RF accelerating structures, Lecture 9

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PHY862 “Accelerator Systems”

September 26, 2023

MICHIGAN STATE UNIVERSITY

RF accelerating structures

Outline:

7. **Multi-cell SRF cavities;**
8. **SRF Cavities for Low β Accelerators;**
9. **Beam-cavity Interaction**

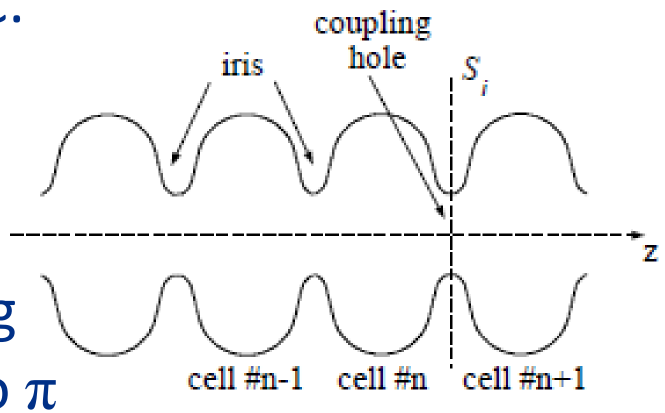
Chapter 7.

Multi-cell SRF cavities.

a. Multi-cell SRF cavities;
b. Why π-mode?
c. Equivalent circuit and normal modes;
d. Parameters of the SRF SW cavity;
c. Cavity efficiency at different particle velocity versus the number of cells;
d. Why elliptical multi-cell cavity does not work at low particle velocity.

Multi-cell SRF cavity:

- Single – cell cavities are not convenient to achieve high acceleration: a lot of couplers, tuners, etc.
- Multi-cell cavities are used in both RT and SRF accelerators.
- Multi-cell SRF cavity is a standing–wave periodic acceleration structure, operating at the phase advance per period equal to π (i.e, the fields in neighboring cells have the same distribution, but opposite sign).



- To provide synchronism with the accelerated particle, period is $\beta\lambda/2$ (in general case it is $\varphi\beta\lambda/2\pi$; φ is phase advance per period).
- The end cells have special design (full length, not half) to provide field flatness along the structure for operation mode with the phase advance π.

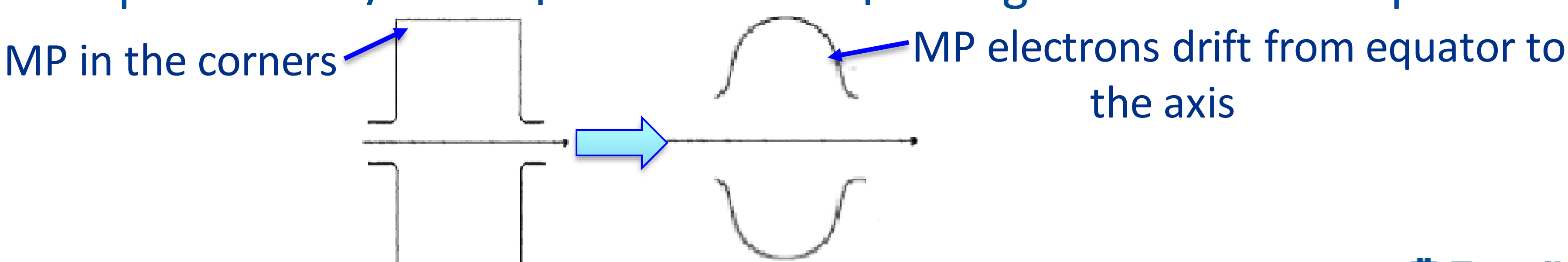
Why SW π – mode?

- The SW modes except π have small acceleration efficiency because most of the cavities have small field (in ideal case $X_{ni} \sim cos\ (\pi qj/N)$, q – mode number, j - cell number).
- Bi-periodic structure π/2-mode does not work because it is prone to multipacting in the empty coupling cells and difficult for manufacturing (different cells) and processing (narrow coupling cells).
- π -mode structure is simple, easy for manufacturing and processing.
- Drawback (see Appendix 11):

  -Big aperture to provide big coupling;

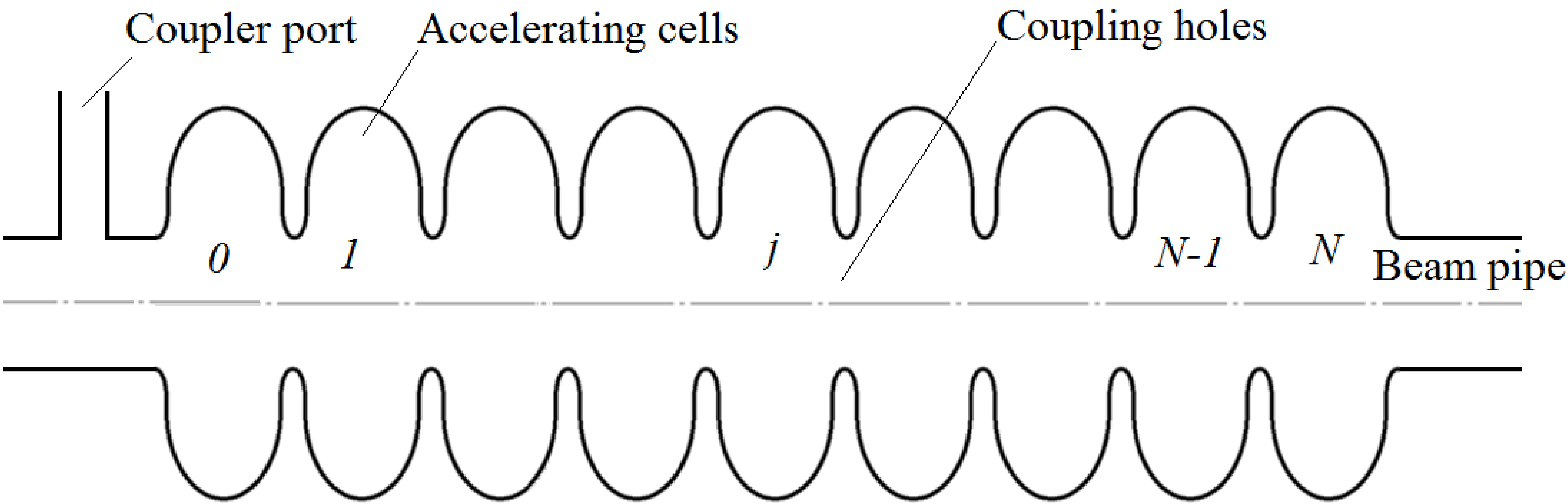
  -Considerably small number of cells N (5-9).

❑ Elliptical cavity is not prone to multipacting in contrast to a pillbox.



Why SW π – mode?

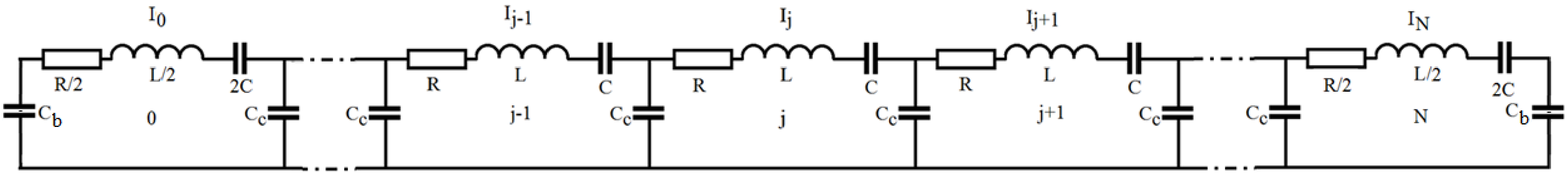
Schematic of the SRF multi-cell cavity



- The cells have elliptical shape to get rid of multipacting;
- The end cells have full length, but the shape is different;
- The coupler is placed in the beam pipe.

Why SW π – mode?

Equivalent circuit of the SRF multi-cell cavity



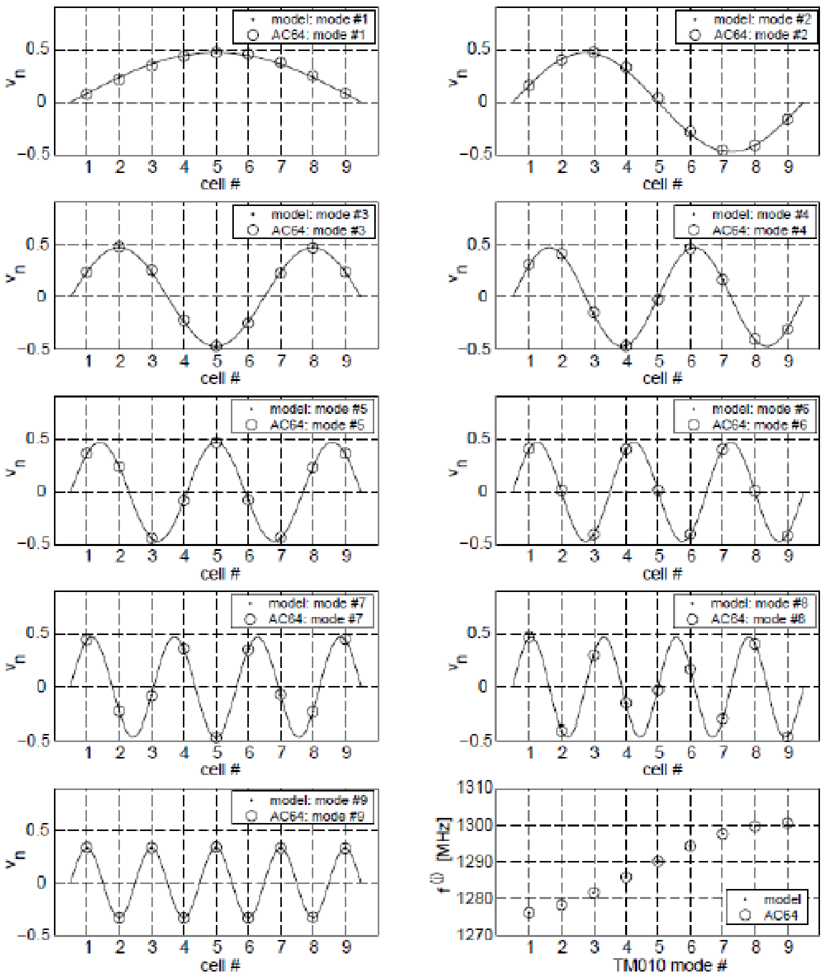
- C_b represents the fringing fields in the beam pipe.
- The shape of the 0^{th} and N^{th} cell are selected to achieve flat field distribution for π-mode only.

$$X_0\left[1-\frac{\omega_0^2}{\omega^2}+i\frac{\omega_0^2}{Q_0\omega^2}\right]+K\frac{\omega_0^2}{\omega^2}X_1+K_1\frac{\omega_0^2}{\omega^2}X_0=0$$

$$X_j\left[1-\frac{\omega_0^2}{\omega^2}+i\frac{\omega_0^2}{Q_0\omega^2}\right]+\frac{1}{2}K\frac{\omega_0^2}{\omega^2}\left[X_{j-1}+X_{j+1}\right]=0$$

$$X_N\left[1-\frac{\omega_0^2}{\omega^2}+i\frac{\omega_0^2}{Q_0\omega^2}\right]+K\frac{\omega_0^2}{\omega^2}X_{N-1}+K_1\frac{\omega_0^2}{\omega^2}X_N=0$$

Multi-cell RF cavity:



An example of calculated eigen modes amplitudes in a 9-cell TESLA cavity compared to the measured amplitude profiles. Also shown are the calculated and measured eigen frequencies.
The cavity has full size end cells especially tuned to get field flatness for the operating mode.

Normal modes in a standing-wave elliptical cavity:

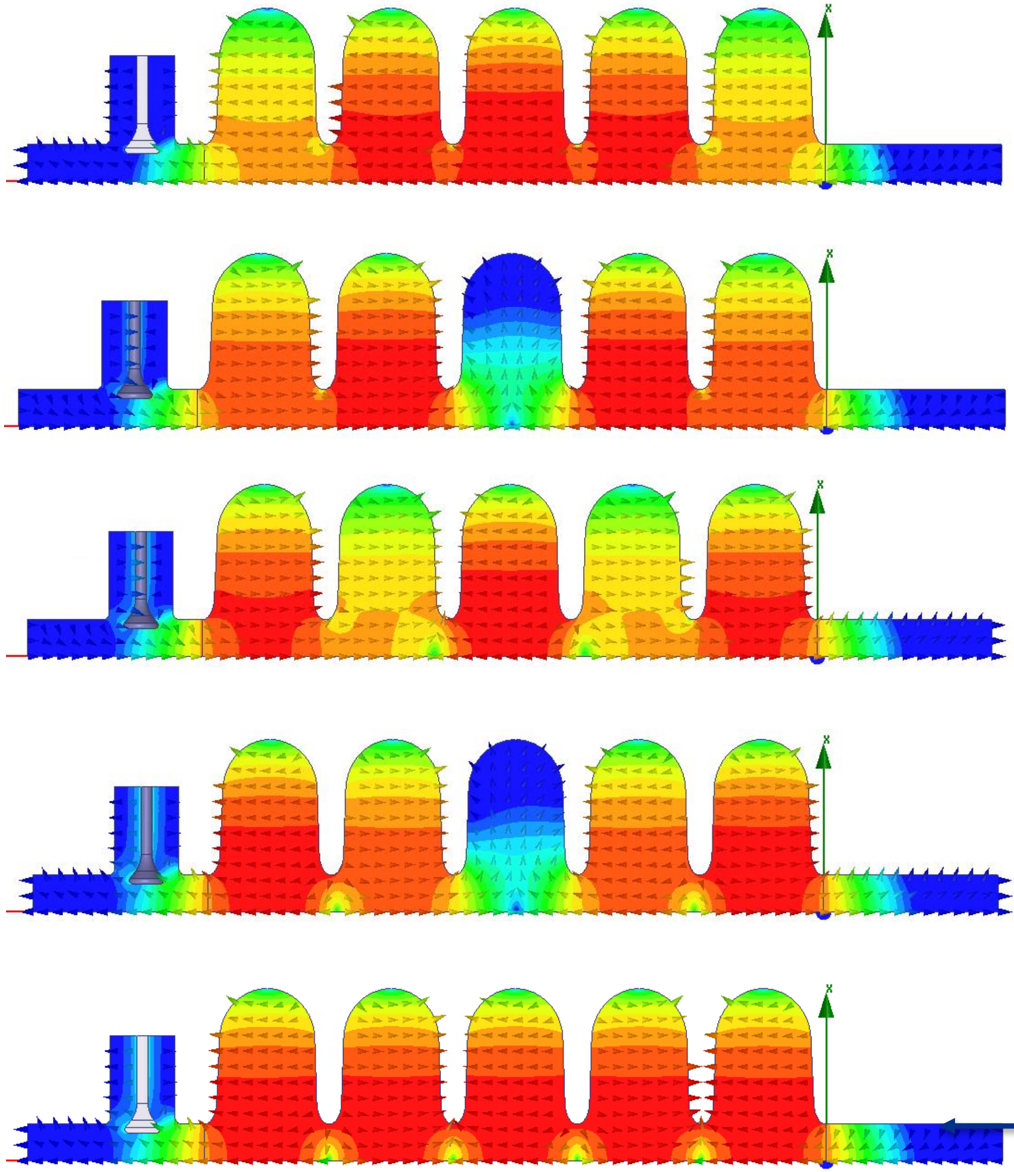
Mode 1
"0"

Mode 2
"π/4"

Mode 3
"π/2"

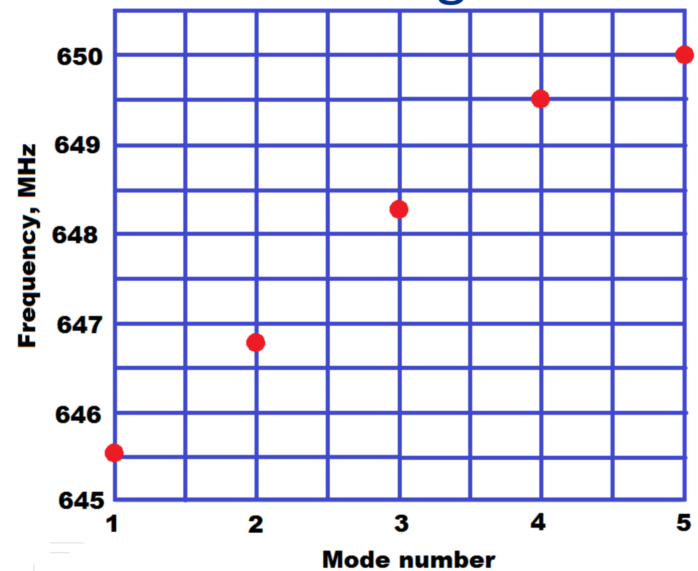
Mode 4
"3π/4"

Mode 5
"π"



PIP II 650 MHz cavity

"Brillouin diagram":



$$f(q) \approx f_0\left(1 - \frac{k}{2}cos\frac{\pi(q-1)}{(N-1)}\right)$$

q–the mode number;

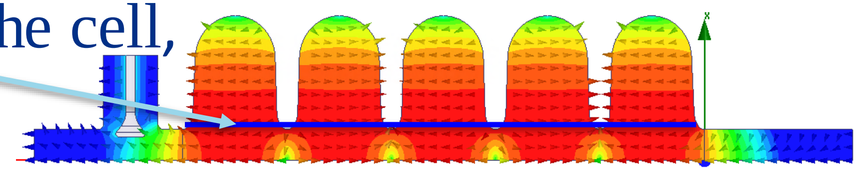
N- the number of cells;

k- coupling: $k=0.7\%$

Operation mode "π"

Axial acceleration field distribution

At the aperture, $E_z(a,z) \sim const$ over the cell,



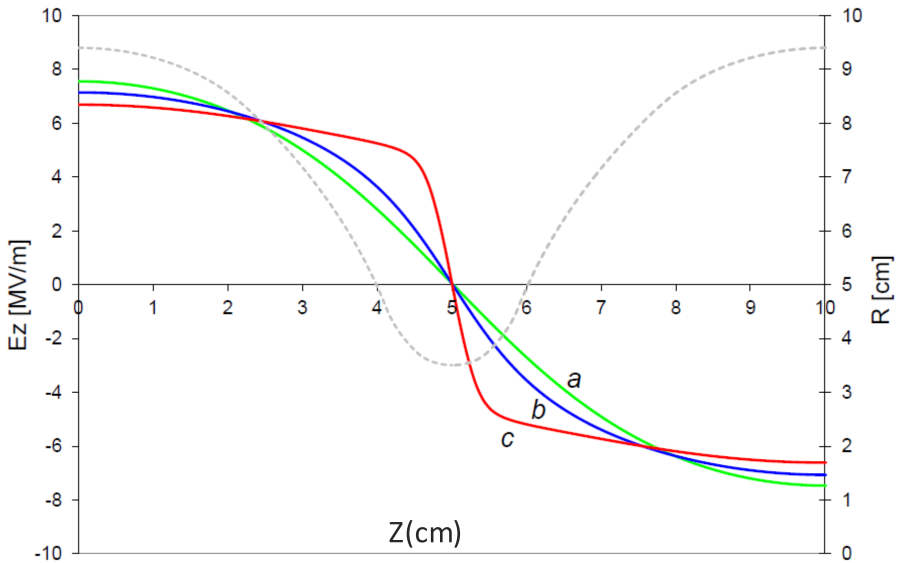
$$E_z(a,z) \sim \sum A_{2n} \cos(2nk_0 z/\beta);$$

$$E_z(0,z) \sim \sum A_{2n} \cos(2nk_0 z/\beta)/I_0[ak_0 a(4n^2/\beta^2-1)^{1/2}] = \sum B_{2n} \cos(2nk_0 z/\beta)$$

$$B_{2n} = A_{2n} I_0[ak_0 a(4n^2/\beta^2-1)^{1/2}];$$

for example for $\beta = 1$ $B_0 = A_0$ *and* $B_{2n} \approx A_{2n} \exp(-2nk_0 a) << B_0$

$E_z(0,z) \sim A_0 \cos(k_0 z)$ – sinusoidal distribution on the axis! Valid for $\beta<1$.



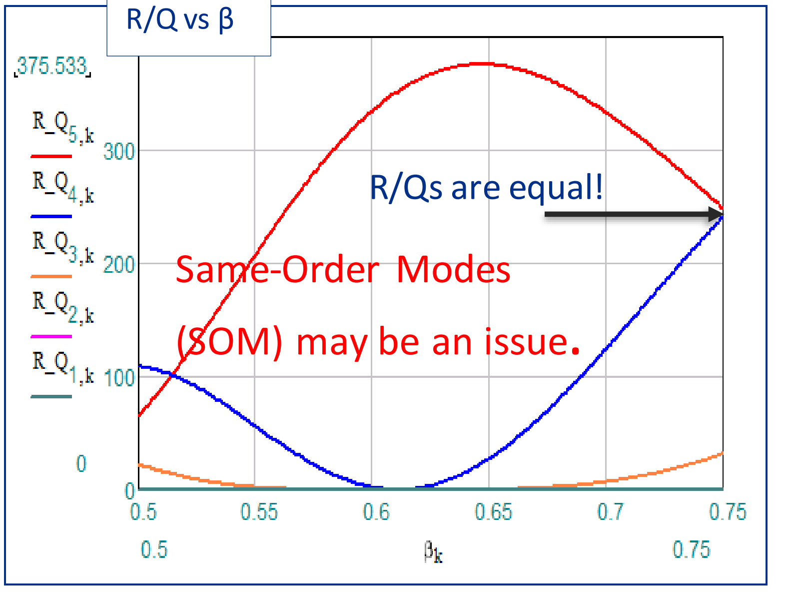
Geometry of an iris of a CEBAF multi-sell cavity (gray line). Longitudinal electric field at a different radial position: r = 0 cm (green line), r = 2.5 cm (blue line), r = 3.45 cm (red line). Fields are normalized to 4 MeV/m accelerating gradient.

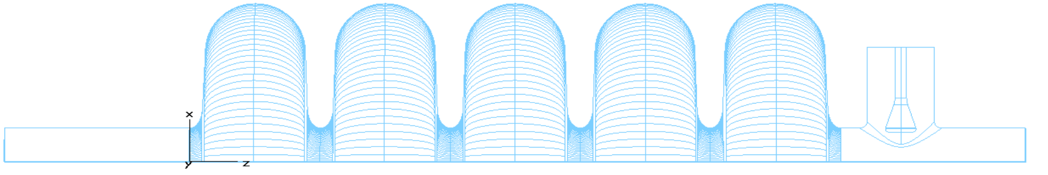
- Field at the aperture close to rectangular
- Field on the axis is close to sinusoidal

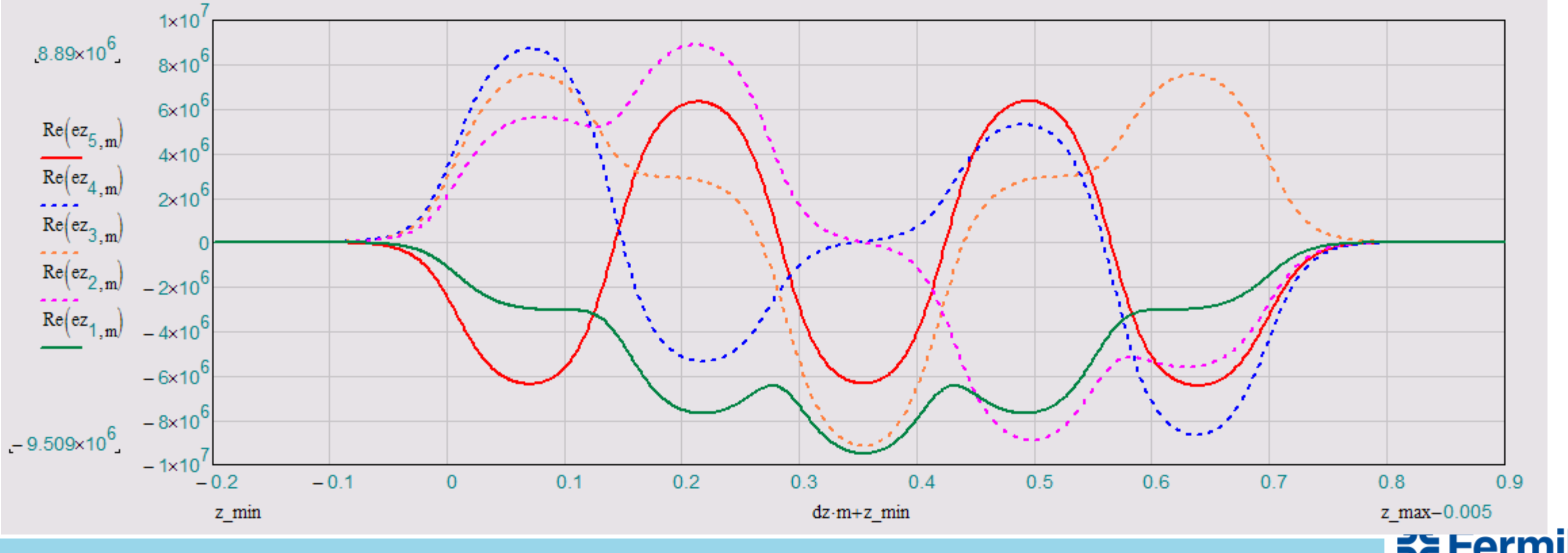
Fermilab

PIP II β_G=0.61, 650 MHz elliptical cavity:

Mode	Freq [GHz]	$(R/Q)_{opt}[\Omega]$	β_{opt}
0	0.6456	0.5	>0.75
¼ π	0.6468	0.4	0.69
½ π	0.6483	32.1	>0.75
¾ π	0.6495	241.0	>0.75
π	0.6500	375.5	0.65

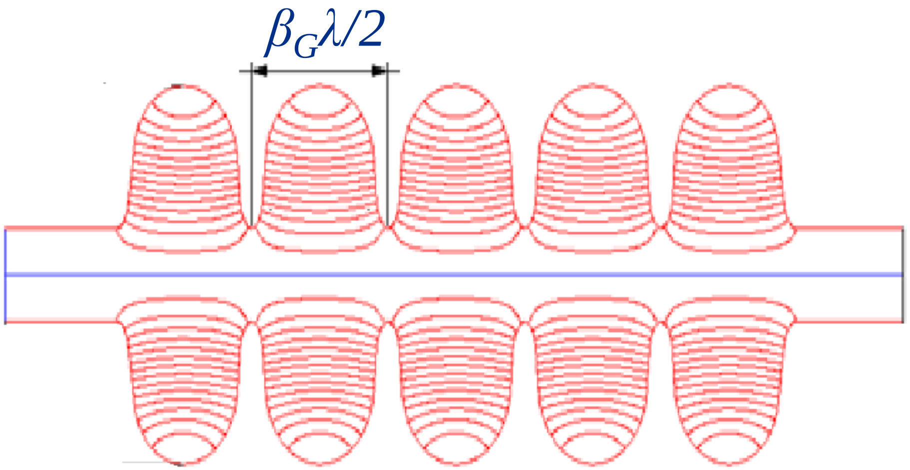






Parameters of a multi-cell cavity:

- "Geometrical beta": $\beta_G = 2l/\lambda$,
  l is the length of a regular cell,
  λ is wavelength.



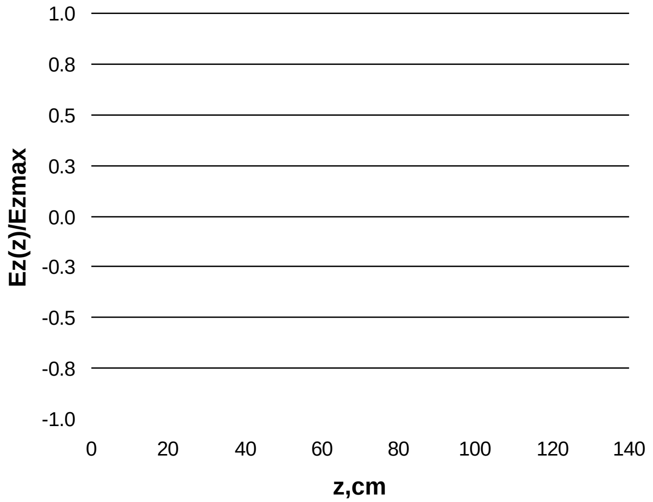
- $R/Q = V^2/\omega U$, V is the energy gain per cavity (in optimal acceleration phase) , $V=V(\beta)$; ω – cyclic operation frequency; U is EM energy stored in a cavity; R/Q is a function of β, as well as V. R/Q is the same for geometrically similar cavities. Decreases when the cavity aperture a increases.
- "Optimal β": value of β, where V (and R/Q) is maximal.
- Acceleration gradient: $E=V/L_{eff}$, $L_{eff} = n\beta_G \lambda/2$ – effective length, n is the number of cells.

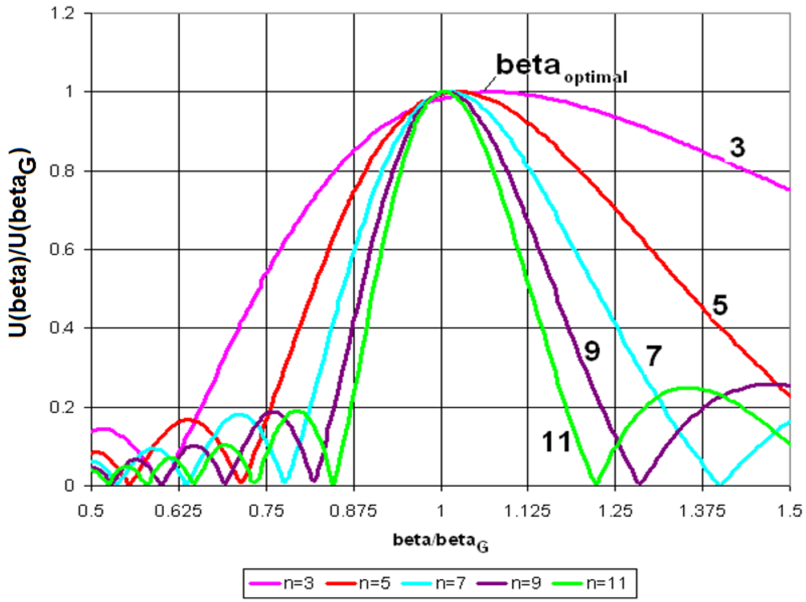
Parameters of a multi-cell cavity (cont)

- Surface electric field enhancement: $K_e = E_{peak}/E$, E_{peak} is maximal surface electric field.
- Surface magnetic field enhancement: $K_m = B_{peak}/E$, B_{peak} is maximal surface magnetic field.
- Unloaded quality factor: $Q_0 = \omega W/P_{loss}$, P_{loss} – surface power dissipation.
- G-factor: $G=Q_0*R_s$, R_s is the surface resistance. G is the same for geometrically similar cavities. At fixed gain the losses are proportional to $G*(R/Q)$.
- Loaded quality factor: $Q_{load} = \omega W/P$, $P= P_{loss} +P_{load}$; P_{load} – power radiated through the coupling port.
- **Coupling:** $\boldsymbol{K=2(f_\pi-f_0)/(f_\pi+f_0)}$**,**

Multi-cell cavity

A multi-cell SRF elliptical cavity is designed for particular $\beta = \beta_G$, but accelerates in a wide range of particle velocities; the range depends on the number of cells in the cavity N. Field distribution for the tuned cavity has equal amplitudes for each cell; longitudinal field distribution for considerably large aperture is close to sinusoidal (see slide 10):





$$\frac{\boldsymbol{V}(\beta)}{\boldsymbol{V}(\beta_{\boldsymbol{optimal}})} = \frac{2\beta}{\pi \boldsymbol{N}} \left(\frac{\sin(\frac{\pi \boldsymbol{N}(\beta - \beta_G)}{2\beta})}{\beta - \beta_G} - (-1)^n \frac{\sin(\frac{\pi \boldsymbol{N}(\beta + \beta_G)}{2\beta})}{\beta + \beta_G} \right)$$

$$\beta_{optimal} \approx \beta_G \left(1 + \frac{6}{\pi^2 N^2} \right)$$

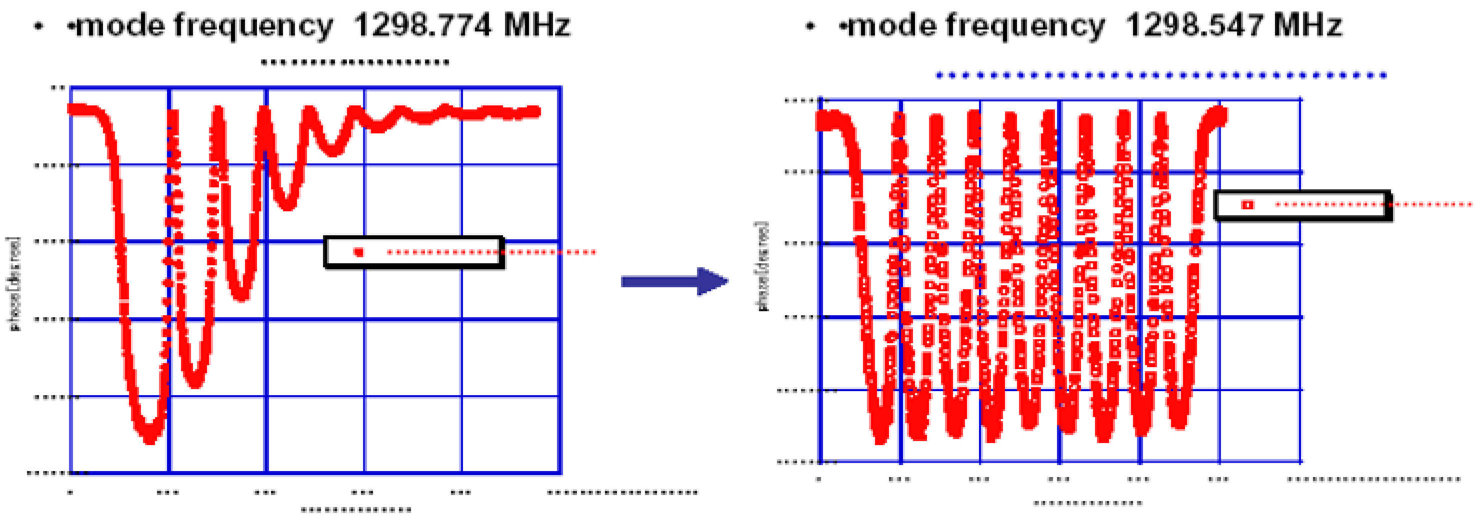
V is the energy gain per cavity.

The cavity containing more cells provides effective acceleration in more narrow particle velocity range!

Why SW π – mode?

Cavity tuning:

- Compensation of the errors caused by manufacturing
- Compensation of the errors caused by cool-down.
- Field flatness
- Tuning the operating mode frequency to resonance.



Field flatness = 0.1 %
(as delivered to KEK)

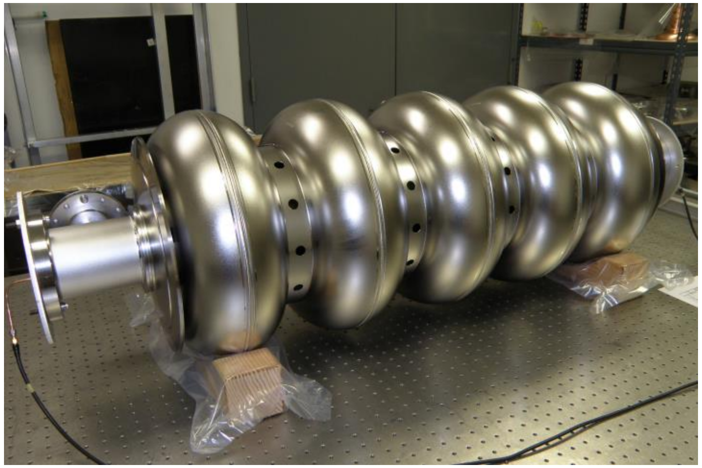
Field flatness = 98 %
(after pre-tuning)

Field flatness in ILC – type cavity before and after pre-tuning.

Elliptical cavities:

INFN Milano, 700 MHz, $\beta_G = 0.5$

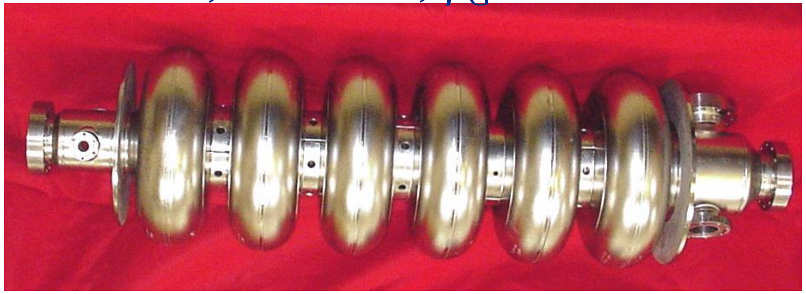
PIP II, 650 MHz, $\beta_G = 0.9$



SNS, 805 MHz, $\beta_G = 0.61$

XFEL, 1300 MHz, $\beta_G = 1$

SNS, 805 MHz, $\beta_G = 0.81$



XFEL, 3900 MHz, $\beta_G = 1$

Multi-cell cavity is not effective for low β*:

During acceleration a particle interacts with cylindrical EM waves,

$$E_z(r,z,t) \sim J_0(k_r r)\exp(ik_z z - i\omega t),$$

where $J_0(x)$ is Bessel function.

For acceleration, the cylindrical wave should be synchronous, i.e., it should have phase velocity equal to the particle velocity:

$\omega/k_z = v = c\beta$, or $k_z = \omega/\beta c = k/\beta$ (k is full wavenumber, $k=\omega/c = 2\pi/\lambda$)

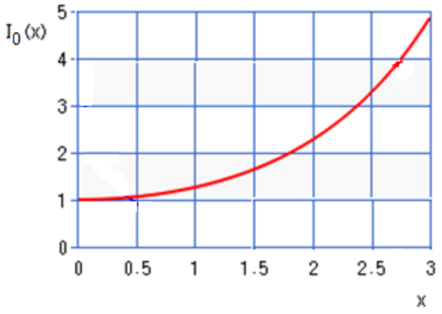
On the other hand, for EM wave one has:

$(k)^2 = (k_r)^2 + (k_z)^2$ or $(k_r)^2 = (k)^2 - (k_z)^2 = (k)^2(1-1/\beta^2)$. Thus, $k_r = ik/\beta\gamma$.

- In ultra-relativistic case $k_r \to 0$
- In non-relativistic case $k_r = ik/\beta$ and the synchronous cylindrical wave is

$$E_z(r,z,t) \sim I_0(2\pi r/\lambda\beta)\exp(ikz/\beta - i\omega t),$$

$I_0(x)$ is modified Bessel function.



*Lecture 7, slide 25

Multi-cell cavity is not effective for low β:

For small β

$I_0(r) \sim exp(2\pi r/\lambda\beta)$.

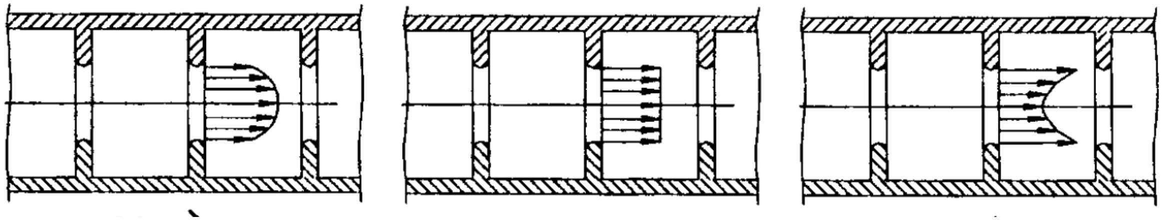
Synchronous EM is concentrated on the cavity periphery, not on the axis! Consequences:

- Small (R/Q):

  $(R/Q) \sim exp(-4\pi a/\lambda\beta)$, a is the cavity aperture radius;
- High K_e

  $K_e \sim exp(2\pi a/\lambda\beta)$;
- High K_m.

  $K_m \sim exp(2\pi a/\lambda\beta)$.



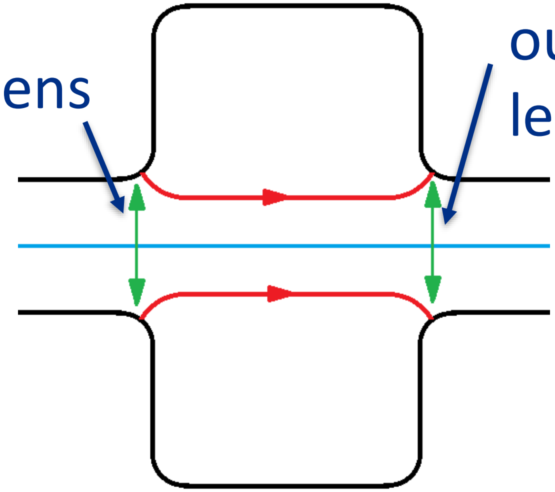
$v>c$ $v=c$ $v<c$

Fermilab

Multi-cell cavity is not effective for low β*:

RF cavity provides the beam focusing,

$$\frac{1}{F} \sim \frac{\pi}{\beta^3\gamma^3}\frac{V}{U_0}\frac{1}{\lambda}\sin(\varphi_s)$$



- For $\varphi_s < 0$ (necessary for longitudinal stability) the cavity provides defocusing!
- Defocusing:

  $\sim 1/\beta^3$;

  $\sim 1/\lambda$.

Defocusing should be compensated by external focusing elements,

-solenoids (low energy);

-quads (high energy).

For small β longer RF wavelength (lower frequency) should be used.

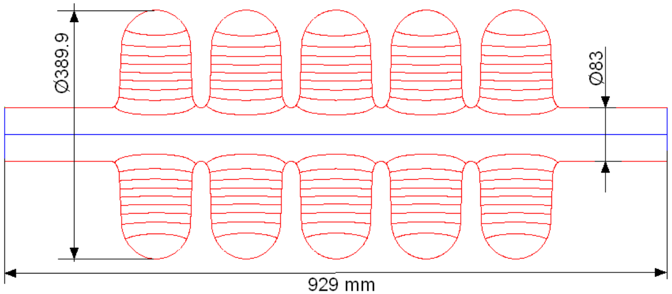
But axisymmetric cavity has very big size, D~3/4 λ

For small β other types of cavities should be used!

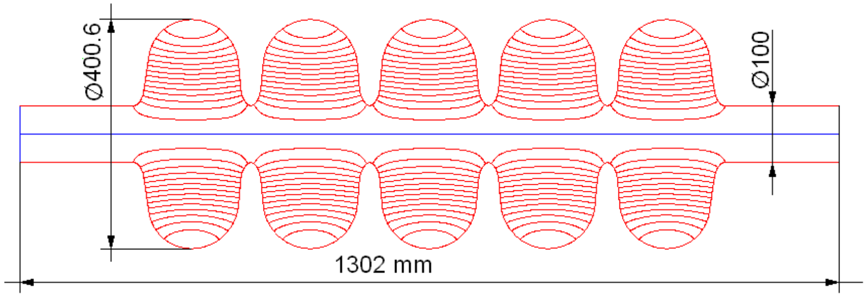
**Lecture 7*

Parameters of an elliptical cavity (cont)

Example for the 650 MHz cavities for PIP II



LB650 (β_G=0.61)



HB650 (β_G=0.9)

Parameter		LH650	HB650
β_G		0.61	0.9
$\beta_{optimal}$		0.65	0.94
Cavity Length = $n_{cell} \cdot \beta_{geom} \lambda/2$	mm	703	1038
R/Q	Ohm	378	638
G-factor	Ohm	191	255
K_e		2.26	2.0
K_m	mT/(MeV/m)	4.22	3.6
Max. Gain/cavity (on crest)	MeV	11.7	17.7
Acc. Gradient	MV/m	16.6	17
Max surf. electric field	MV/m	37.5	34
Max surf. magnetic field,	mT	70	61.5
Q_0 @ 2K	$\times 10^{10}$	2	3
P_{2K} max	[W]	24	24

Summary:

- ❑ Single – cell cavities are not convenient in order to achieve high acceleration: a lot of couplers, tuners, etc; multi-cell π–mode elliptical cavities are used in SRF accelerators;
- ❑ Why π–mode?

- The SW modes except π have small acceleration efficiency because most of cavities have small field;
- Bi-periodic structure π/2-mode does not work because it is prone to multipacting in the empty coupling cells and difficult for manufacturing (different cells) and processing (narrow coupling cells).
- π -mode structure is simple, easy for manufacturing and processing.

- ❑ Elliptical cavities are used to mitigate multipacting;
- ❑ End cells have the same length as regular ones, but a bit different shape to keep field flatness for operation π–mode.
- ❑ Range of acceleration efficiency strongly depends on the number of cells: cavities with smaller number of cells operate in wider β range.
- ❑ Elliptical cavities are not effective for low particle velocity.

Chapter 8.

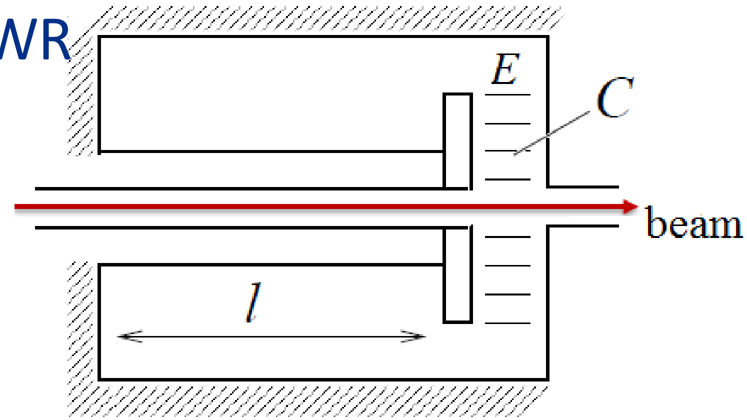
SRF Cavities for Low β Accelerators.

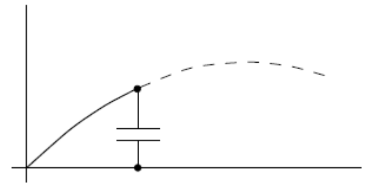
a. Why TEM-type cavities work at low particle velocities;
b. Types of TEM cavities;
c. Velocity range of TEM-type cavities.

RF cavity types

Quarter-wave resonator (QWR) concept:

One-gap QWR





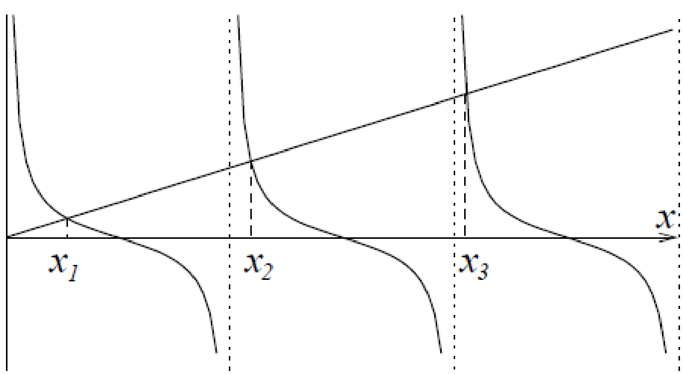
Resonance:

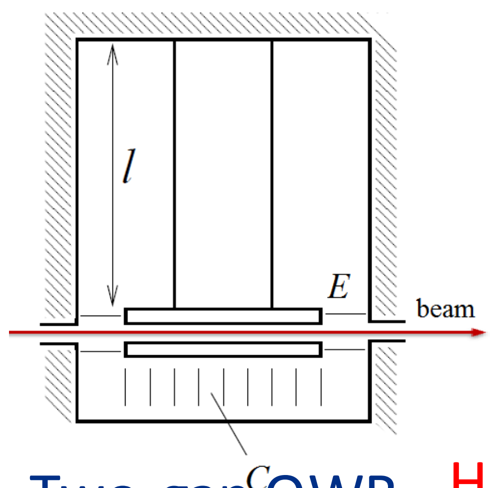
$$\frac{1}{i\omega C} + iZ_c tan\left(\frac{\omega l}{c}\right) = 0 \quad \text{or} \quad cot\left(\frac{\omega l}{c}\right) = \omega C Z_c.$$

Z_c here is the coaxial impedance

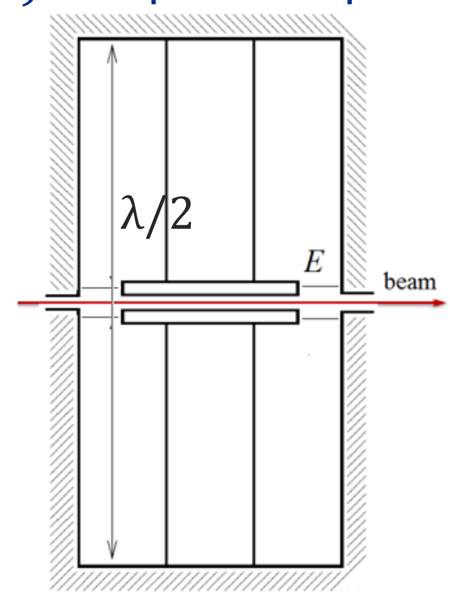
$$\frac{\omega l}{c} = x, \frac{CZ_c c}{l} = A \rightarrow \cot(x) = Ax.$$

Compact ($L \gtrsim \lambda/4$) compared to pillbox ($D \lesssim 3/4\lambda$).





Two-gap QWR



Half-wave resonator (HWR)

Fermilab

RF cavities for low β:

TEM-like cavities:
- Split-ring resonator;
- Quarter-wave resonator;
- Half-wave resonator;
- Spoke resonator.

Split-ring

QWR

HWR

SSR (single-spoke)

- Narrow acceleration gap (~βλ) allows concentrate electric field near the axis;
- Aperture ~ 0.02-0.03λ allows acceptable field enhancement;
- Number of gaps in modern cavities is 2 for small beta which allows operation in acceptably wide beta domain. For $\beta > 0.4$ multi-gap cavities are used –double- and triple-spoke resonators;
- Focusing elements (typically, solenoids) are placed between the cavities.

RF cavities for low β:

Quarter-wave resonator:

•Allows operate at very low frequency ~50 MHz, (and thus, low beta) having acceptable size;

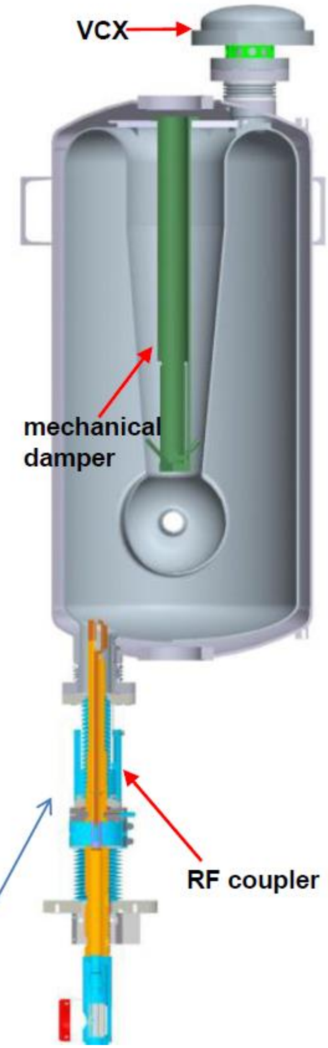
•Has a good (R/Q);

•Low cost and easy access.

But:

•Special means needed to get rid of dipole and quadrupole steering, and

•Provide mechanical stability

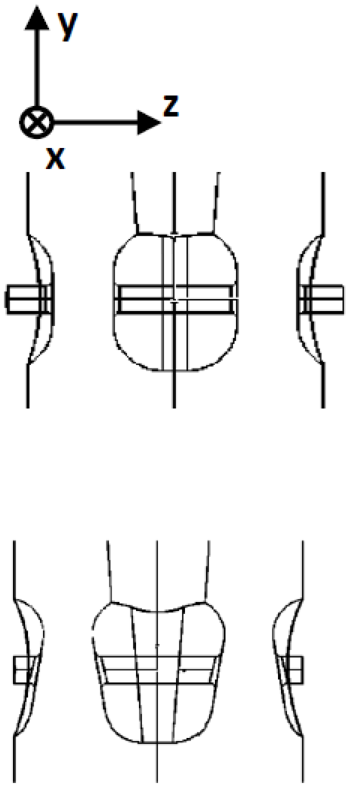


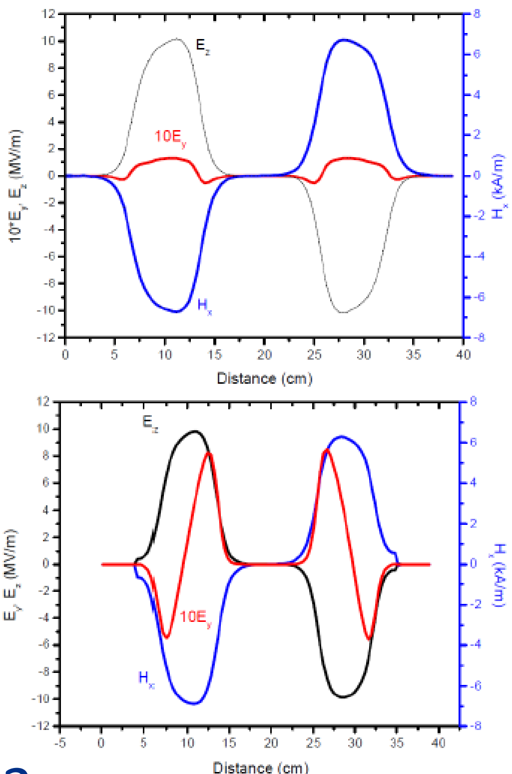
beta=0.14, 109.125 MHz QWR(**Peter N. Ostroumov)**

RF cavities for low β

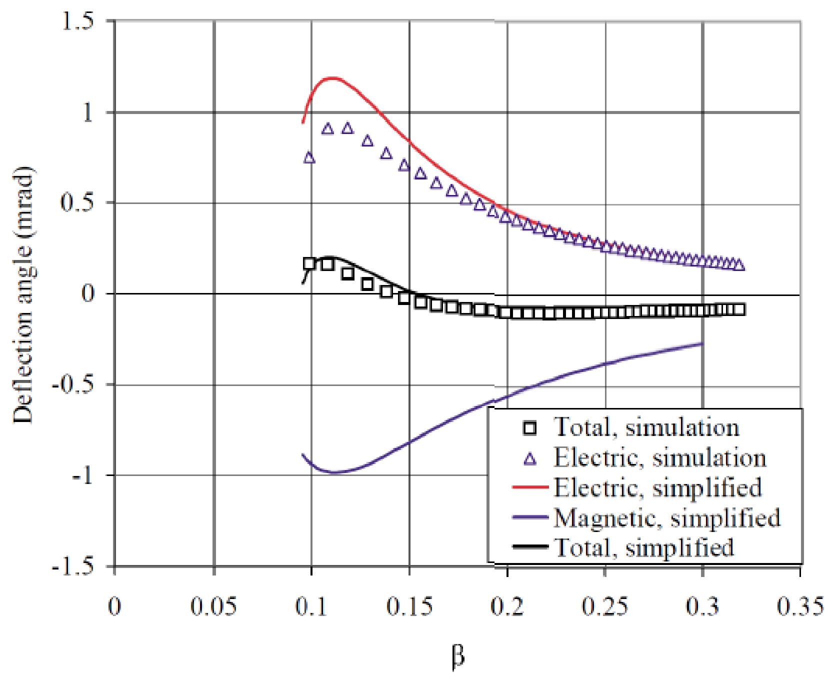
Beam Steering in Quarter-wave Cavities*

- Beam steering due to unavoidable magnetic field on the beam axis.
- One remedy: The vertical field E_y, normally small, may be modified by the cavity geometry to cancel magnetic steering due to H_x.





$$\Delta p_y \sim \frac{1}{\beta^2} \int_{L/2}^{-L/2} E_y\, cos(kz+\varphi) + \beta c \cdot B_x\, sin(kz+\varphi) dz$$

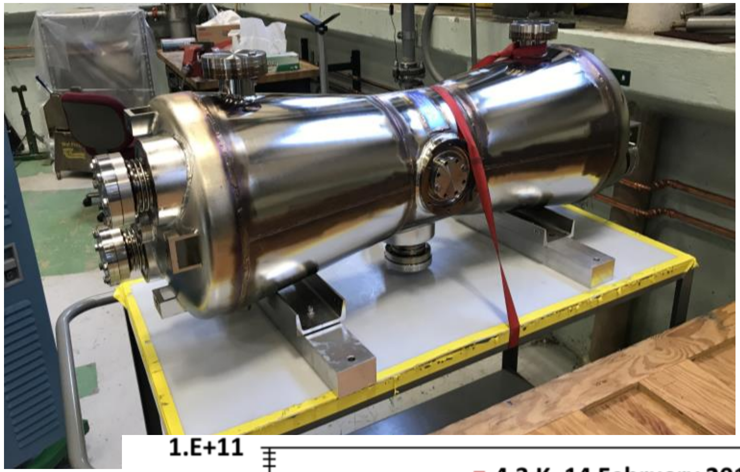


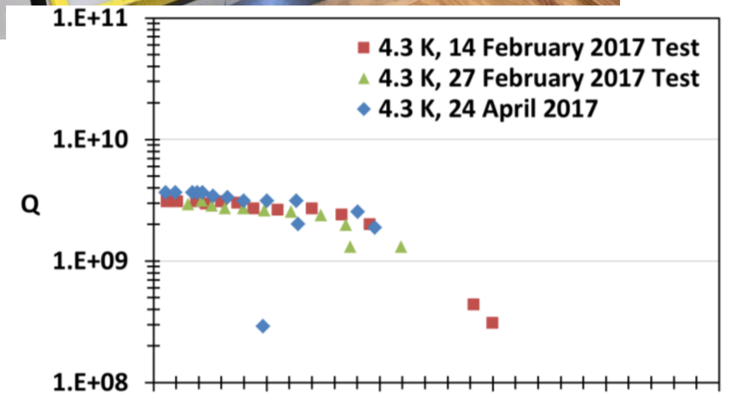
M. Kelly, 2013

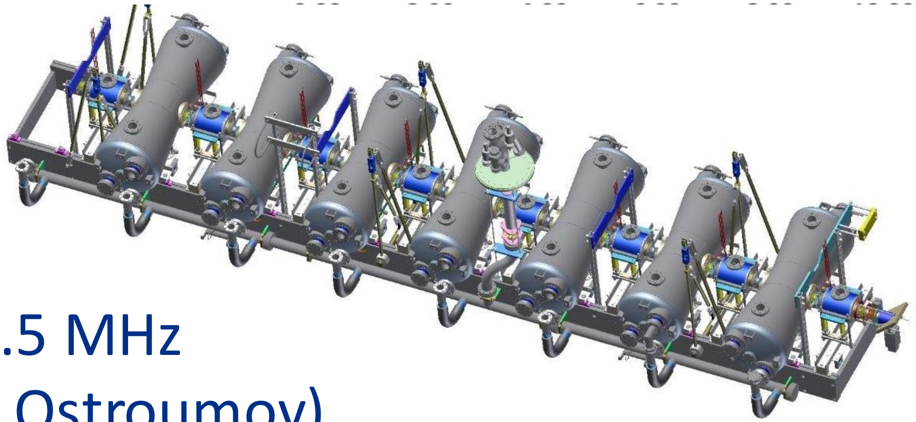
RF cavities for low β:

Half-wave resonator (HWR):
•No dipole steering;
•Lower electric field enhancement;
•High performance;
•Low cost;
•Best at ~200 MHz.

But:
• Special means needed in some cases to get rid of quadrupole effects;
• Two times lower R/Q

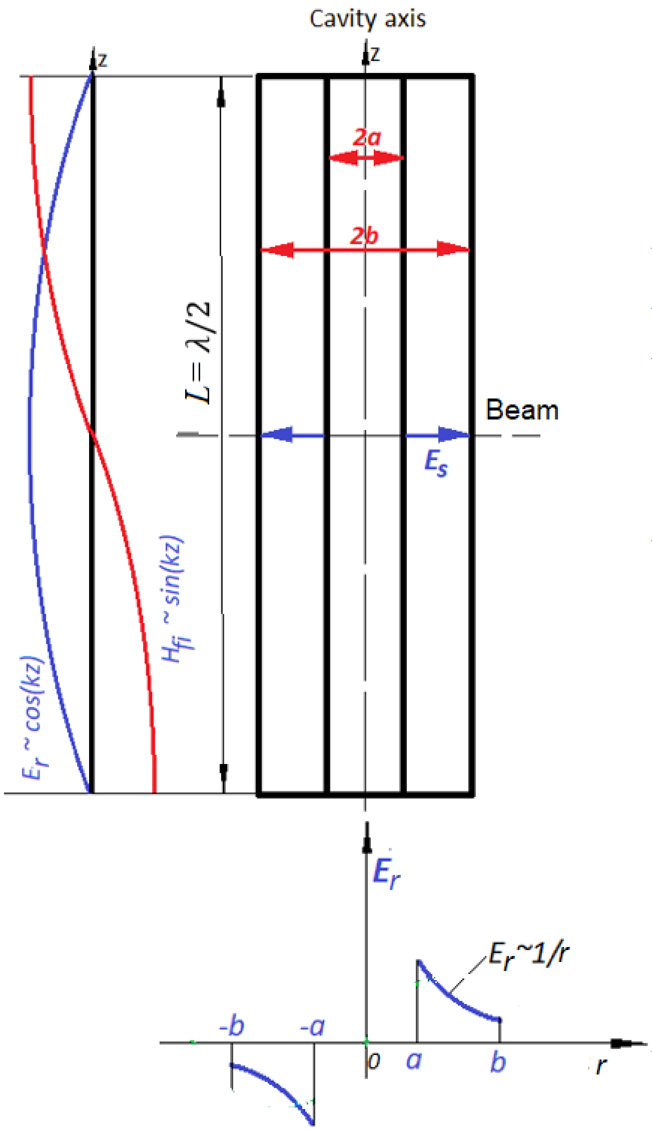






PIP II HWR cavity, 162.5 MHz
(M.Kelli, Z. Conway, P. Ostroumov)

Ideal HWR



- The cavity is a TEM coaxial line shortened at $z=\pm L/2$
- Electric field in a coaxial line:

$$E_r(r,z) = \frac{C}{r}\cos(kz),\ a \le r \le b;$$
$$E_r(r,z) = 0, \qquad r < a\,;$$
$$E_r(r,z) = 0,\ r > a\,, k = \frac{\omega}{c}$$

$$E_r(r,z) = \frac{C}{r}\cos(kz),$$
$$U = \int_a^b E_r(r,0)\,dr = C \cdot ln\left(\frac{b}{a}\right)$$
$$\to C = \frac{U}{ln\left(\frac{b}{a}\right)}$$
$$E_r(r,z) = \frac{U}{ln\left(\frac{b}{a}\right)} \cdot \frac{1}{r}\cos(kz),$$

- Magnetic field:

From Maxwell equations:

$$H_\varphi(r,z) = \frac{i}{\omega\mu_0} \cdot \frac{\partial E_r}{\partial z} = \frac{iC}{Z_0 r}\sin(kz) =$$

$= \frac{iU}{2\pi Z_c r}\sin(kz)\,;\ Z_c = \frac{1}{2\pi} Z_0 ln\left(\frac{b}{a}\right)$ - the line impedance; $Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi Ohm$

- Resonance frequency:

$kL = \pi \to k = \frac{\pi}{L} \to \omega = \frac{\pi c}{L} \to f = \frac{c}{2L},$ c is speed of light.

- Stored energy:

$$W = \frac{\mu_0}{2}\int |H|^2 dV = \frac{\pi}{4}\frac{U^2}{\omega Z_c}$$

- Ohmic loss:

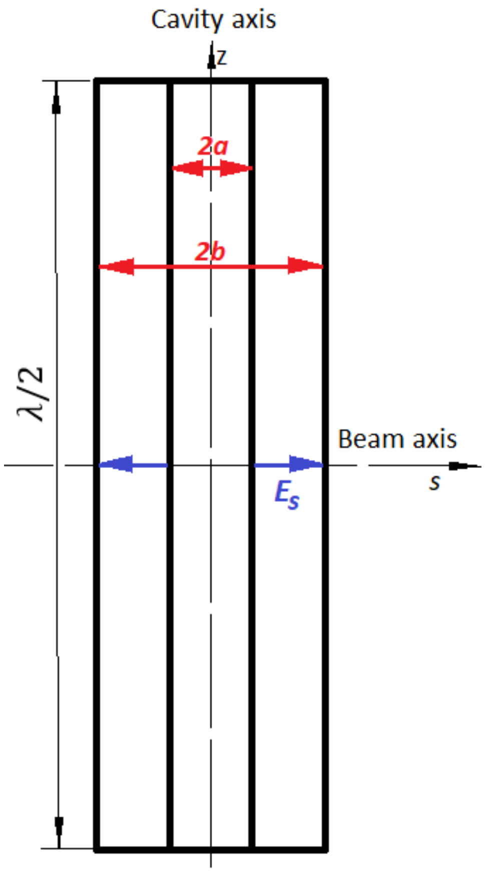
$$P = \frac{1}{2}\oint R_s H^2 dS = \frac{R_s U^2}{8\pi Z_c^2}\left[L\left(\frac{1}{b} + \frac{1}{a}\right) + 4ln\left(\frac{b}{a}\right)\right]$$

- Unloaded quality factor:

$$Q_0 = \frac{\omega W}{P}.$$

Fermilab

Ideal HWR



- Acceleration field on the beam axis: $E_s(s) = \frac{U}{ln\left(\frac{b}{a}\right)} \cdot \frac{1}{s}$
- Acceleration voltage:

$$V = \frac{2U}{ln\left(\frac{b}{a}\right)} \int_a^b \frac{\sin\left(\frac{ks}{\beta}\right)}{s} ds = \frac{2U}{ln\left(\frac{b}{a}\right)} \left[Si\left(\frac{kb}{\beta}\right) - Si\left(\frac{ka}{\beta}\right) \right]$$

where $Si(x) = \int_0^x \frac{\sin(x)}{x} dx$. We have two gaps → factor "2" in the nominator.

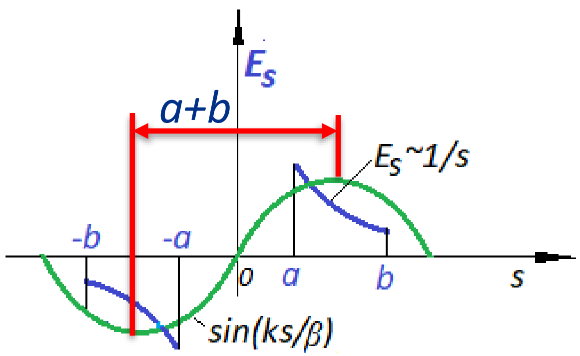
Optimal acceleration:

$$\frac{dV}{d\beta} = 0 \rightarrow sin\left(\frac{kb}{\beta}\right) - sin\left(\frac{ka}{\beta}\right) = 2sin\left(\frac{k(b-a)}{2\beta}\right) cos\left(\frac{k(a+b)}{2\beta}\right) = 0$$

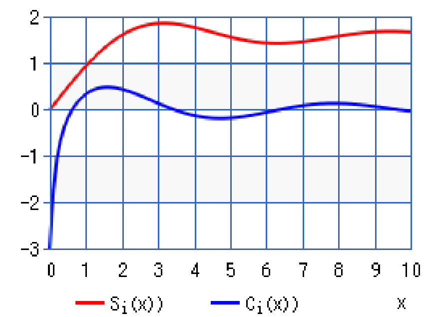
$$\rightarrow \frac{k(a+b)}{2\beta} = \frac{\pi}{2} \quad \rightarrow \boxed{\frac{a+b}{2} = \frac{\beta\lambda}{4}}$$

- "Effective cavity length": $L_{eff} = \beta\lambda$

  (compare to multi-cell elliptical cavity: $L_{eff} = \frac{\beta\lambda}{2} n$, n is number of gaps)



Si(x) calculator:
https://keisan.casio.com/exec/system/1180573420



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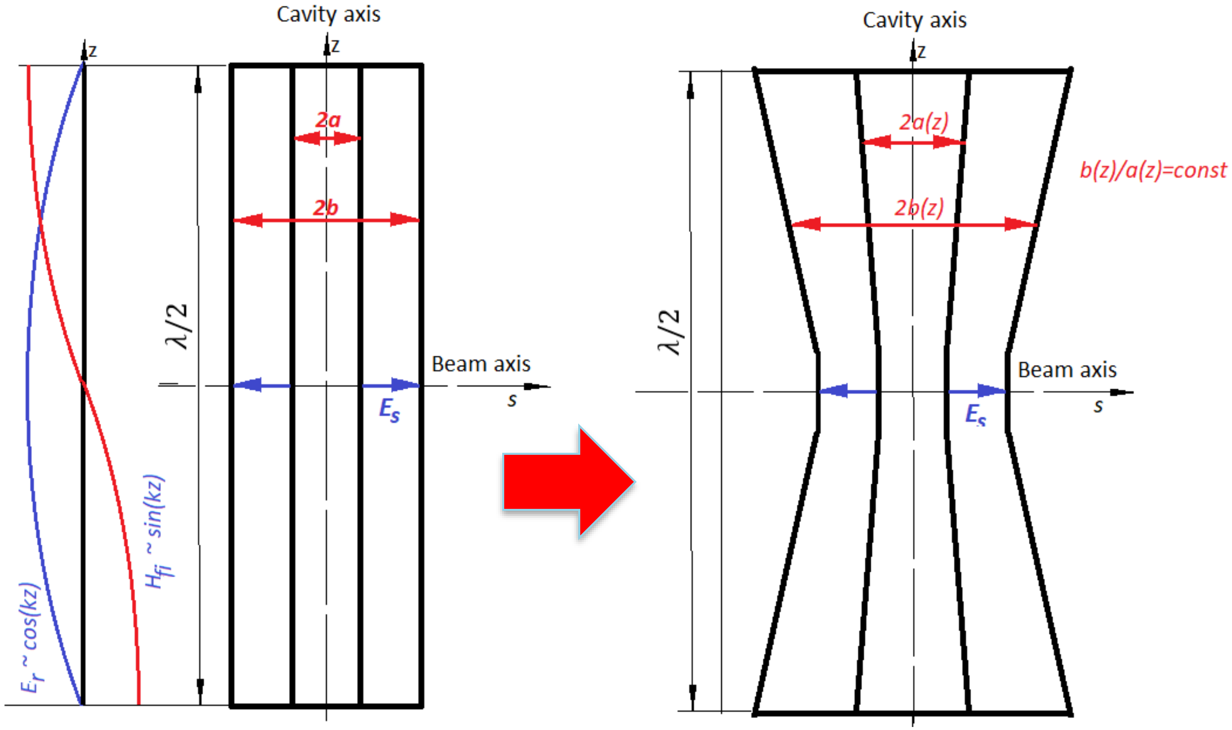
Loss reduction in HWR: conical HWR*



Increase the cavity transverse size at the ends keeping about the same ratio *b(z)/a(z)* helps to decrease loss without change of the *R/Q*, which is determined in high degree by Z_c

$$H_\varphi(r,z) = \frac{I}{2\pi r}\sin(kz) = \frac{U}{2\pi Z_c r}\sin(kz);$$

$$Z_c = \frac{Z_0}{2\pi}\ln\left(\frac{b(z)}{a(z)}\right) = \text{const. } b(z) = b(0) + \frac{b(L/2)-b(0)}{L/2}z;\ a(z) = a(0) + \frac{a(L/2)-a(0)}{L/2}z$$

$$P = \frac{1}{2}\oint R_s H^2 dS = \frac{R_s U^2}{8\pi Z_c^2}\left[4\int_0^{L/2} sin^2(kz)\left(\frac{1}{b(z)}+\frac{1}{a(z)}\right)dz + 4ln\left(\frac{b(L/2)}{a(L/2)}\right)\right] \sim \frac{R_s U^2}{8\pi Z_c^2}\left[L\left(\frac{1}{b(L/2)}+\frac{1}{a(L/2)}\right) + 4ln\left(\frac{b(L/2)}{a(L/2)}\right)\right]$$

$$\frac{P_{ideal}}{P_{con}} = \frac{G_{con}}{G_{ideal}} \lesssim \frac{a(L/2)}{a(0)};\ \frac{K_{Mcon}}{K_{Mideal}} \lesssim \frac{a(0)}{a(L/2))};\ \frac{K_{Econ}}{K_{Eideal}} \sim 1$$

If $\frac{a(L/2)}{a(0)} \sim 3$, one may expect reduce loss and surface magnetic field 2 – 3 times.

*P. Ostroumov, E. Zaplatin

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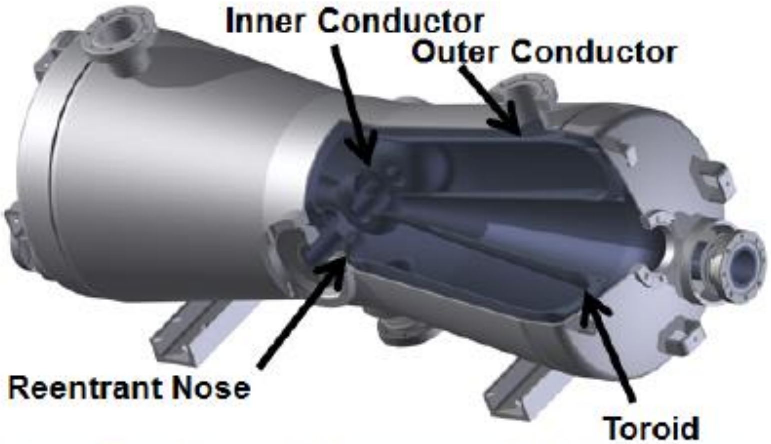
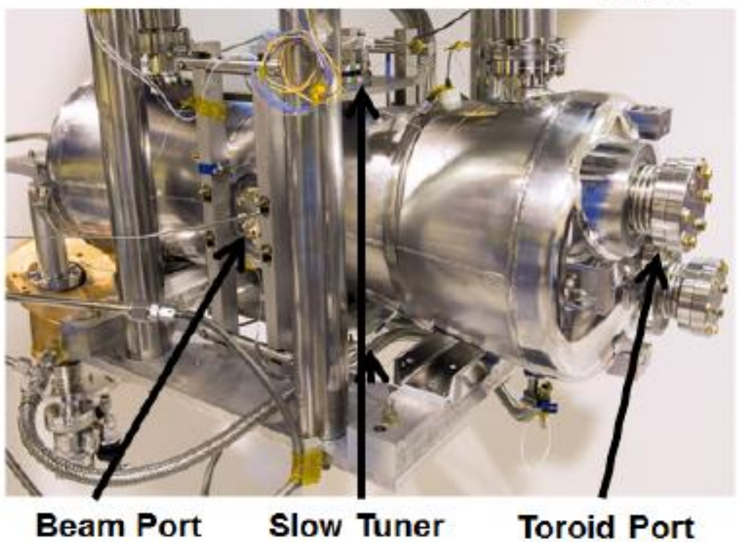
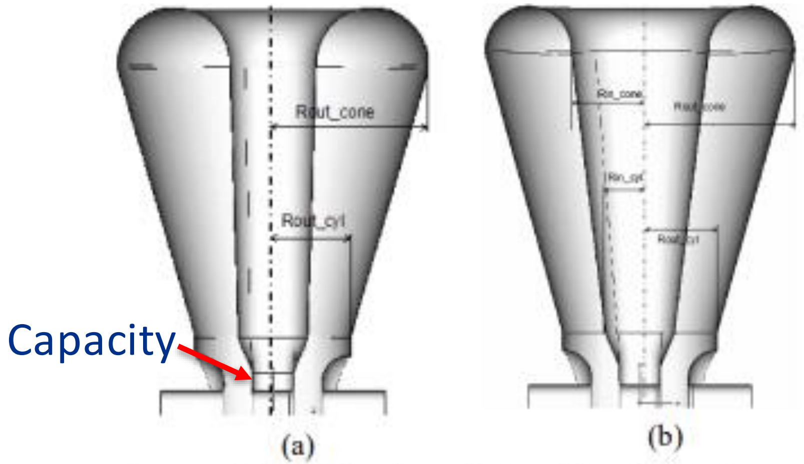
Loss reduction in HWR: conical HWR*

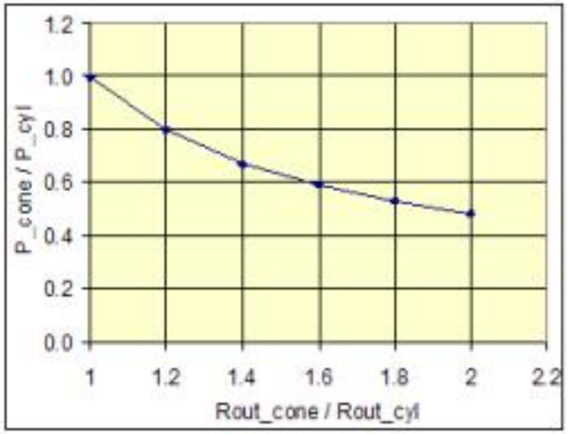




PIP II HWR
(P. Ostroumov)



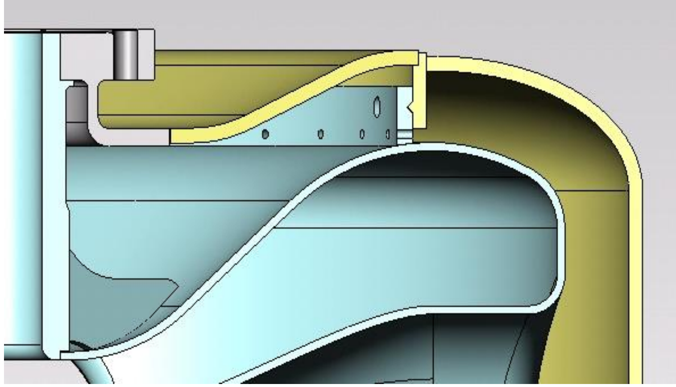
HWR with enlarged outer (a) and central conductor (b) dome diameters.



Power dissipation in conical HWR relative to cylindrical shape.

RF cavities for low β:

Spoke resonator

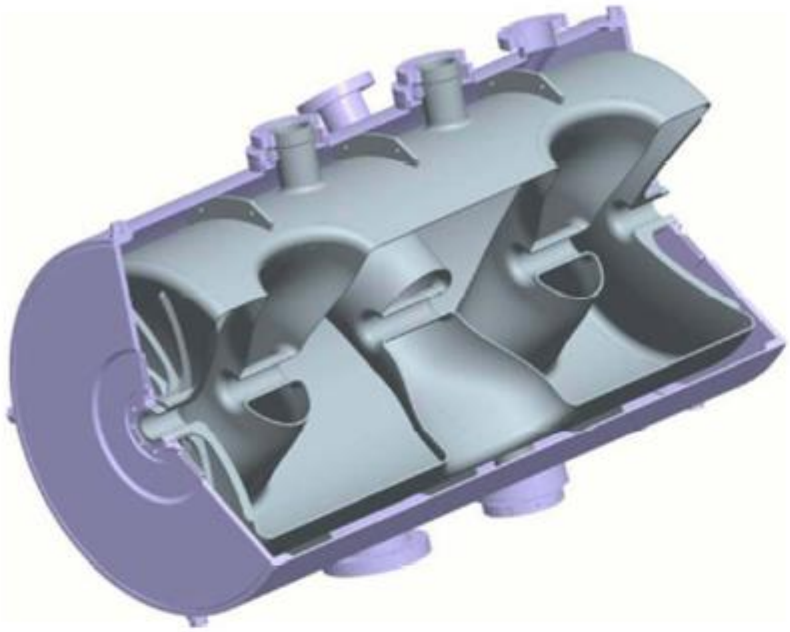


Mechanical coupling of the cavity to the He vessel in order to improve mechanical stability.

FNAL 325 MHz SSR1 cavity layout and photo. β=0.22

RF cavities for low β:

Multi-spoke resonators

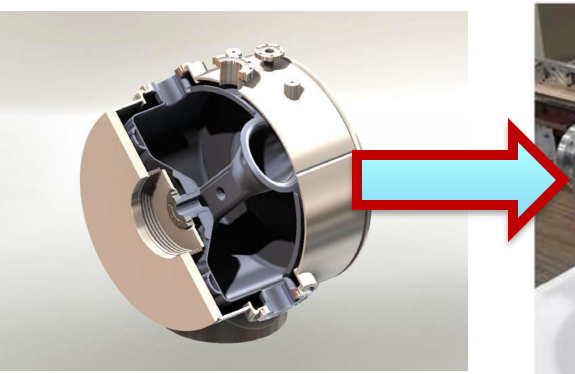


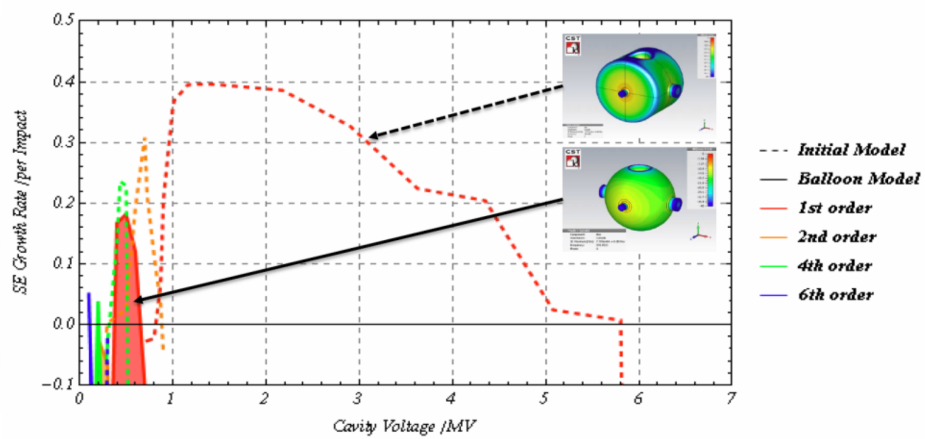
Triple-spoke cavity

345 MHz, β=0.4,
3-gap spoke cavity
for ion beam acceleration
ANL

RF cavities for low β:

- TEM-type cavities are prone to multipacting;
- Elliptical cavities have much better performance (MP electrons drift towards the axis)
- Idea (R. Laxdal): combine SR and elliptical cavity →balloon cavity.



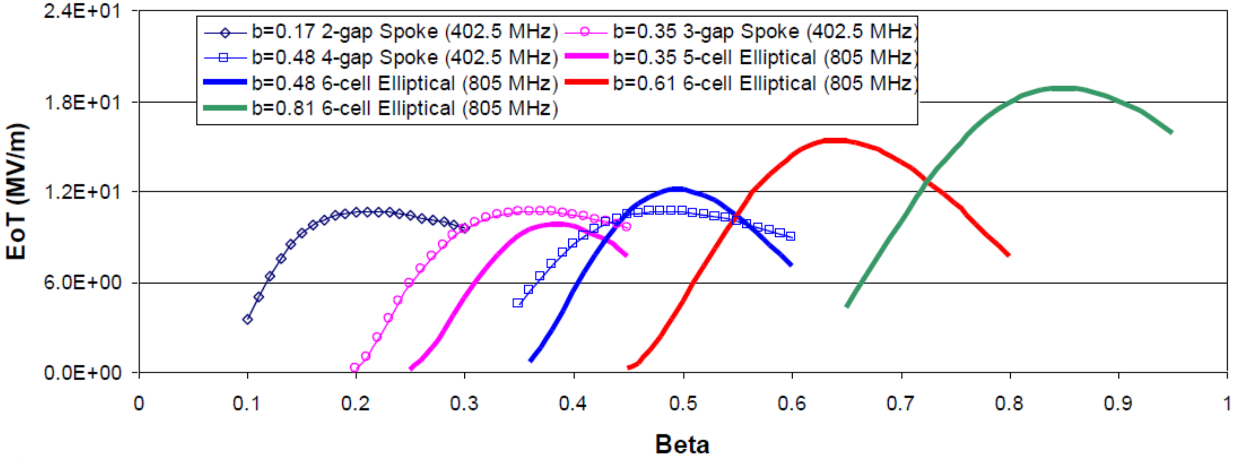


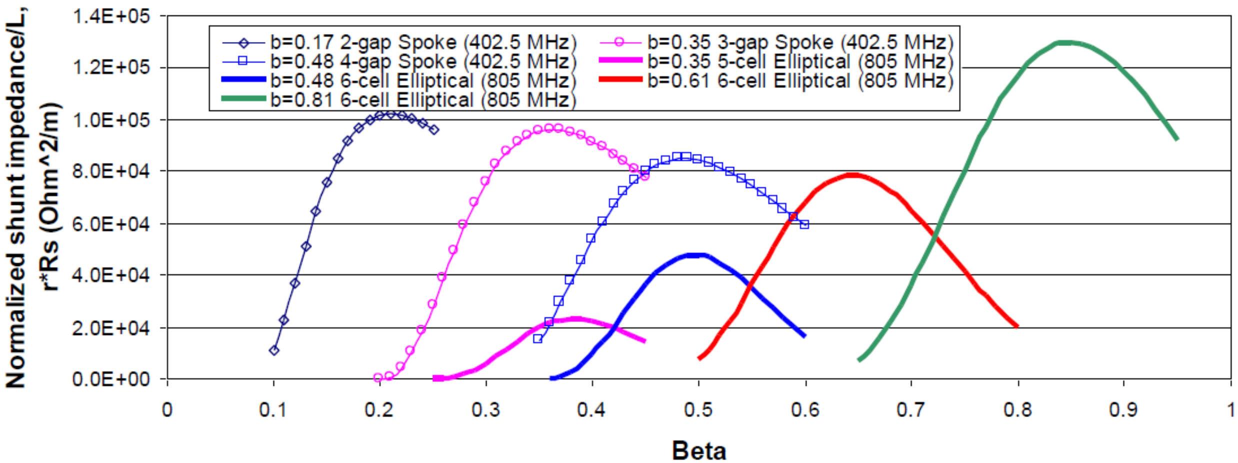
- Balloon cavity is successfully tested:
condition time reduced from ~10 hours to ~30 mins!

Why not multi-spoke for β>0.5?

Comparison of RF properties (elliptical cavity versus spoke cavity)*

Spoke cavities (402.5 MHz) and elliptical cavities (805 MHz) are optimally designed under the same criteria: E_{peak} ≈40 MV/m and B_{peak} ≈ 85 mT. Here EoT is gradient, and r*Rs is R/Q*G per unit length.

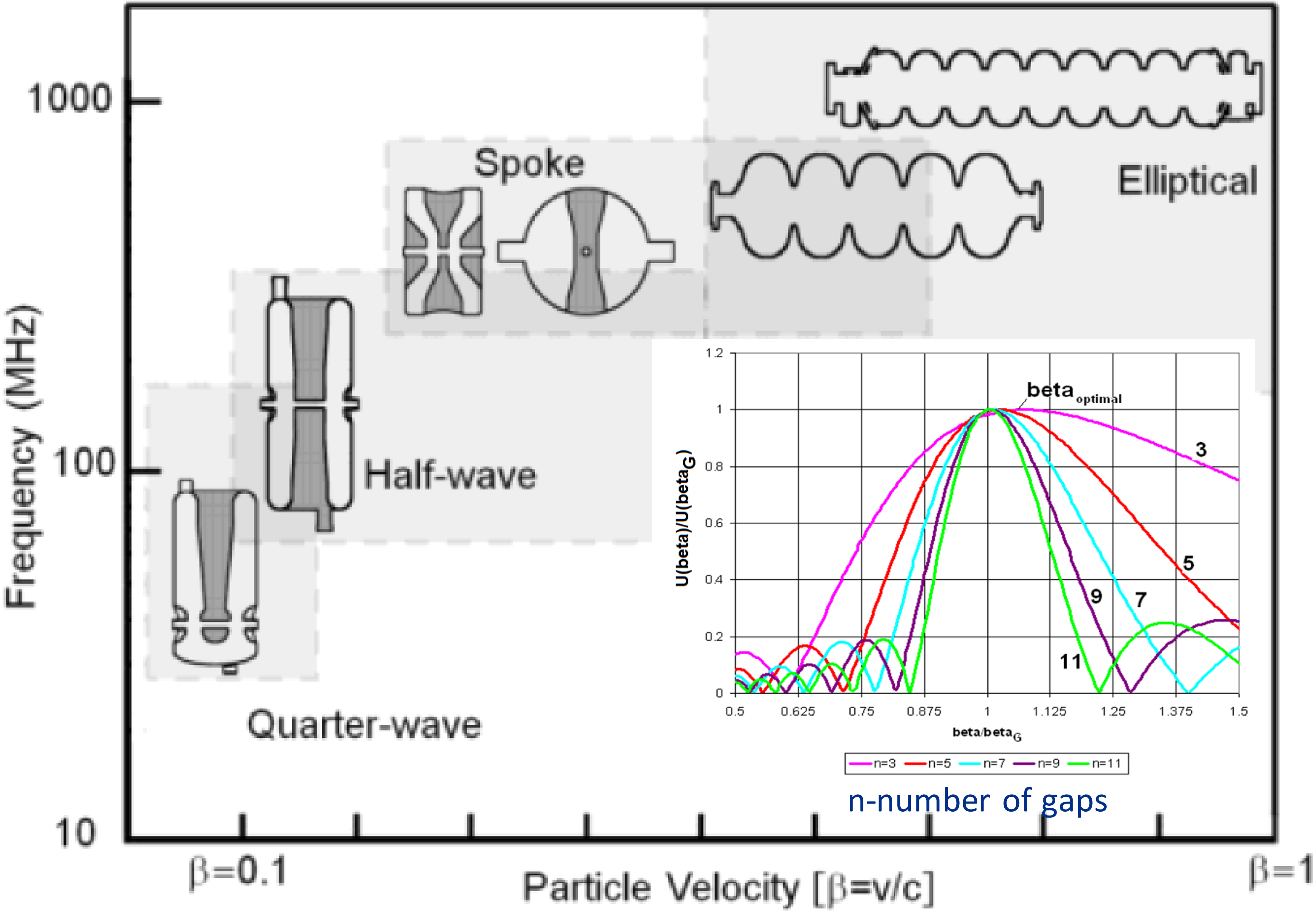




For β>0.5-0.6 elliptical cavity is preferable!

*Sang-Ho Kim, Mark Doleans, USPAS, January 2013, Duke University

SRF Cavity types depending on particle velocity



Summary:

- For acceleration of the particles having low velocity, QWR, HWR and spoke cavities are used in modern RT and SRF accelerators, which have high R/Q at low β .
- Double and triple-spoke resonators are also used up to β =0.5.
- QWR, HWR and SR are prone to MP; Balloon cavity has no MP.
- TEM-type cavities are used up to β =0.5. For higher β elliptical cavities are used in SRF accelerators.

Chapter 9.

Beam-cavity Interaction

a. Beam loading;
b. Optimal coupling;
c. Wake potential;
d. HOM excitation effects.

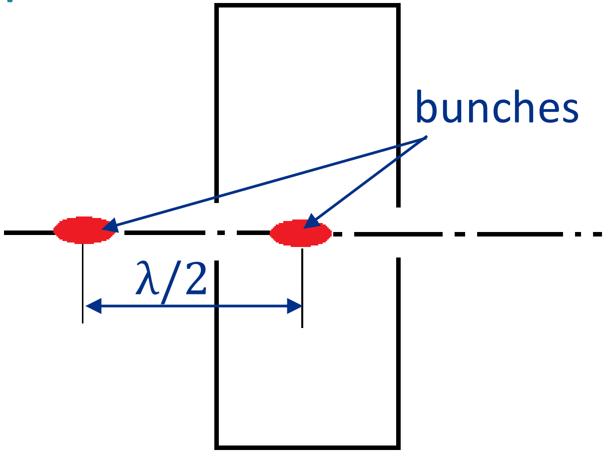
Beam Loading

Wilson's Theorems:

Perry B. Wilson
1927-2013

1. The bunch exiting the empty cavity, decelerates by $V_i/2$, where V_i is the voltage left in the cavity.

Two bunches with the distance between them of $\lambda/2$ excite total zero voltage.
If on bunches "sees" fraction α of V, one has:



$$q_b(V_i - \alpha V_i) = q_b \alpha V_i \rightarrow \alpha = 1/2.$$

Beam Loading

Wilson's theorems:

2. The voltage V exited by the bunch with the charge q_b is

$$V_i = 1/2 \cdot R/Q \cdot \omega \cdot q_b$$

Energy conservation law:

$$1/2 \cdot V_i \cdot q_b = V_i^2/(R/Q \cdot \omega) \rightarrow V_i = 1/2 \cdot R/Q \cdot \omega \cdot q_b$$

The energy loss of the bunch is equal to

$$U = 1/2 \cdot V_i \cdot q_b = 1/4 \cdot R/Q \cdot \omega \cdot q_b^2 = k \cdot q_b^2$$

here k is loss factor,

$$k = 1/4 \cdot R/Q \cdot \omega.$$

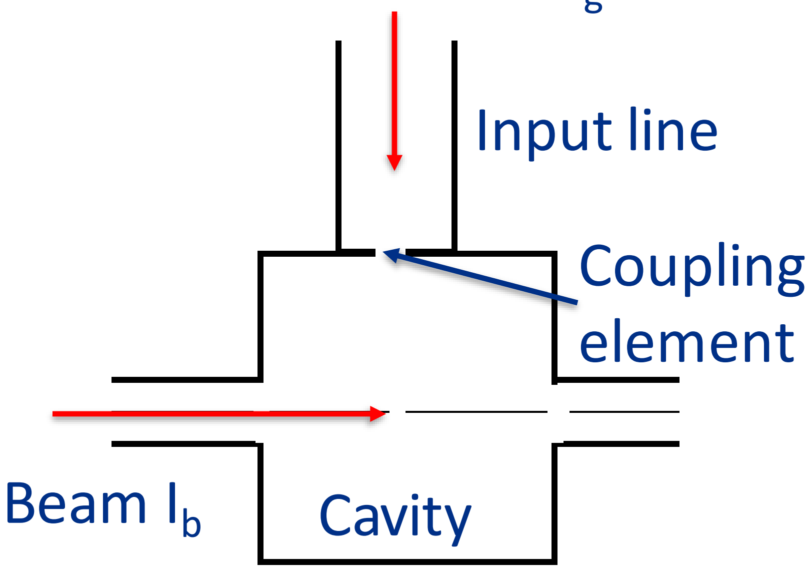
If the beam pulse is short compared to time constant τ (field decay time),

$$V_I = 1/2 \cdot R/Q \cdot \omega \cdot q$$

is a total voltage induced by the beam pulse in the cavity,

q is a total charge, $q = \Sigma q_b = I \cdot t_{beam}$.

Beam Loading

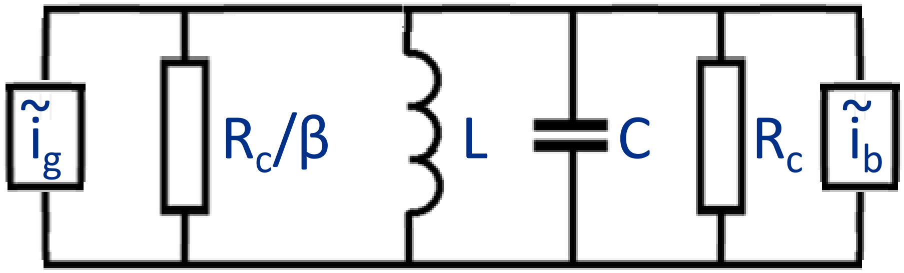


- RF source and beam $\omega_g=\omega_b= \omega$;
- Cavity: ω_0
- Cavity voltage : V_c
- Shunt impedance: R_{sh}
- Losses: $P_c = V_c^2/R_{sh} = V_c^2/(Q_0 \cdot R/Q)$
- Radiation to the line: $V_c^2/(Q_{ext} \cdot R/Q)$
- Coupling: $\beta = Q_0/ Q_{ext}$
- Loaded Q: $Q_L= Q_0/(1+ \beta)$
- Average beam current: I_b
- Synchronous phase: φ
- Power consumed by the beam: $P_b = I_b V_c cos\varphi$
- Input power P_g
- Reflected power: $P_r = P_g - P_c - P_b$

Details are in Appendix 8

Beam Loading

Equivalent circuit for the beam-loaded cavity transformed to the resonance circuit:



$L = R/Q/(2\omega_0)$
$C = 2/(R/Q\cdot\omega_0)$
$R_c = R/Q\cdot Q_0/2$
$\tilde{i}_b = -2I_b$

From this equivalent circuit we have:

$$P_g = \frac{V_c^2(1+\beta)^2}{4\beta Q_0(R/Q)}\left[\left(1+\frac{I_{\mathrm{Re}}(R/Q)Q_0}{V_c(1+\beta)}\right)^2+\left(\frac{Q_0}{1+\beta}\frac{(\omega^2-\omega_0^2)}{\omega_0^2}+\frac{I_{\mathrm{Im}}(R/Q)Q_0}{V(1+\beta)}\right)^2\right]$$

where $I_{Re} = I_b\cos\varphi$ and $I_{Im} = I_b\sin\varphi$

Fermilab

Beam Loading

If the cavity is detuned by Δf versus the RF source frequency and r.m.s. microphonics amplitude is δf, the required power is the following :

$$P_g=\frac{V_c^2(1+\beta)^2}{4\beta Q_0(R/Q)}\left[\left(1+\frac{I_{\mathrm{Re}}(R/Q)Q_0}{V_c(1+\beta)}\right)^2+\left(\frac{Q_0}{1+\beta}\frac{2\delta f}{f}+\left|\frac{Q_0}{1+\beta}\frac{2\Delta f}{f}+\frac{I_{\mathrm{Im}}(r/Q)Q_0}{V_c(1+\beta)}\right|\right)^2\right]$$

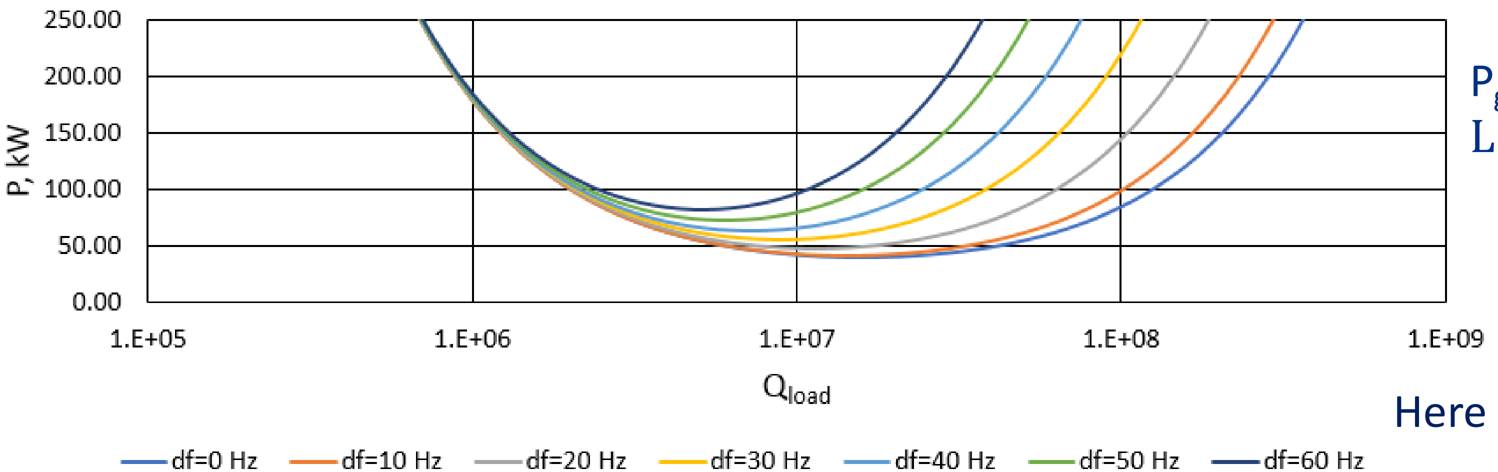
Typically, the cavity has “static detune”:

$$\Delta f=-f\frac{I_{\mathrm{Im}}(R/Q)}{2V_c}$$

In this case.

$$P_g=\frac{V_c^2(1+\beta)^2}{4\beta Q_0(R/Q)}\left[\left(1+\frac{I_{\mathrm{Re}}(R/Q)Q_0}{V_c(1+\beta)}\right)^2+\left(\frac{Q_0}{1+\beta}\frac{2\delta f}{f}\right)^2\right]$$

The optimal coupling providing minimal power: $\beta_{opt}=\left[\left(1+\frac{I_{\mathrm{Re}}(R/Q)Q_0}{V_c}\right)^2+\left(\frac{2\delta f Q_0}{f}\right)^2\right]^{1/2}$



$P_g(\delta f)$ for LB 650 (PIP II)

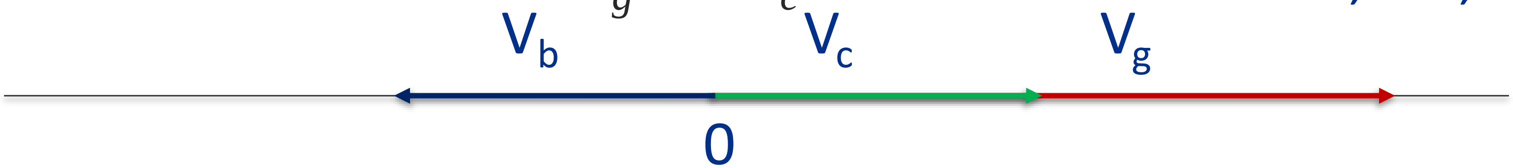
$$Q_{load}=\frac{Q_0}{1+\beta}$$

$$df=\frac{f}{Q_{load}},$$

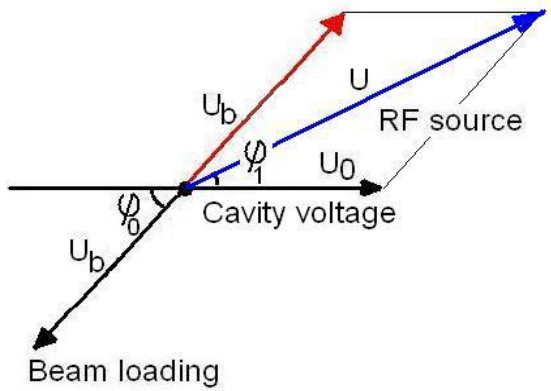
Here df-cavity bandwidth

Beam Loading

- In resonance for a SRF cavity $\beta_{opt} >> 1$ and $\beta_{opt} = I_b \cdot R/Q \cdot Q_0/V_c$ and $Q_L = V_c/(I_b \cdot R/Q)$. The cavity bandwidth $\Delta f = f/Q_L = f \cdot I_b \cdot (R/Q)/V_c$.
- for optimal coupling for the SRF cavity $V_b = -V_c$ and $P_g = |V_c \cdot I_b|$ Note that in this case $V_g = 2V_c$ and reflection is zero, i.e., $P_r = 0$.



- Without the beam in order to maintain the same voltage in the SRF cavity at the same coupling $P_{g0} = 1/4 \cdot P_q$. For SRF cavity reflection in this case is ~100%.
- Phase shift between the bunch and the cavity voltage



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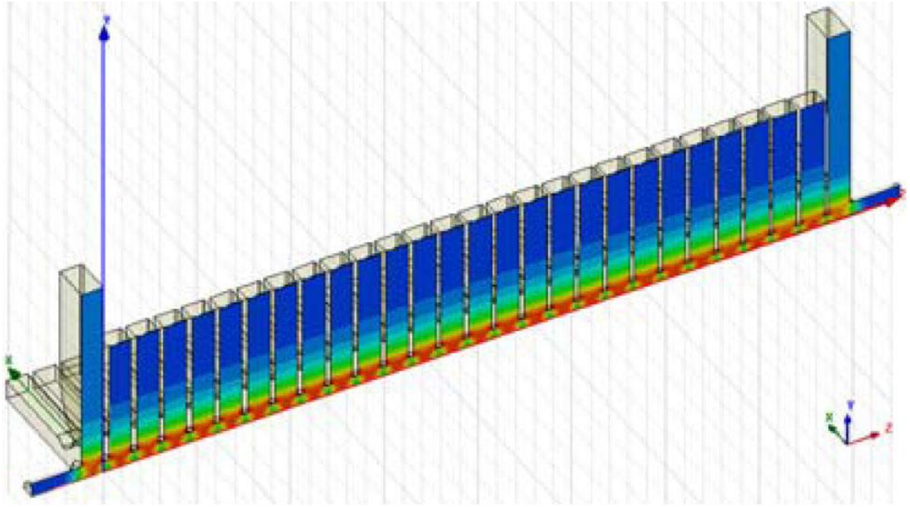
Beam Loading, Travelling Wave

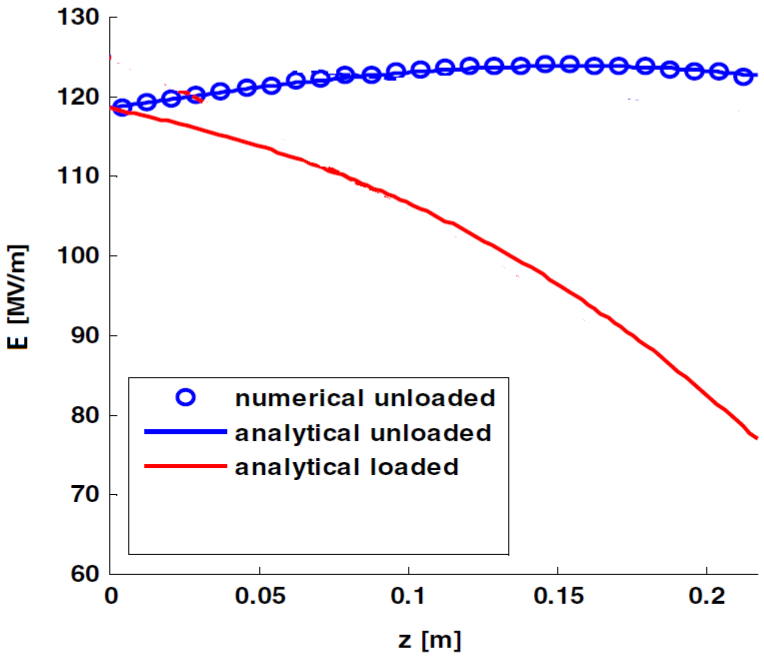
In presence of a beam (see Lecture 8, slide 14):

$$\frac{dW_{0,j}}{dt} = -P_j + P_{j-1} - \frac{\omega_0 W_{0,j}}{Q_0} - V_c I_b, \qquad (1)$$

Beam loading changes the field distribution along the structure.



CLIC 12 GHz structure

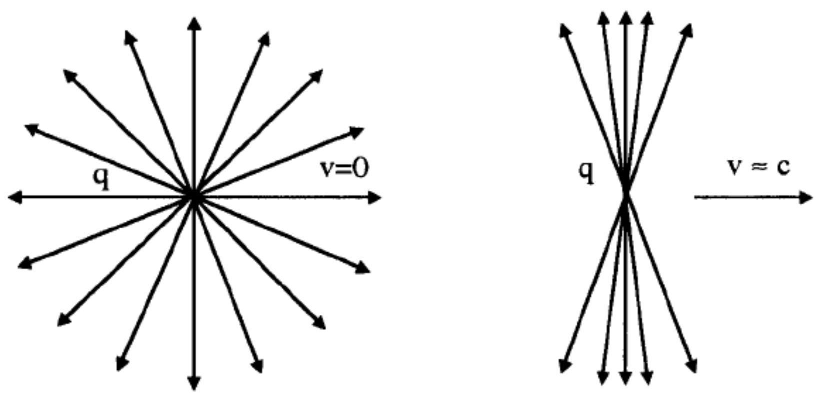


Wake potentials

1. Fields of a moving charge in free apace:

$$\mathbf{E} = \frac{q\hat{r}}{4\pi\varepsilon_0 r^2\gamma^2(1-\beta^2\sin^2\psi)^{3/2}}$$

For $\gamma \longrightarrow \infty$ $\quad E_r = \dfrac{q\delta(z-ct)}{2\pi\varepsilon_0 r} \qquad B_\theta = \dfrac{\mu_0 c q\delta(z-ct)}{2\pi r}$



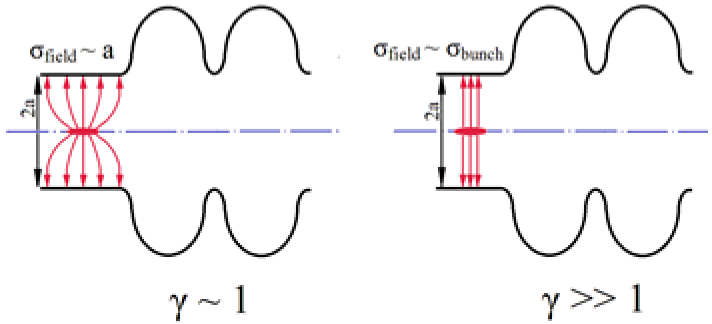
$$F_r = q'(E_r - cB_\theta) = \frac{q'q\delta(z-ct)}{2\pi\varepsilon_0 r} - \frac{q'q\mu_0 c^2\delta(z-ct)}{2\pi r} = 0$$

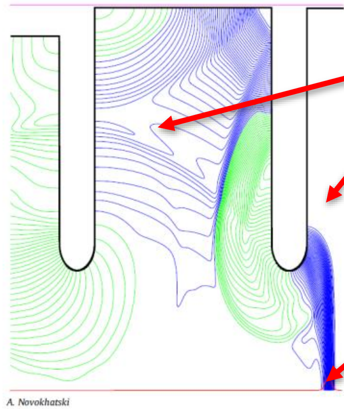
Wake potentials

1. Fields in the smooth waveguide with ideally conducting walls:
   No radiation (lack of synchronism: $v_{ph}>c$)
   Coulomb forces (including image):
   $F_r = q(E_r - vB_\phi) \sim 1/\gamma^2$
   $F_z \sim 1/\gamma^2$ (static field is compensated by eddy field).

2. In presence of obstacle radiation takes place
   - change of cross section,
   - finite conductivity
   - dielectric wall

Wake potentials

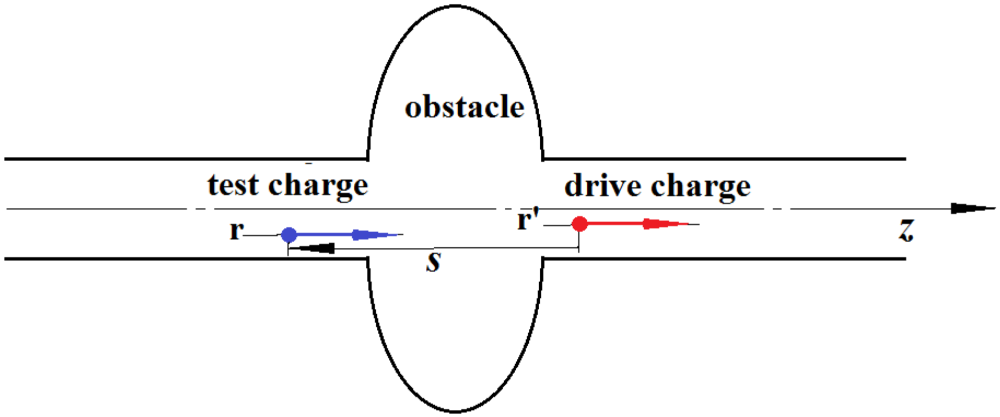




Blue – deceleration, green – acceleration

- Energy lost by the bunch:
  $W = kq^2$
  k is the loss factor.
- Transverse momentum kick:
  $\Delta p_{\perp} c = r q^2 k_{\perp}$
  $k_{\perp}$ is a kick factor

Radiation fields in the TW acceleration structure



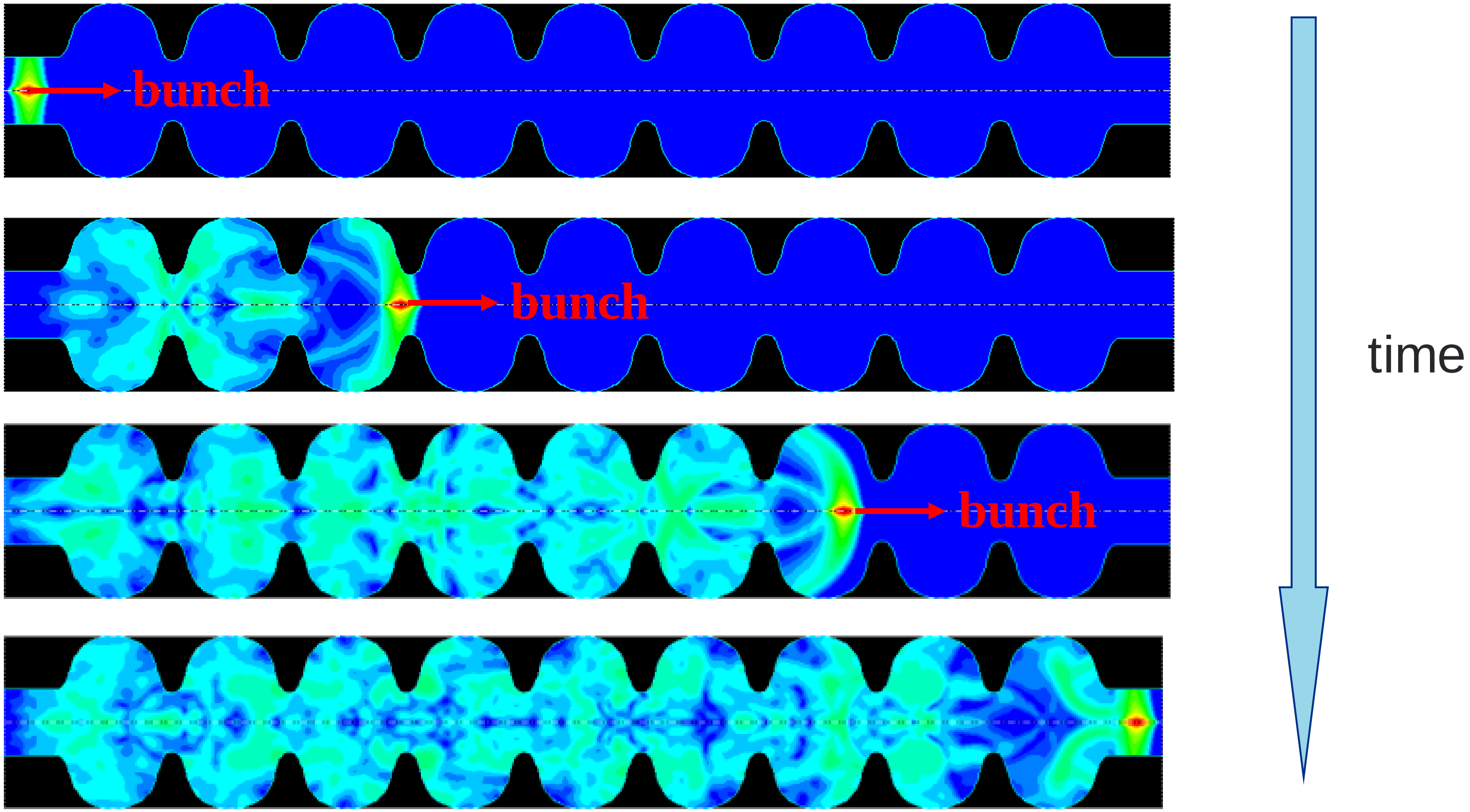
$$W_z(\vec{r},\vec{r}',s) = -\frac{1}{q}\int_{z_1}^{z_2} dz\,[E_z(\vec{r},z,t)]_{t=(z+s)/c},$$

$$\vec{W}_{\perp}(\vec{r},\vec{r}',s) = \frac{1}{q}\int_{z_1}^{z_2} dz\,\left[\vec{E}_{\perp} + c(\hat{z}\times\vec{B})\right]_{t=(z+s)/c}.$$

$W_z=0,\ W_{\perp}=0$ for $s<0$

More details: Appendix 13

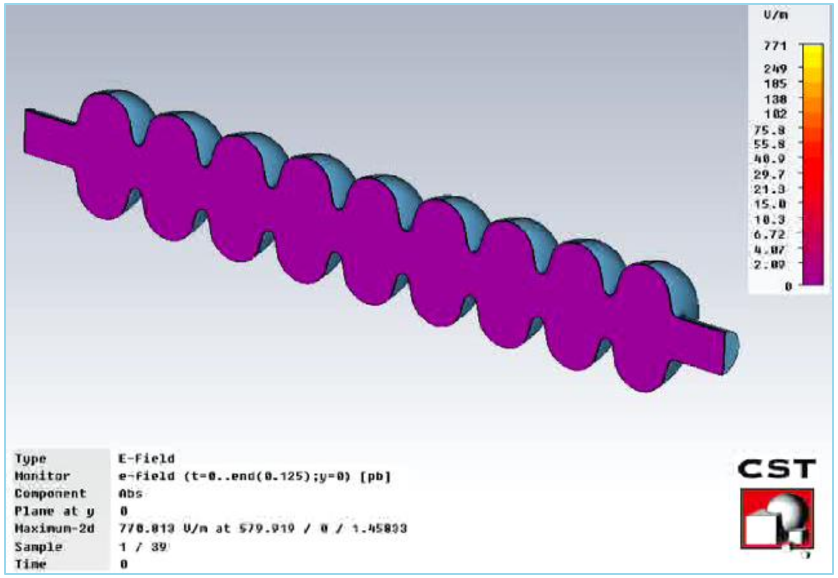
Electromagnetic field excited by bunch



The bunched beam excites electromagnetic field inside an originally empty cavity.

Short- and long-range wakefields

- Short range wake-field → Fields along the bunch and just behind it:
  - Cause bunch energy loss and energy spread along the bunch
  - Single bunch break up instability
  - Cooper pair breaking in the case of extremely short bunches
- Long range wakes (HOMs):
  - Monopole modes: Longitudinal coupled bunch instabilities; RF heating; Longitudinal emittance dilution …
  - Dipole modes: Transverse transverse coupled bunch instabilities; Emittance dilution; beam break-up instabilities …

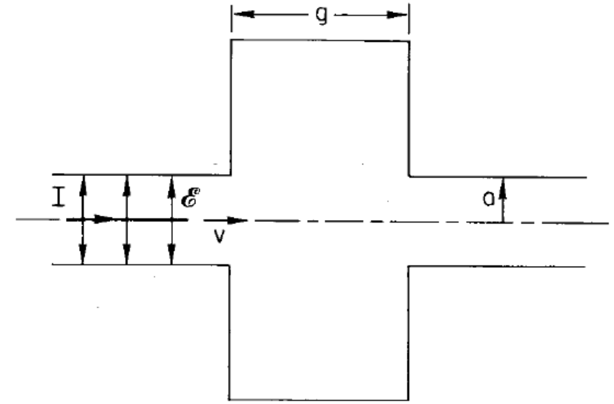


Wake potentials

Pillbox with the holes:

$$k_\ell(\sigma) = \frac{Z_0 c}{\pi^{5/2} a}\sqrt{\frac{g}{\sigma}}\left[\Gamma(1/4)/4 - \left(\frac{\omega_c \sigma}{c}\right)^{1/2}\right]$$

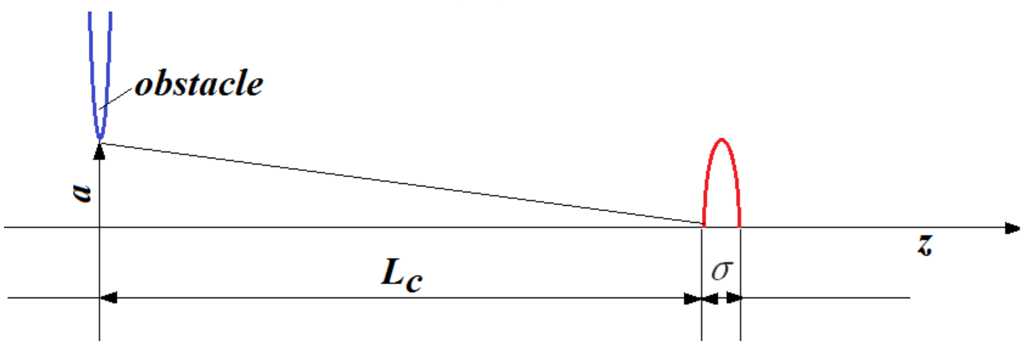
$$k_\perp(\sigma) = (4.36..)\frac{Z_0 c}{\pi^3 a^3}\sqrt{g\sigma}$$



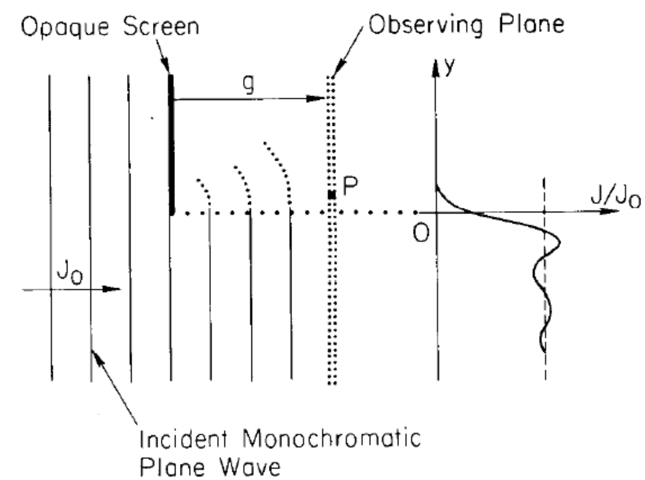
For Gaussian bunch, $\Gamma(1/4)/4 = 0.908..$

Loss and kick factors depend on the cavity geometry and the bunch Length.

Catch-up problem:



$(L_c^2+a^2)^{1/2} = L_c+\sigma \qquad L_c \approx a^2/2\sigma$

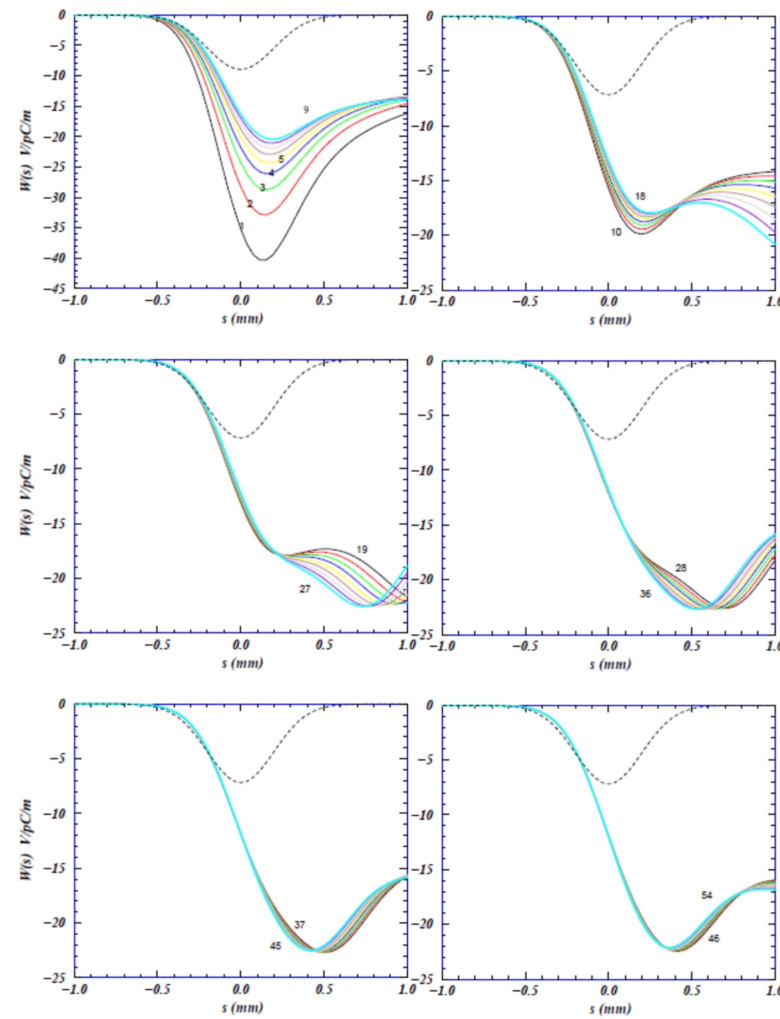


Diffraction model:

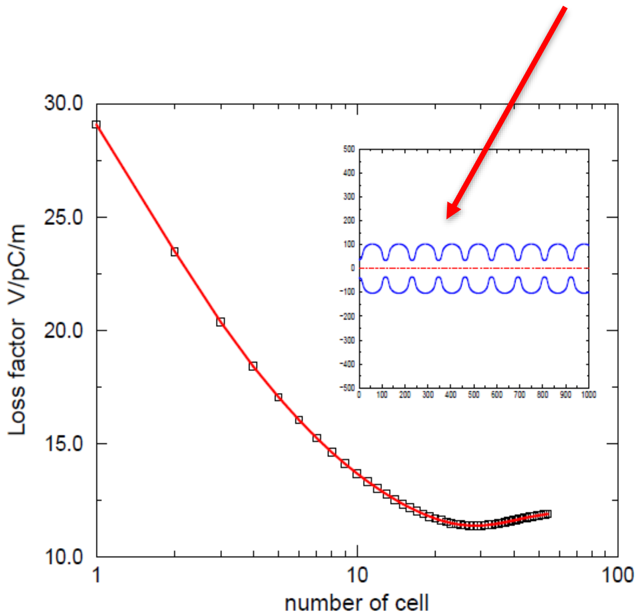
Wake potentials

Transition of the wakefield in semi-infinite periodic structure:

- Calculations of the loos distribution for a chain of TESLA cells. The loss factor and wake amplitude decrease with the cell number. The shape of the wake does not change significantly after the bunch exceeds the catch-up distance, which is ~3 m (27 TESLA cells) for this case (σ = 0.2 mm, a = 35 mm)



$$L = \frac{a^2}{2\sigma_z}$$



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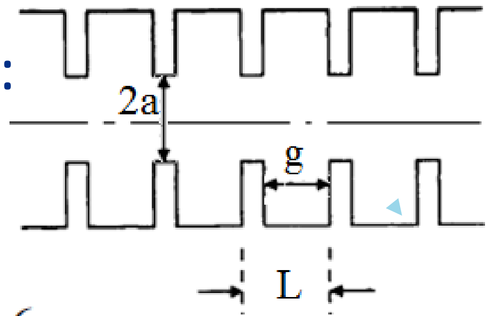
Wake potentials



Semi-Infinite periodic structure (steady-state wake):
→wakes per unit length.

- Karl Bane model (KB) :

$$W_L(s, s_0) = \frac{Z_0 c}{\pi a^2} \exp\left(-\sqrt{s/s_0}\right); \quad \text{where } s_0 = 0.41 \frac{a^{1.8} g^{1.6}}{L^{2.4}},$$

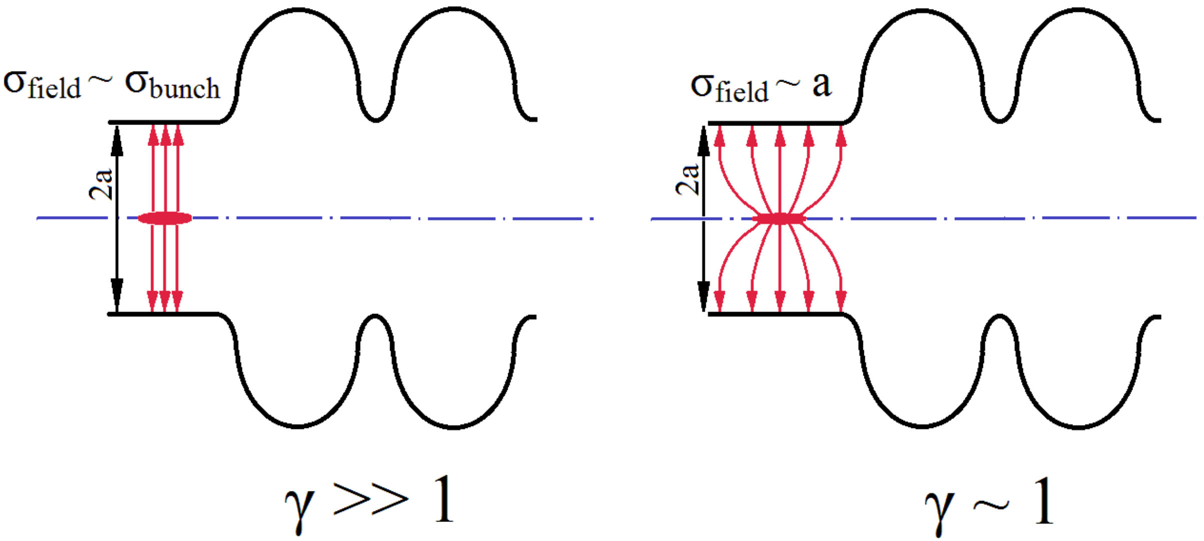
when $\sigma \to 0,\ k_l = \frac{Z_0 c}{\pi a^2}$

Steady-state wake – when the structure length > catch-up distance.

❖ Wake potentials limit the cavity aperture and therefore, determine the cavity design, especially for SRF electron linacs for FELs, where the bunch length is small.

Wake potentials

Wakefiled for non-relativistic bunch:



HE electron linac
(ILC, XFEL or LCLS II)

$f_{max} \sim \; c/\sigma_{bunch}$

Proton linac

$f_{max} \sim \; c/a$

$for\ \sigma_{bunch} = 50\mu, f_{max} < 6\ THz$ $for\ a = 50mm, f_{max} < 6\ GHz$

Diffraction losses are determined by $\sigma_{field},\ P \sim \; (\sigma_{field})^{-1/2}$

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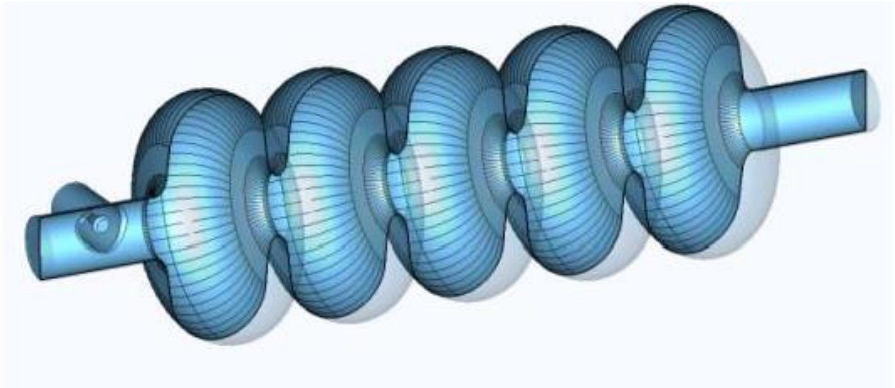
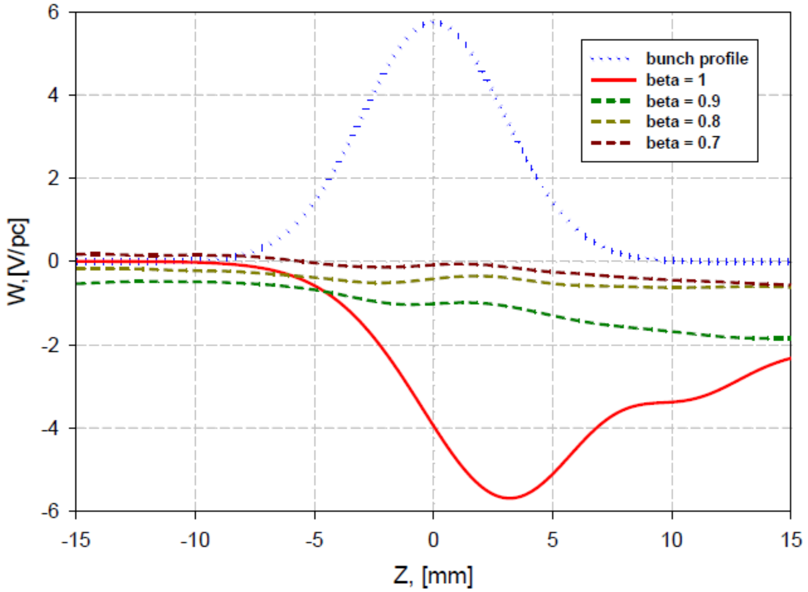
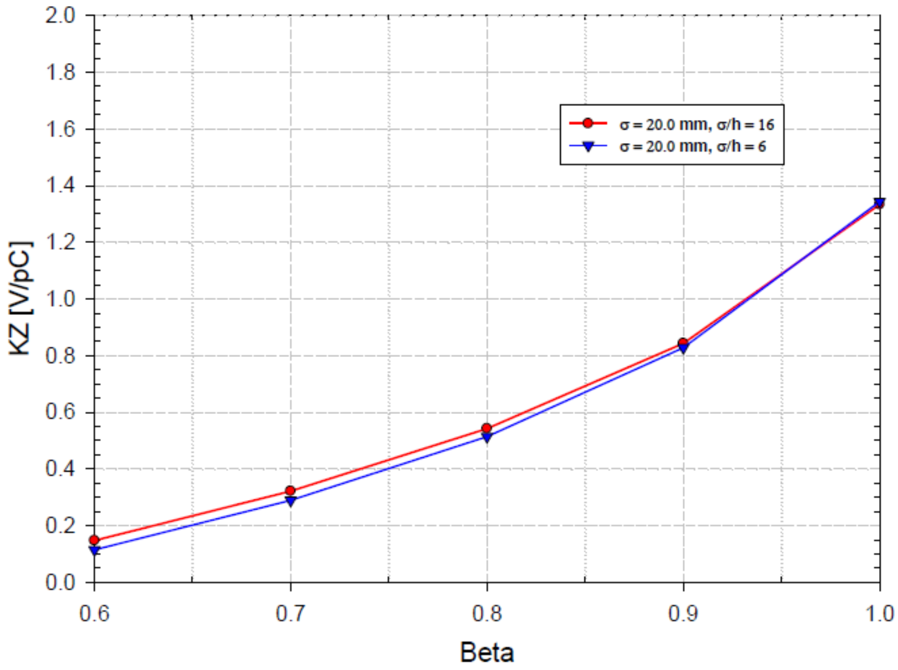
Wake potentials

Wakefiled for non-relativistic bunch*:



The 650 MHz, β=0.9 elliptical accelerating cavity for PIP II





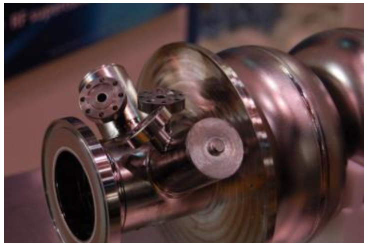
*S. Kurennoy

High-Order Modes in elliptical SRF cavities (long-range wakes)

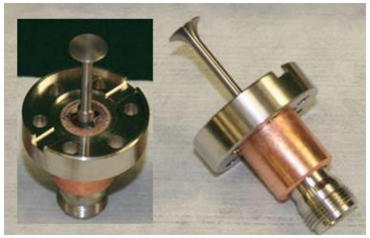
- ❑ Possible issues:
- Trapped modes;
- Resonance excitation of HOMs;
- Collective effects – transverse (BBU) and longitudinal (klystron-type instability);

↓

- Additional cryo-losses;
- Emittance dilution (longitudinal and transverse).



- ❑ HOM damper is a vulnerable, expensive and complicated part of SC acceleration cavity (problems – heating, multipacting, etc; additional hardware – cables, feed-through, connectors, loads). HOM dampers may limit a cavity performance and reduce operation reliability;



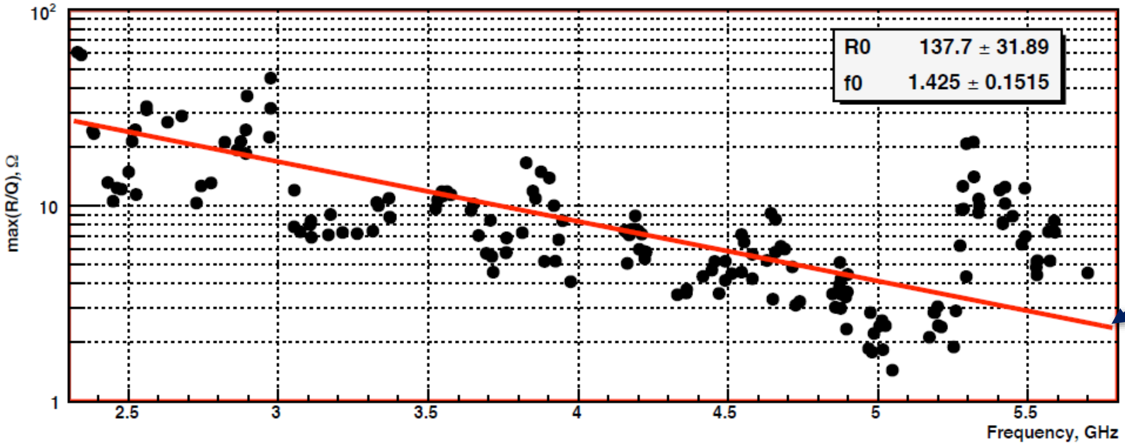
"To damp, or not to damp?"

High-Order Modes in elliptical SRF cavities

- HOM energy gain dependence on frequency (see Lecture 7, slide 25)

  $V(r) \sim e^{kr/\beta\gamma}$; or $V(0)/V(a) \sim e^{-ka/\beta\gamma}$. It means that for HOMs $\frac{R}{Q} = \frac{V(0)^2}{\omega W} \sim e^{-2ka/\beta\gamma}$

Example: β =0.9, 650 MHz, 5-cell cavity.



$$\frac{R}{Q} = R_0 e^{-\frac{f}{f_0}}$$

- In proton linacs the HOM spectrum is limited in contrast to SRF cavities for electron linacs;
- R/Q of propagating modes having high frequencies is considerably small.

* **A. Sukhanov**, et al., "Higher Order Modes in Project-X Linac", Nuclear Instruments and Methods in Physics Research, Vol. 734, Part A, January 2014

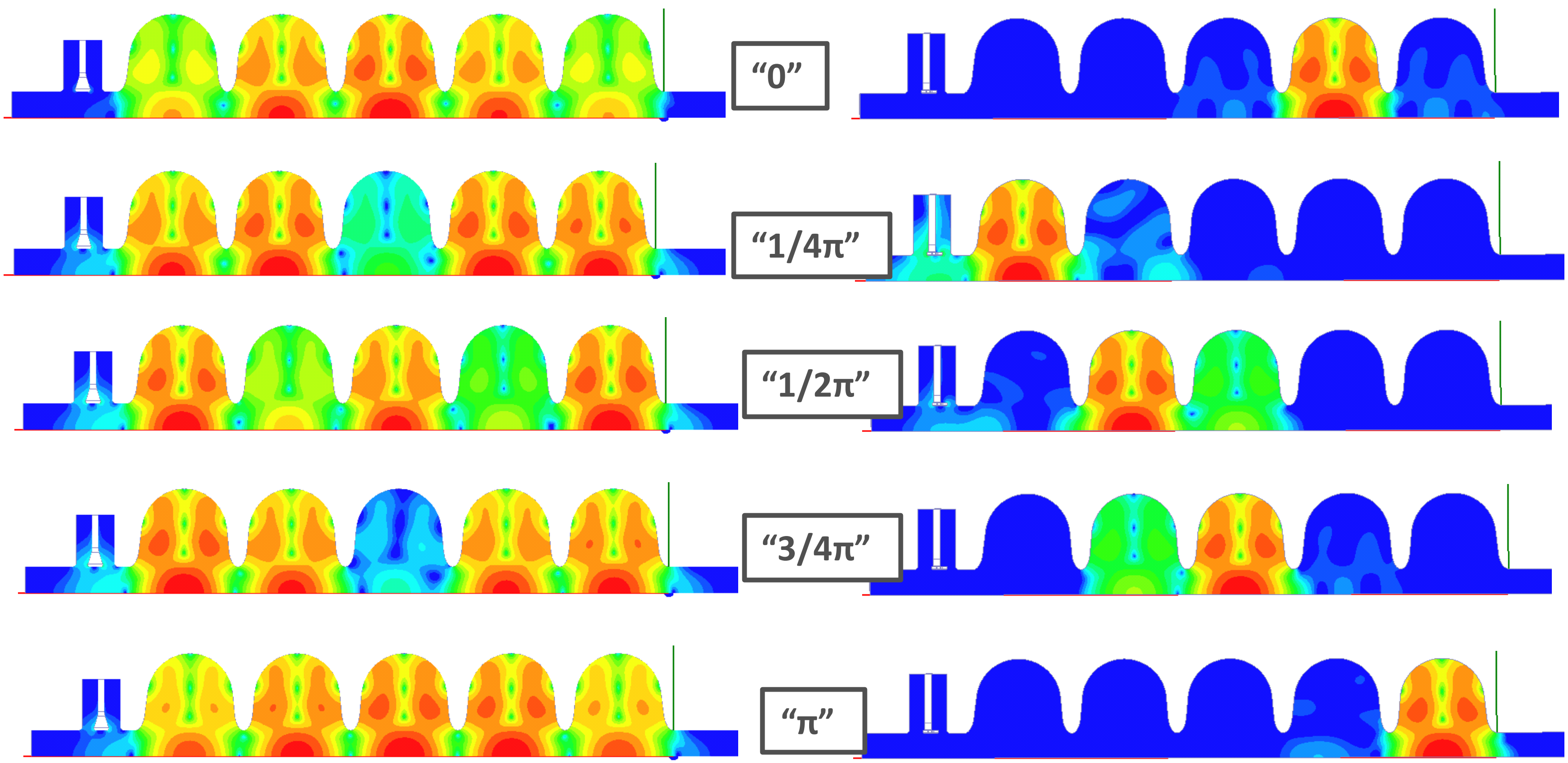
High-Order Modes in elliptical SRF cavities (long-range wakes)

- Specifics of Higher Order Mode effects in the elliptical cavities of proton linacs:
  - Non-relativistic beam;
  - Small current and small bunch population;
  - No feedback (linac);
  - Complicated beam timing structure (dense frequency spectrum).

Trapped Modes in elliptical SRF cavities

For some modes *k (coupling)* may be very small (electric coupling is compensated by magnetic coupling). Because of manufacturing errors, the field distribution may change, the mode will not be coupled to the FC or beam pipe and have high Q_{load} – so called trapped modes.

An example of a bad cavity design containing a trapped mode:

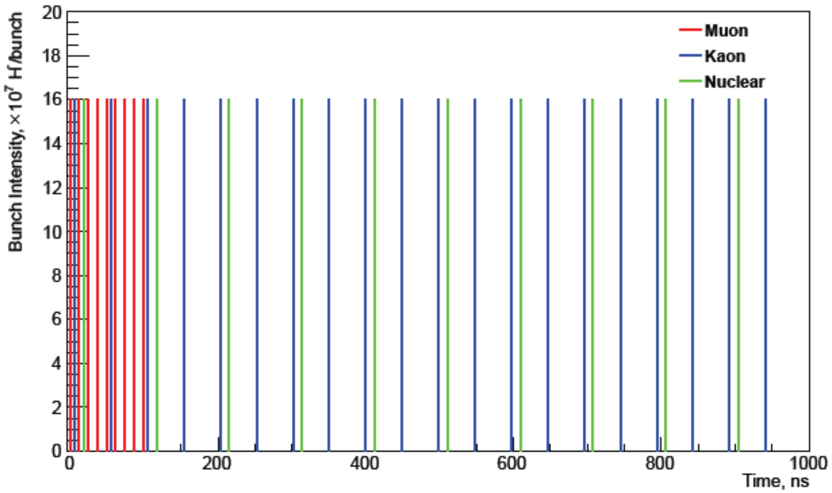


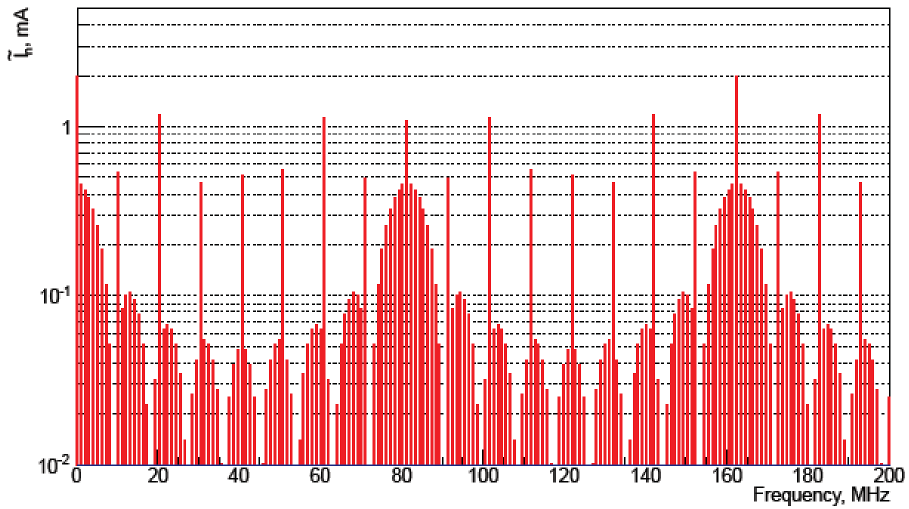
HOMs in an ideal cavity.

HOMs in a “realistic” cavity, i.e., in presence of misalignments.

In both cases the operating mode is tuned to correct frequency and field flatness.

Resonance excitation of HOMs in elliptical SRF cavities



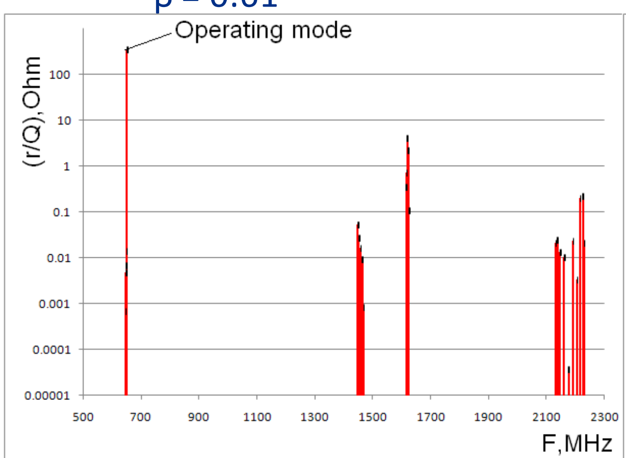


Example of the beam structure for multi-experimental proton driver (PIP II)

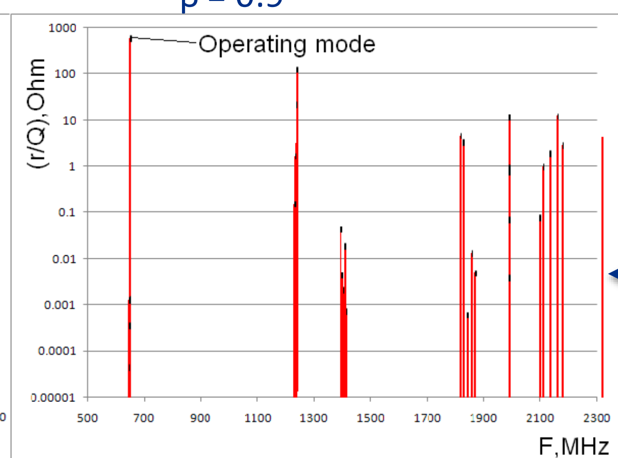
The beam current spectrum may be dense: for PIP II it contains

- ● harmonics of the bunch sequence frequency of 10.15 MHz and
- ● sidebands of the harmonics of 81.25 MHz separated by 1 MHz.

β = 0.61



β = 0.9



R/Q spectrum of the cavities

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Resonance excitation of HOMs in elliptical SRF cavities

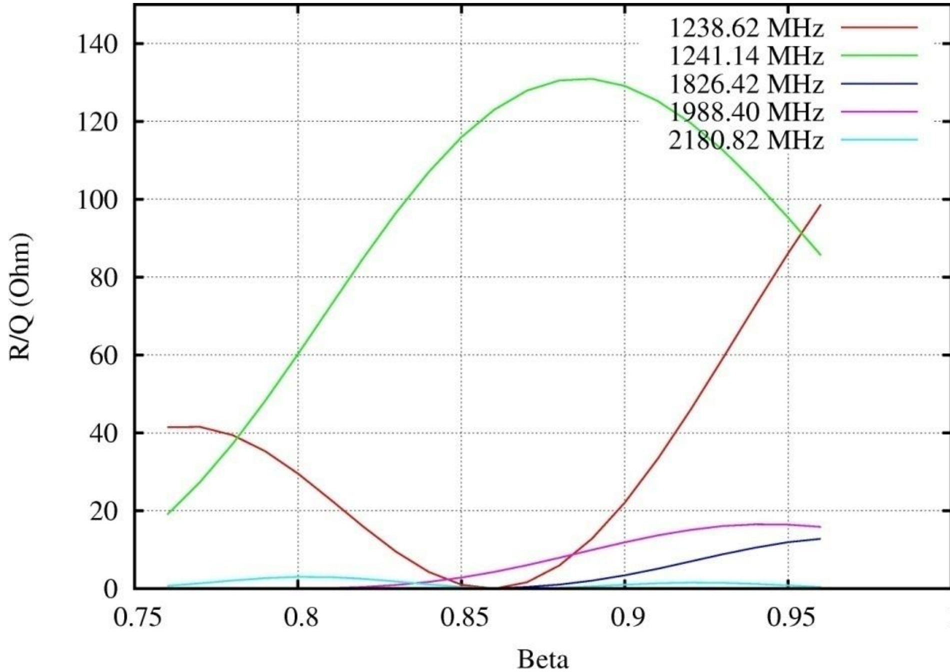
HOM have frequency spread caused by manufacturing errors:

❖For 1.3 GHz ILC cavity r.m.s. spread σ_f of the resonance frequencies is 6-9 MHz depending on the pass band;

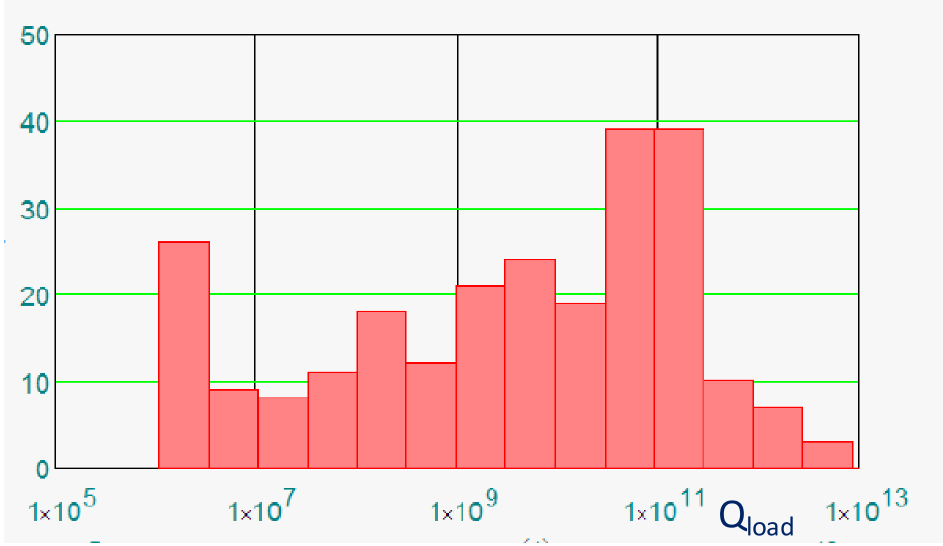
❖Cornell: $\sigma_f \approx 10.9 \cdot 10^{-4} \times (f_{HOM} - f_0)$,

❖SNS: $\sigma_f \approx (9.6 \cdot 10^{-4} - 13.4 \cdot 10^{-4}) \times (f_{HOM} - f_0)$;

$\Delta f_{max} = |f_{HOM,calculated} - f_{HOM,measured}| \sim \sigma_f$



(R/Q) for HOM modes depends on the particle velocity β (650 MHz, β=0.9 cavity)



Variation of Q_{load} for 5th passband (650 MHz, β=0.9 cavity)

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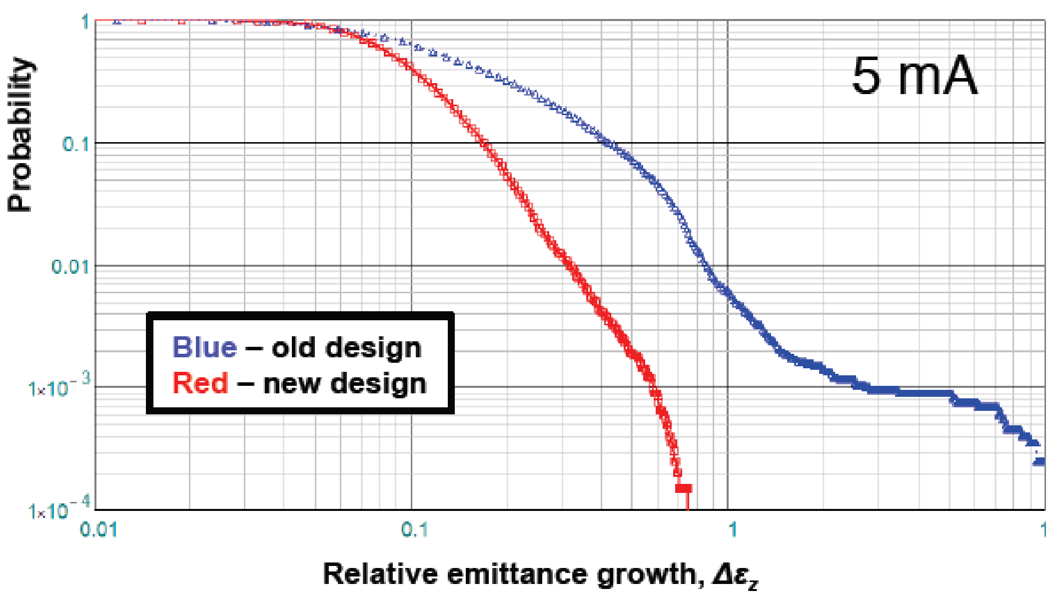
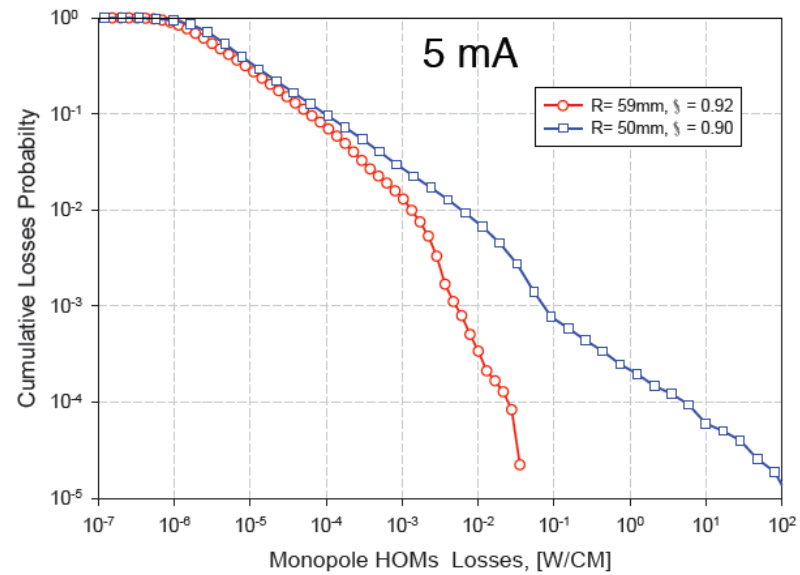
Resonance excitation of HOMs in elliptical SRF cavities (cont)

- Longitudinal emittance dilution does not take place if $\delta f >> f \dfrac{\tilde{I}(R/Q)\sigma_t}{4\sqrt{2}\varepsilon_z}$
  For typical parameters for proton linacs $\delta f >>$ 10-100 Hz.

- Transverse emittance dilution does not take place if $\delta f \ll \dfrac{c x_0 \tilde{I}(R/Q)_1}{8\sqrt{2}\pi\beta\gamma U_0\sqrt{\varepsilon/\beta_f}}$ Not an issue!
  For typical parameters for proton linacs $\delta f >>$ 1-10 Hz.



PIP II SRF linac



Cryo load caused by HOMS

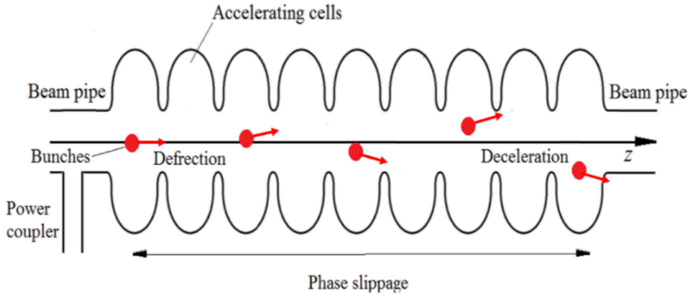
Resonance excitation of HOMs in elliptical SRF cavities (cont)

❑ Regenerative instability, transverse (BBU):

- Regenerative BBU develops in one cavity and requires feedback; in SRF multi-cell cavities this feedback is caused by dipole partial travelling waves reflections from the cavity ends, which compose a HOM standing wave.
- The excitation mechanism :
  - Transverse kick in the beginning of the cavity; the $U_{kick} \sim V_{HOM}$, V_{HOM} is the HOM amplitude.
  - Deflecting in the cavity; deflection $x \sim V_{HOM}$
  - Phase slippage → deceleration in the cavity end, $P_{beam} \sim x\, I_{beam} V_{HOM} \sim I_{beam}\,(V_{HOM})^2$
  - Instability condition: average power lost by the beam is equal or less to the loss power P_{loss} in the cavity: $P_{loss} \sim (V_{HOM})^2/(r_\perp/Q \cdot Q_{load})$:

$$<P_{beam}> = P_{loss} \qquad (1)$$

  - It gives the critical beam current: $I_{crit} = \kappa/(r_\perp/Q \cdot Q_{load})$. κ depends on the beam velocity and HOM field distribution, it is determined numerically. **It does not depend on the relationship between the HOM frequency and bunch spectrum line! "Instability selects its own frequency"**



For ultra-relativistic beam*:

$$I_{crit} \gtrsim \frac{\pi^3 pc/e}{2k_{HOM} L \left(\frac{r_\perp}{Q}\right) Q_{load}}$$

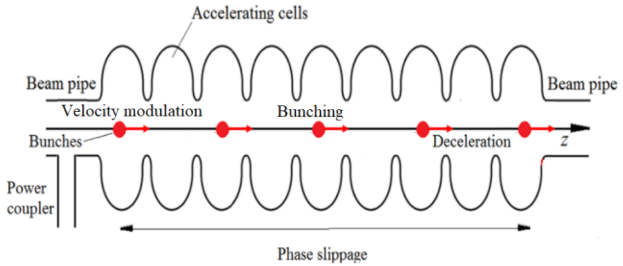
*R.L. Gluckstern and H.S. Butler, "Transverse Beam Blow-Up in a Standing Wave Linac Cavity," IEEE Transactions on Nuclear Science, vol. 12N, No3, 1965, pp. 605 – 612.

Resonance excitation of HOMs in elliptical SRF cavities (cont)

- Regenerative instability, longitudinal (monotronic):
- The excitation mechanism :
  - Velocity modulation in the beginning of the cavity; $\Delta\beta \sim V_{HOM}$, V_{HOM} is the HOM amplitude.
  - Bunching in the cavity at the HOM frequency; the current harmonic $I_{HOM} \sim \Delta\beta I_{beam} \sim V_{HOM} I_{beam}$
  - Phase slippage → deceleration in the cavity end, $P_{beam} \sim I_{HOM} V_{HOM} \sim I_{beam} (V_{HOM})^2$
  - Instability condition: average power lost by the beam is less or equal to the loss power P_{loss} in the cavity: $P_{loss} \sim (V_{HOM})^2/(R/Q \cdot Q_{load})$:

$$\langle P_{beam} \rangle = P_{loss} \qquad (2)$$

  - It gives the critical beam current: $I_{crit} = \kappa_{||} /(R/Q \cdot Q_{load})$. $\kappa_{||}$ depends on the beam velocity and HOM field distribution, it is determined numerically from (2).
- **Similar to BBU, it does not depend on the relationship between the HOM frequency and bunch spectrum line! “Instability selects its own frequency”**
- **Typically, in proton accelrators I_{crit} is > 1 A. <u>But it should be always checked!</u>**



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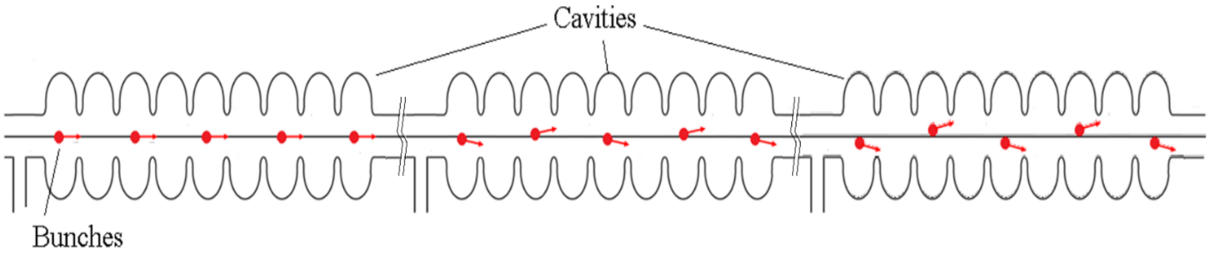
Resonance excitation of HOMs in elliptical SRF cavities (cont)

❑ Cumulative BBU

- In contrast to regenerative BBU, cumulative BBU develops in the chain of the cavities.



- Mechanism:
  - Because of the initial transverse modulation on the beam, a dipole HOM is excited in the first accelerator cavity, which in turn provides transverse momentum modulation of the later bunches.
  - As a result, transverse momentum modulation converts in the downstream cavities to displacement modulation exciting there the dipole HOM, which in turn lead to further transverse displacement modulation, and therefore to the beam transverse emittance dilution, or even to the beam lost.
- The effect is coherent and cumulative as a function of length and time. As well as the regenerative instability, the cumulative instability is determined by the beam current I_{beam} and the cavity HOM parameters: resonance frequency f_{HOM}, transverse impedance $r_{\perp}/Q$ and Q_{load}.

Resonance excitation of HOMs in elliptical SRF cavities (cont)

Why collective effects is not an issue for SRF proton linacs with elliptical cavities:

- No feedback as in ERLs (or CEBAF);
- Different cavity types with different frequencies and different HOM spectrum are used;
- Frequency spread of HOMs in each cavity type, caused by manufacturing errors;
- Velocity dependence of the (R/Q);
- Small – compared to electron linacs -beam current.

- No HOM dampers in SNS upgrade cavities (I_{beam} =26 mA);
- No HOM dampers in ESS cavities (I_{beam} =50 mA);
- No HOM dampers in PIP II cavities (I_{beam} up to 5 mA);
- Probably, HOM dampers will be necessary for future high – current drivers for ADS.

$$U_{kick} = ix_0 I_0 Q_{ext}\left(\frac{r_\perp}{Q}\right)$$

- Alignment!

Summary:

- Accelerated beam excites RF field in the cavity, which should be compensated by the RF source'
- The required power is determined by the beam current, voltage, cavities R/Q, loaded Q, cavity detune and the synchronous phase. There is the optimal coupling which provides minimal input power.
- Ultra-relativistic bunch radiates the field in the cavity, which may cause energy spread and transverse instability. Short-range wake changes the beam dynamics in the same bunch.
- Loss factor and kick factor limit the cavity aperture, that should be taken into account during the cavity design.
- Long-range wakes (= HOMs) may affect the beam dynamics (cumulative instabilities). The cavity should be optimized to get rid of trapped modes, and modes with high R/Q and Q_{load}. For proton linacs (pulsed and CW) HOMs typically are not the issue; for electron SRF linacs HOMs should be damped.
- Proper cavity alignment should be provided to mitigate or get rid of cumulative instabilities.