_{beam} \cdot R_{HOM}$,   Longitudinal impedance: $R_{HOM} = (R/Q)_{HOM} \cdot Q_{load}$

- Design of the cavities with small R/Q (poor beam-cavity interaction)
- HOM dampers – special coupling elements connected to the load (low Q_{load}) .



LHC main cavity

LHC HOM coupler, Q_{ext} <200 for most "dangerous" modes

- Long wide waveguides between the cavity cells:
- HOMs propagate in the WGs and interact with the beam;
- No synchronism in the WGs (phase velocity >speed of light) → reduced R/Q_{HOM} for HOMs

High-Order Modes in cavities (Appendix 9):

- Transverse modes:

The beam interacts with the longitudinal component of the HOM electric field and provides transverse kick. For axisymmetric cavity for dipole TM-mode longitudinal field is proportional to the transverse coordinate next to the cavity axis.

Let's consider a cavity excited by a beam current I_0 having offset x_0. The kick caused by the dipole mode excited by the beam:

$$U_{kick} = i x_0 I_0 Q_{ext}\left(\frac{r_\perp}{Q}\right)$$ where $$\left(\frac{r_\perp}{Q}\right) \equiv \frac{\left|\int_{-\infty}^{\infty}\left(\frac{\partial E_z(x,0,z)}{\partial x}\right)_{x=x_0} e^{ikz}dz\right|^2}{kW\omega_0}$$

is transverse impedance, $k=\omega_0/c$ and $W = \frac{\varepsilon_0}{2}\int|\vec{E}|^2 dV$ - stored energy.

Compare to "longitudinal" (R/Q):

$$\left(\frac{R}{Q}\right) \equiv \frac{\left|\int_{-\infty}^{\infty} E_z(0,0,z)\, e^{ikz}dz\right|^2}{W\omega_0}$$

β=1 is considered.

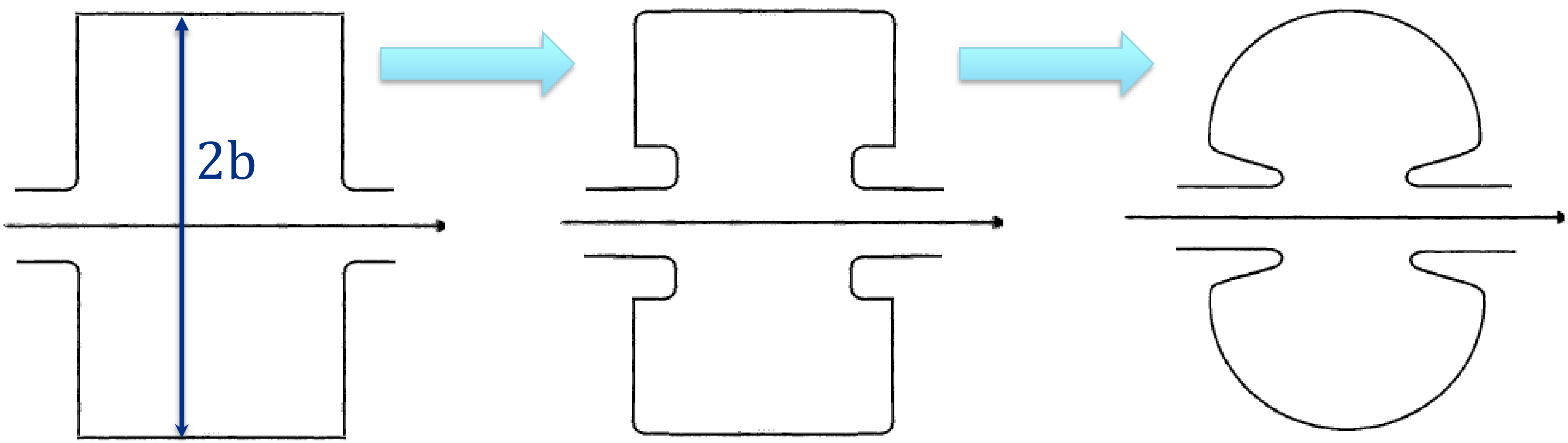
Note that $\left(\frac{r_\perp}{Q}\right)$ is measured in Ohm/m.

*Note that sometimes they use other transverse impedance, that is determined as:

$\left(\frac{r_\perp}{Q}\right)_1 = \frac{|U_{kick}|^2}{\omega_0 W_0} = \left(\frac{r_\perp}{Q}\right)\times\frac{1}{k}$. In this case, $U_{kick} = i(kx_0)I_0 Q_{ext}\left(\frac{r_\perp}{Q}\right)_1$, $\left(\frac{r_\perp}{Q}\right)_1$ **is measured in Ohm.**

RF cavity types

Pillbox RT cavities:



Pillbox with the beam pipes

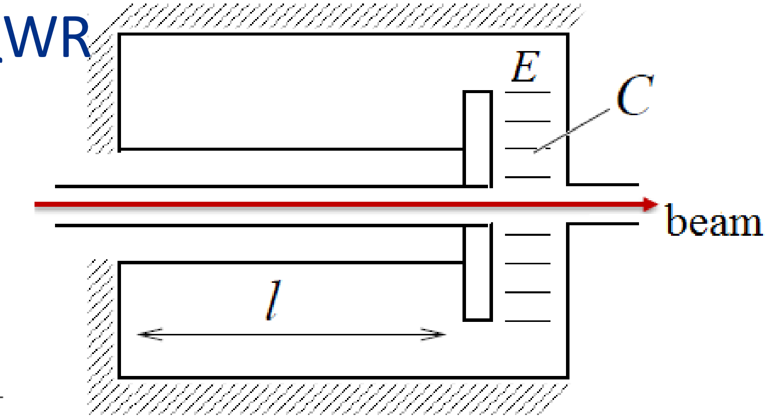
Drift tubes
T improvement

Rounding
R_{sh} improvement

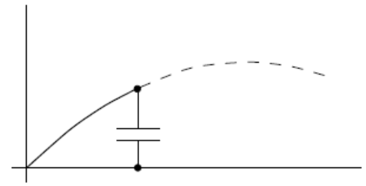
$$2b \lesssim 3/4\lambda$$

One-gap QWR

RF cavity types



Quarter-wave resonator (QWR) concept:

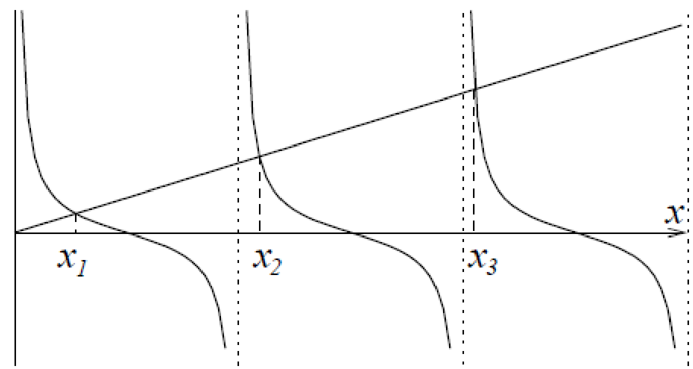


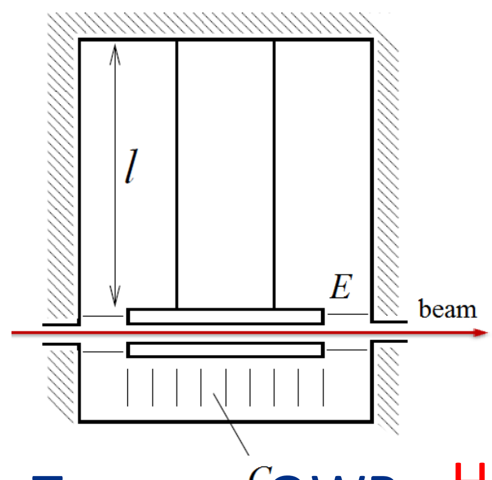
Resonance:

$$\frac{1}{j\omega C} + jZ_0 \tan\frac{\omega}{c}l = 0. \quad \text{or} \quad \cot\frac{\omega}{c}l = Z_0\omega C.$$

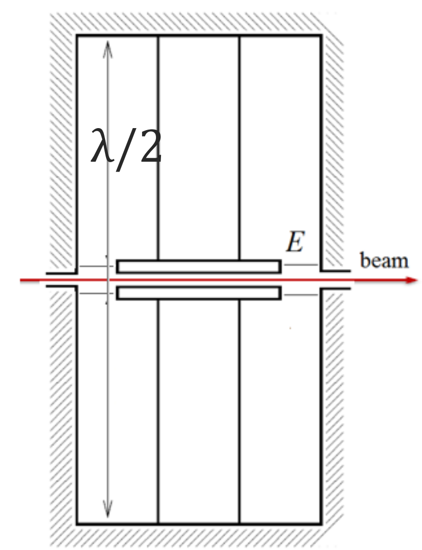
Z_0 here is the coaxial impedance. Compact ($L \gtrsim \lambda/4$) compared to pillbox ($D \lesssim 3/4\lambda$).

$$\frac{\omega}{c}l = x,\ A = \frac{Z_0 C\, c}{l} \longrightarrow \cot x = Ax.$$





Two-gap QWR



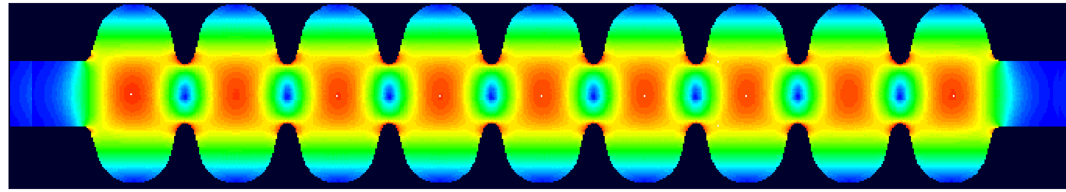
Half-wave resonator (HWR)

Tools for RF cavity simulations:

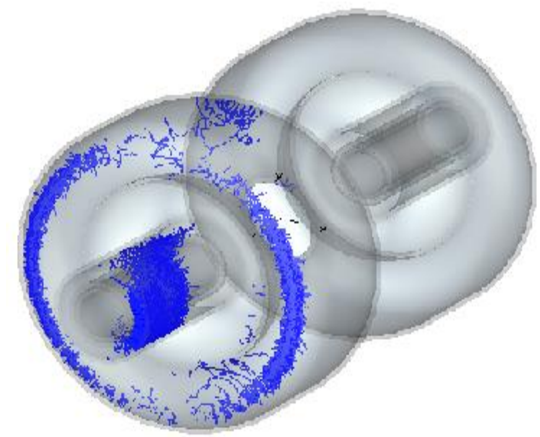
I. Field calculations:

-Spectrum, (r/Q), G, β (coupling)

-Field enhancement factors



- HFSS (3D);
- CST (3D);
- Omega-3P (3D);
- Analyst (3D);
- Superfish (2D)
- SLANS (2D, high precision of the field calculation).

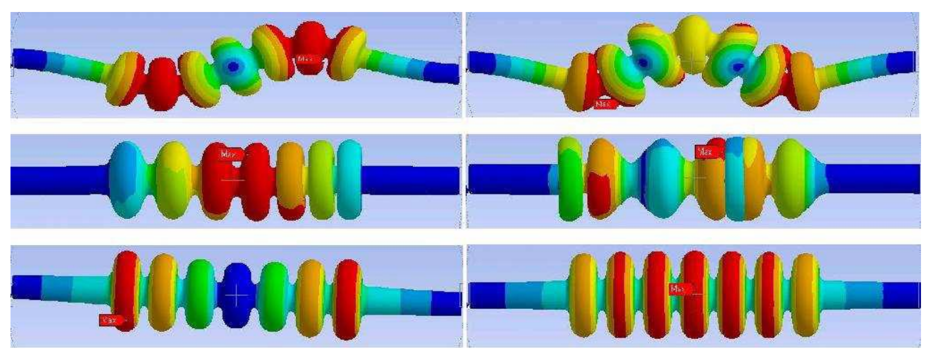


II. Multipactoring (2D, 3D)

- Analyst;
- CST (3D);
- Omega-3P

III. Wakefield simulations (2D, 3D):

- GdfidL;
- PBCI;
- ECHO.



IV. Mechanical simulations:

Lorenz force and Lorenz factor,

Vibrations,

Thermal deformations.

a. ANSYS

Fermilab

Summary:

- To create acceleration RF field, resonance RF cavities are used;
- The cavities typically have axisymmetric field distribution near the beam axis. Most of cavities have geometry close to axisymmetric.
- There are infinite number of resonance modes in an RF cavity having different radial, azimuthal and longitudinal variations. The modes are orthogonal;
- In axisymmetric cavities there are two types of modes, TM and TE;
- For acceleration, TM_{010} mode is used, which has axial electric field on the axis.
- Other modes, HOMs, are parasitic, which may caused undesirable effects.

Summary (cont):

- The cavity mode is characterized by the following parameters:
  - Resonance frequency;
  - Acceleration gradient (energy gain/cavity length);
  - Unloaded Q, Q_0, which characterize the losses in the cavity;
  - *G*-factor, which relates Q_0 and surface resistance;
  - *(R/Q)*, which relates the energy gain and the energy stored in the cavity;
  - Shunt impedance R, which relates the gain and total losses in the cavity;
  - Electric and magnetic field enhanced factors, which relate maximal surface fields and the acceleration gradient;

Summary (cont):

- The cavity coupled to the input port is characterized by the following parameters:
  - Coupling to the feeding line, β (do not mix with the relative particle velocity☺)
  - External Q, Q_{ext}
  - Loaded Q, Q_{load}
- The beam excites the cavity creating decelerating voltage, which is proportional at resonance to the shunt impedance and the beam current. This voltage should be compensated by the RF source to provide acceleration.
- High-Order Modes excited by the beam may influence the beam dynamics and lead to additional losses in the cavity.
  - Dipole modes are characterized by transverse impedance, $(r_{\perp}/Q)$, which relates transverse kick and stored energy.
  - Both monopole and dipole HOMs should be taken into account during the cavity design process, and damped if necessary.