

Light Sources and Free Electron Lasers

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Outline

- Introduction
- Insertion device radiation
 - Time dilation effects
 - Bending magnet, undulators, wigglers
- Synchtrotron light sources
 - Energy trade-offs
 - Experiments
 - Achieving higher brightness
- Free electron lasers (FELs)
 - Gain and spectrum
 - Fully coherent x-rays





Laser Technologies Span THz to UV





Facility for Rare Isotope Beams

Laboratory Radiation Source Development





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Radiation from Charged Particles

The treatment of the radiation field emitted from charged particles follows classically from Larmor and Lienard-Wiechert.

$$P_{Larmor} = \frac{2}{3} \frac{e^2}{4\pi\varepsilon_0 c^3} a^2 \qquad \frac{dP}{d\Omega} = \frac{1}{4\pi} \frac{e^2}{4\pi\varepsilon_0 c^3} a^2 \sin^2\theta \qquad \text{'a' is apparent} acceleration$$

$$\frac{dP(t')}{d\Omega} = \frac{1}{4\pi} \frac{e^2 a^2}{4\pi\varepsilon_0 c^3} \frac{\sin^2\theta}{(1-\beta\cos\theta)^5} \qquad \text{Acceleration || velocity}$$

$$\frac{dP(t')}{d\Omega} = \frac{1}{4\pi} \frac{e^2 a^2}{4\pi\varepsilon_0 c^3} \frac{(1-\beta\cos\theta)^2 - (1-\beta^2)\sin^2\theta\cos^2\theta}{(1-\beta\cos\theta)^5} \qquad \text{Acceleration \perp} \text{ velocity}$$

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Relativistic effects in time dilation





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K.J. Kim, AIP Conf. Proc. 184, 565 (1989).

Undulators and Wiggler Motion

We increase the number of emitters by creating specialized magnet devices, undulators and wigglers, with multiple poles.



Pure permament magnet (PPM) scaling of peak field with full gap, *g*, between jaws

$$B_0[T] = 3.44 \exp\left[-\frac{g}{\lambda_u} \left(5.08 - 1.54 \frac{g}{\lambda_u}\right)\right]; NdFeB$$
$$B_0[T] = 3.33 \exp\left[-\frac{g}{\lambda_u} \left(5.47 - 1.8 \frac{g}{\lambda_u}\right)\right]; SmCo$$



Insertion Devices

LCLS PM Undulator Prototype









Lorentz Force Governs Motion

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \Rightarrow \frac{d\vec{p}}{dt} = q(\vec{v} \times \vec{B})$$

$$\frac{d\vec{p}}{dt} = \frac{d}{dt}(\gamma \vec{\beta} mc) = \gamma mc \frac{d\vec{\beta}}{dt} \qquad q(\vec{v} \times \vec{B}) = -ec(\vec{\beta} \times \vec{B})$$

$$\frac{d\vec{\beta}}{dt} = \frac{-e}{\gamma m}(\vec{\beta} \times \vec{B}) \qquad \frac{d\beta_x}{dt} = \frac{-e}{\gamma m}(\beta_y B_z - \beta_z B_y) \Rightarrow \frac{d\beta_x}{dt} = \frac{e}{\gamma m}\beta_z B_y$$
Oth order - constant β_z

$$1^{st} \text{ order - oscillatory } \beta_x$$

$$\frac{d\beta_x}{dt} = \frac{e}{\gamma m}\beta_z B_y = \frac{e\beta_z B_o}{\gamma m} \sin(2\pi z/\lambda_u)$$

$$\frac{d\beta_x}{dt} \Rightarrow \beta_z c \frac{d}{dz} \qquad \frac{d\beta_x}{dz} = \frac{eB_o}{\gamma mc} \sin(2\pi z/\lambda_u)$$

Equations of Motion from Lorentz Force

Electron transverse velocity

$$\beta_x = \frac{K}{\gamma} \cos(2\pi z/\lambda_u)$$
 where $K = \frac{eB_0\lambda_u}{2\pi mc} = 0.934\lambda_u [cm]B_0[T]$

- The maximum slope in the trajectory is $\delta = K/\gamma$
- We differentiate between 'undulator' ($K \leq 1$) and 'wiggler' motion (K>1)
- The longitudinal velocity, $c\beta_z$, is found

$$\beta_{z} = \sqrt{1 - \frac{1}{\gamma^{2}} - \beta_{x}^{2}} \approx 1 - \frac{1 + \frac{K^{2}}{2}}{2\gamma^{2}} - \frac{K^{2}}{4\gamma^{2}}\cos(4\pi z/\lambda_{u})$$

• And displacement (with $\omega_u = 2\pi c/\lambda_u$)

$$\frac{1}{c}\vec{r}(t') = \left[\frac{K}{\gamma\omega_u}, 0, \left(1 - \frac{1 + \frac{K^2}{2}}{2\gamma^2}\right)t' - \frac{K^2}{8\gamma^2\omega_u}\sin(2\omega_u t')\right]$$

'Figure-8' motion

Longitudinal 'slippage'



Facility for Rare Isotope Beams U.S. Department of Energy Office of Science Michigan State University B.M. Kincaid, J. Appl. Phys., 48 (1977), 2684-2691. K.J. Kim, AIP Conf. Proc. 184, 565 (1989).

Beam-frame and Lab-frame Motion





FIG. 2. Schematic representative of radiation produced in strong- and weak-field cases viewed both in the lab and in the moving frame.





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Equations of Motion - Longitudinal

- The unit observation vector, \boldsymbol{n} , is approximated by (with $\theta, \phi, \psi \sim \frac{1}{\gamma} \ll 1$) $\boldsymbol{n} \approx (\varphi, \psi, 1 - \theta^2/2)$, and $\theta^2 = \phi^2 + \psi^2$
- The observer time (t) is then

t

$$= t'\frac{1+\frac{K^2}{2}+\gamma^2\theta^2}{2\gamma^2} + \frac{K^2}{8\omega_u\gamma^2}\sin(2\omega_u t') - \frac{K\phi}{\omega_u\gamma}\sin(\omega_u t')$$

• Multiply by
$$\omega_1(\theta) = \frac{2\gamma^2}{1 + \frac{K^2}{2} + \gamma^2 \theta^2} \omega_1$$

Figure 4.2 Coordinate system for the undulator radiation.

- To obtain $\omega_1(\theta)t = \omega_u t' + \frac{K^2}{4\left(1 + \frac{K^2}{2} + \gamma^2 \theta^2\right)} \sin(2\omega_u t') \frac{2\gamma K\phi}{1 + \frac{K^2}{2} + \gamma^2 \theta^2} \sin(\omega_u t')$
- The apparent motion is **periodic** in time with period $2\pi/\omega_1(\theta)$ but **not sinusoidal**.
- The period is shorter than the period in the electron emitter time by a large factor.
- Undulator radiation exhibits sharp spectral peaks at <u>odd</u> multiples of $\omega_1(\theta)$ <u>on-axis</u>.

K.J. Kim, AIP Conf. Proc. 184, 565 (1989).



Intensity and Spectral Characteristics



The radiation from an undulator has bandwidth $\Delta \omega / \omega = 1/2N_u$ (where N_u is the number of periods in the undulator), and is emitted in a cone with opening angle $1/\gamma$.

The on-axis intensity in the 1st harmonic is

$$\frac{dI}{d\Omega d\omega}(\theta = 0) = N_u^2 \frac{e^2}{c} \gamma^2 \frac{K^2/2}{(1 + K^2/2)^2} [JJ]^2$$

Where
$$[JJ] = J_1\left(\frac{K^2/4}{(1+K^2/2)^2}\right) - J_0\left(\frac{K^2/4}{(1+K^2/2)^2}\right)$$

Figure 4.9 The angle-integrated undulator spectrum for K = 1. The dotted lines are individual harmonics and the solid line is their sum, including up to the 4th harmonic.

K.J. Kim, AIP Conf. Proc. 184, 565 (1989).

Brightness and Coherence [1] Electron beam size effects



 $\sigma'/\theta_{cen} = 0$ 1.0 $\sigma'/\theta_{cen} = \frac{1}{3}$ Photon flux per unit bandwidth (relative units) 0.8 $\sigma'/\theta_{cen} = \frac{2}{3}$ 0.6 $\sigma'/\theta_{cen} = 1$ 0.4 $\sigma'/\theta_{cen} = 2$ 0.2 0 400 300 350 450 500 Photon energy (eV)

Beam angular divergence (σ')



Preserving the spectral line shape of undulator radiation requires

$$\sigma'^2 \ll \theta_{\rm cen}^2$$
 (5.55b)

Define effective, or total central cone half-angles

$$\theta_{Tx} = \sqrt{\theta_{\text{cen}}^2 + {\sigma'_x}^2} \text{ and } \theta_{Ty} = \sqrt{\theta_{\text{cen}}^2 + {\sigma'_y}^2} \quad (5.56)$$

S. Lidia, Light Sources and Free Electron Lasers, Slide 14

Courtesy D. Attwood

Bending Magnet, Wiggler, Undulator



Optical Beamline





Due to high absorption, soft and hard x-ray optics are typically made from multi-layer Bragg reflectors, bent for focusing - "Kirkpatrick-Baez" geometry

Some EUV focusing optics are transmissive, based on high-order Fresnel lenses.

See https://www.cxro.lbl.gov/



Undulator

Brightness and Coherence [2] Transverse Coherence

- Diffraction limited radiation fields can be described by modes (cf. Yariv, Siegman, Loudon, etc.)
- Coherent modes have propagation characteristics:
 - Wavelength λ , Rayleigh length Z_R , waist σ_r
 - Divergence $\sigma_{r'}$ $Z_R = \sigma_r / \sigma_{r'}$
 - Lowest order mode is Gaussian

$$\sigma_r(z) = \sqrt{\frac{\lambda}{4\pi} \left(Z_R + \frac{z^2}{Z_R} \right)} \ \sigma_{r'}(z) = \sigma_{r'}$$
 At z=0: $\sigma_r \sigma_{r'} = \frac{\lambda}{4\pi} = \varepsilon_{r,min}$



Photon beam emittance

• Overlap with electron beam ($\sigma_{\chi}, \sigma_{\chi'}$) produces the resultant radiation pattern

• $\Sigma_x = \sqrt{\sigma_x^2 + \sigma_r^2}$ $\Sigma_{x'} = \sqrt{\sigma_{x'}^2 + \sigma_{r'}^2}$

• If the electron beam moments are negligible with respect to the radiation mode

» $\Sigma_{\chi}\Sigma_{\chi'} \approx \sigma_r \sigma_{r'} = \frac{\lambda}{4\pi}$ 'diffraction-limited radiation', transversely coherent

• If the electron beam moments dominate the radiation mode

 $\gg \Sigma_{\chi} \Sigma_{\chi'} \gg \frac{\lambda}{4\pi}$

'incoherent radiation'

Intermediate case is 'partially coherent' radiation
 » No. of coherent modes is represented by

»
$$M_T = \frac{\Sigma_x \Sigma_{x'}}{\lambda/4\pi} \approx \frac{\varepsilon_x}{\varepsilon_r}$$





Brightness and Coherence [3] Longitudinal Coherence

- Assume each electron in the bunch emits photons independently and randomly
 - $E(t) = \sum_{j=1}^{N_e} E_0(t-t_j) = e_0 \sum_{j=1}^{N_e} exp\left[-\frac{(t-t_j)^2}{4\sigma_r^2} i\omega_r(t-t_j)\right]$ total radiation electric field • $E(\omega) = \frac{e_{0\sqrt{\pi}}}{\sigma_{0}} \sum_{j=1}^{N_e} exp\left[-\frac{(\omega-\omega_r)^2}{4\sigma_{0}} + i\omega_r t_j\right]$ frequency description (Fourier transform), with $\sigma_{\omega} = 1/\sigma_{\tau}$
- Define coherence time, t_{coh} , of optical field $E_0(t) = e_0 exp \left| -\frac{(t-t_j)^2}{4\sigma_\tau^2} i\omega_r(t-t_j) \right|$ via
 - $t_{coh} = \int d\tau |C(\tau)|^2 = 2\sqrt{\pi}\sigma_{\tau}$ where the 1st order correlation function is • $C(\tau) \equiv \frac{\langle \int dt E(t)E^*(t+\tau) \rangle}{\langle \int dt E(t)E^*(t) \rangle}$ where $\langle \rangle$ indicates the ensemble average of sources
- Consider an observation time T_{obs} • The no. of *longitudinal modes*, M_L , is $M_L \approx \frac{T_{obs}}{t_{obs}} \approx \frac{T_{obs}}{4\sigma_c}$

 $\sigma_{N_{ph}}^{2} = \frac{\langle N_{ph} \rangle^{2}}{M_{*}}$ Fluctuation in photon number in observation

- Considering quantum statistics (degeneracy) $\sigma_{N_{ph}}^2 = \frac{\langle N_{ph} \rangle^2}{M} + \langle N_{ph} \rangle M = M_L M_T^2$
- Back to the electrons . . . $\langle |E(\omega)|^2 \rangle = |E_{\omega 0}|^2 \langle \left| \sum_{j=1}^{N_e} exp[-i\omega_r t_j] \right|^2 \rangle = |E_{\omega 0}|^2 \langle N_e + \left| \sum_{j\neq k}^{N_e} exp[-i\omega_r(t_j t_k)] \right| \rangle$
- Electrons longitudinal distribution $f(t), \tilde{f}(\omega) \rightarrow \left\langle \sum_{j \neq k}^{N_e} exp[i\omega_r(t_j t_k)] \right\rangle = N_e(N_e 1) \left| \tilde{f}(\omega) \right|^2$
- $\langle |E(\omega)|^2 \rangle = N_e |E_{\omega 0}|^2 \left[1 + (N_e 1) |\tilde{f}(\omega)|^2 \right]$



Coherent enhancement if electron distribution has structure (microbunching) at frequency ω

Electron Rings As Light Sources

	Elettra	ALBA	DLS	ESRF	APS	SPring-8
Energy	2 GeV	3 GeV	3 GeV	6 GeV	7 GeV	8 GeV
Circumference	259 m	269 m	562 m	845 m	1104 m	1436 m
Lattice type	DBA	DBA	DBA	DBA	DBA	DBA
Current	300 mA	400 mA	300 mA	200 mA	100 mA	100 mA
Hor. emittance	7.4 nm	4.4 nm	2.7 nm	4 nm	3.1 nm	3.4 nm



Ring parameters can be adjusted to meets specific requirements. Presented parameters are typical operational parameters.



Choice of Beam Energy

Advantages of higher electron beam energy:

- Easier to produce high-energy photons (hard x-rays).
- Better beam lifetime.
- Easier to achieve higher current without encountering beam instabilities.

Disadvantages of higher beam energy:

- Higher energy beams have larger emittances (reduced brightness) for a given lattice.
- Stronger (more expensive) magnets are needed to steer and focus the beam.
- Larger rf system needed to replace synchrotron radiation energy losses.
- Many modern machines settle around 3 5 GeV



Typical Applications of Light Sources Protein Crystallography

SPRING 8 Japan



Protein crystallography

The best-established method to reveal the three-dimensional structure of proteir



The ribbon model represents the norovirus 3C-like protease; una uses determined at CDring 0

Preparation and Execution Of Experiments









Result Processing







Developing High Brightness Synchrotrons



Development of Multi-Bend Achromat (MBA) lattices to replace 3rd Generation Triple Bend Achromats (TBA)



MBA Upgrade Increases Beam Brightness for Diffraction Limited X-rays



Parameter	Units	ALS	ALS-U
Electron energy	GeV	1.9	2.0
Horiz. emittance	pm 🕻	2000	~50
Vert. emittance	pm	30	~50
Beamsize @ ID center (σ_x / σ_y)	μm 🤇	251/9	<10 / <10
Beamsize @ bend (σ_x / σ_y)	μm	40 / 7	<5 / <7
bunch length (FWHM)	ps 🄇	60-70 (narmonic cavity)	120-200 (harmonic cavity)
RF frequency	MHz	500	500
Circumference	m	196.8	~196.5
		А	LS-U

Free Electron Lasers

- Free electron lasers operate on the principle of electromagnetic instability involving three components:
 - A relativistic electron beam
 - A magnetic undulator (K<1)
 - A co-propagating radiation (EM) field
- The undulator field couples with the transverse electron motion and allows a transverse current to interact with the transverse EM electric field.
- The EM field modulates the beam energy, producing bunching and debunching over the EM wavelength.



- Microbunching further enhances the coherent coupling between electron and photon field → gain
- These system can operate in oscillator or amplifier configurations
- Lack of available optics for hard x-rays means that only amplifier configurations are possible
 - Self-Amplified Spontanaeous Emission (SASE) and Cascaded High Gain Harmonic Generation (HGHG) processes



Storage Ring and Linac FELs



1-D Gain and Scaling in High Gain Regime

- Linear gain regime
 - Microbunches form and become well defined
 - Slippage carries photons to different microbunches
 - Radiated power is increasing exponentially

 $P_{rad}(z) = P_0 e^{z/L_{gain}}$ -- power amplifier

- Microbunch energy spread increases
- 'Cooperation' length (I_c) is the amount of slippage that occurs over a gain length
- Saturation
 - The field saturates after a length $L_{sat} \approx 20 L_{gain}$
 - Radiated power is $P_{sat} = \rho_{FEL} I_{beam} E_{beam}$
- Caveats (ie. what are the hidden assumptions?)
 - Optimal phase space overlap between electron and photon beams » $\varepsilon_{\perp} < \lambda/4\pi$
 - Relative energy spread should be smaller than FEL parameter $\sigma_{\delta} < \rho_{FEL}$



 $L_{gain} \approx \frac{\lambda_u}{4\sqrt{3}\rho_{FEL}}$

 $\rho_{FEL} = \left[\frac{K}{4\nu} [JJ]^2 \frac{\omega_p}{\omega_u}\right]^{2/3}$

 $\omega_p^2 = 4\pi r_e c^2 n_e / \gamma$

 $l_c \approx \frac{\lambda_{rad}}{\lambda_{rad}} L_{gain}$

Linac Coherent Light Source (LCLS)



Table1 | Design and typical measured parameters for both hard (8.3 keV) and soft (0.8–2.0 keV) X-rays. The 'design' and 'hard' values are shown only at 8.3 keV. Stability levels are measured over a few minutes.

Parameter	Design	Hard	Soft	Unit
Electrons				
Charge per bunch	1	0.25	0.25	nC
Single bunch repetition rate	120	30	30	Hz
Final linac e ⁻ energy	13.6	13.6	3.5-6.7	GeV
Slice [†] emittance (injected)	1.2	0.4	0.4	μm
Final projected [†] emittance	1.5	0.5-1.2	0.5-1.6	μm
Final peak current	3.4	2.5-3.5	0.5-3.5	kA
Timing stability (r.m.s.)	120	50	50	fs
Peak current stability (r.m.s.)	12	8-12	5-10	%
X-rays				
FEL gain length	4.4	3.5	~1.5	m
Radiation wavelength	1.5	1.5	6-22	Å
Photons per pulse	2.0	1.0-2.3	10-20	10 ¹²
Energy in X-ray pulse	1.5	1.5-3.0	1-2.5	mJ
Peak X-ray power	10	15-40	3-35	GW
Pulse length (FWHM)	200	70-100	70-500	fs
Bandwidth (FWHM)	0.1	0.2-0.5	0.2-1.0	%
Peak brightness (estimated)	8	20	0.3	10 ³² *
Wavelength stability (r.m.s.)	0.2	0.1	0.2	%
Power stability (r.m.s.)	20	5-12	3-10	%

*Brightness is photons per phase space volume, or photons s^{-1} mm⁻² mrad⁻² per 0.1% spectral bandwidth.

¹Slice' refers to femtosecond-scale time slices and 'projected' to the full time-projected (that is, integrated) emittance of the bunch.

Lasing at 1.5 Angstroms



P. Emma, et al, Nature Photonics, v.4, Sept 2010.



Self Amplified Spontaneous Emission

- From 1-D gain $P_{rad}(z) = P_0 e^{z/L_{gain}}$
- What happens when P₀ = 0?
- Startup from noise
 - Electron bunch is composed of distinct particles with randomized positions. Schottky noise exists over broad band.
 - Noise spectrum components at the fundamental wavelength (and harmonics) couple to the EM field
- Temporal coherence is lost

$$\frac{\Delta\omega}{\omega} \sim 2\rho \sim 10^{-3}$$

$$\left(\frac{\Delta\omega}{\omega}\right)_{spike} \sim \frac{1}{\sigma_T \omega} \sim 10^{-5}$$



Figure 5.2.4 Temporal (top) and spectral (bottom) structure for 12.4 keV XFEL radiation from SASE 1. Smooth lines indicate averaged profiles. Right side plots show enlarged view of the left plots. The magnetic undulator length is 130 m.

Source: The European XFEL TDR - DESY 2006-097 (2006)



Facility for Rare Isotope Beams

Two-Stage Cascaded Harmonic Generation Seeded amplifiers can limit the growth of the noise spectrum





Facility for Rare Isotope Beams

Electron Beam Quality Determines Performance

- Electron beam injectors are pushing on brightness frontiers
 - Short bunches (< ps) and lower bunch charges (10s-100s pC)
- Emittance exchange methods (swap warm for cold phase spaces when needed)
- Increase incoherent energy spread to prevent longitudinal microbunching instabilities ('laser heater')





Thank You!



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Additional topics

- Undulator topics
 - Transverse multipoles
 - Phase errors
 - Variable polarization
- KMR equations
 - Simulation of bunching and beam intensity (1D)
 - SASE
 - Self-seeded hard xray
- 3D effects
 - Electron emittance slice/projected, matching parameter
 - Optical diffraction, gain guiding
 - wakefields

