Phys 862 Iron Dominated Magnets

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Maxwell Basics

What is an Iron Dominated Magnet?

- A magnet is a material or object that produces a magnetic field.
- Iron Dominated means that iron is used to shape the fields inside the magnets (ex. Iron poles shape the fields).
- Guides the charged particles consistent with $F = qv \times B$.
- The Maxwell equations describe the field in the vacuum aperture and magnet assembly.
- The Magnetic Fields can resolve into components x, y, z and multipole expanded.

Maxwell Equations in Vacuum &Matter

Maxwell's Equations in Matter:

$$\nabla \cdot \mathbf{D} = \rho \tag{1}$$
$$\nabla \cdot \mathbf{B} = 0 \tag{2}$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{3}$$
$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \tag{4}$$

where:

$\mathbf{D}$: Electric displacement field
$\mathbf{B}$: Magnetic flux density
$\mathbf{E}$: Electric field
$\mathbf{H}$: Magnetic field
ρ : Free charge density
$\mathbf{J}$: Free current density

Maxwell's Equations in Vacuum:

$$\nabla \cdot \mathbf{E} = 0 : \text{Guass's Law} \tag{5}$$
$$\nabla \cdot \mathbf{B} = 0 : \text{Gauss's Law for Magnetism} \tag{6}$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} : \text{Faraday's law} \tag{7}$$
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} : \text{Apere's law} \tag{8}$$

where:

$\mathbf{E}$: Electric field
$\mathbf{B}$: Magnetic field
μ_0 : Permeability of free space
ϵ_0 : Permittivity of free space
$\mathbf{J}$: Free current density

B field

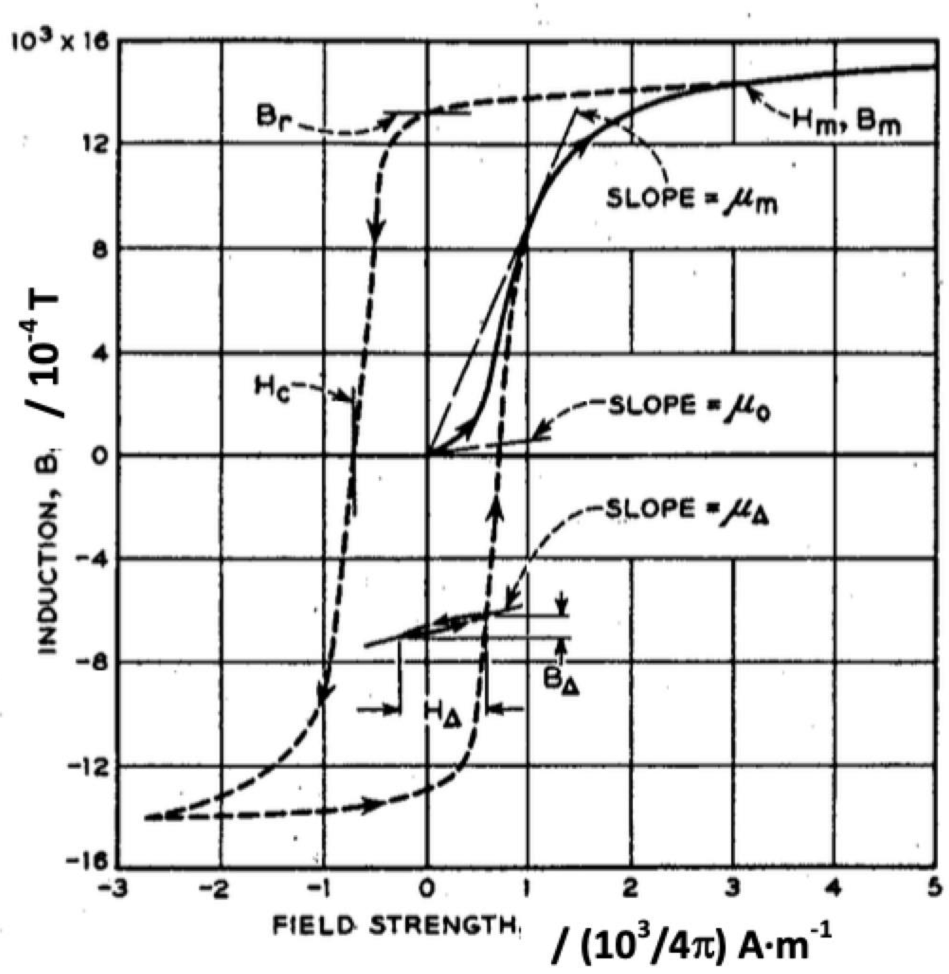
H is the magnetic field
B is the magnetic flux density or magnetic induction

$$B = \mu_r \mu_0 H$$

$\mu_0 = 4\pi \cdot 10^{-7}$ T — Permeability of free space

μ_r — Relative permeability

Unlike the permeability of free space, the relative permeability is not a constant but varies with applied field strength. In some materials like Iron there is a curve that represents B density vs H strength



Magnetization curve and hysteresis loop of iron [1]

Development of B fields

B field: Taylor expansion to obtain Multipole Terms

- If the fields are static or sufficiently slowly varying where the time derivatives can be neglected, then the maxwell equations for B in the aperture will satisfy:

$$\nabla \cdot \mathbf{B} = 0 \qquad \beta_b c = v$$
$$\nabla \times \mathbf{B} = 0$$

- The Perpendicular Components of the F and B field are as follows:

Force: $\mathbf{F}_\perp^a \simeq q\beta_b c\hat{\mathbf{z}} \times \mathbf{B}_\perp^a$ $\qquad F_x^a \simeq -q\beta_b c B_y^a$

Field: $\mathbf{B}_\perp^a = \hat{\mathbf{x}} B_x^a + \hat{\mathbf{y}} B_y^a$ $\qquad F_y^a \simeq q\beta_b c B_x^a$

- We can Expand the B field components using a Tylor series in vacuum aperture
- Terms in expansion will depend on the magnet system being analyzed and Initial Conditions
- Non-linear terms will have effects. Goal is to minimize in linear optics.
- Taylor coefficients not all independent, they must obey Maxwell equations in vacuum

$$B_x^a = \overset{1}{B_x^a(0)} + \overset{2}{\frac{\partial B_x^a}{\partial y}(0)y} + \overset{3}{\frac{\partial B_x^a}{\partial x}(0)x}$$

Nonlinear Focus

$$\boxed{+\frac{1}{2}\frac{\partial^2 B_x^a}{\partial x^2}(0)x^2 + \frac{\partial^2 B_x^a}{\partial x \partial y}(0)xy + \frac{1}{2}\frac{\partial B_x^a}{\partial y^2}(0)y^2 + \cdots}$$

$$B_y^a = \overset{1}{B_y^a(0)} + \overset{2}{\frac{\partial B_y^a}{\partial x}(0)x} + \overset{3}{\frac{\partial B_y^a}{\partial y}(0)y}$$

Nonlinear Focus

$$\boxed{+\frac{1}{2}\frac{\partial^2 B_y^a}{\partial x^2}(0)x^2 + \frac{\partial^2 B_y^a}{\partial x \partial y}(0)xy + \frac{1}{2}\frac{\partial B_y^a}{\partial y^2}(0)y^2 + \cdots}$$

Terms:
1: Dipole Bend
2: Normal Quad Focus
3: Skew Quad Focus

Cauchy Reiman Conditions in B field. B field is an Analytic Function

This B equations satisfy the Maxwell Equations

$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{B} = 0$$

B(x,y,z)

$$\overline{B_x}(x,y) = \frac{1}{\ell}\int_{-\infty}^{\infty} dz\ B_x^a(x,y,z)$$

$$\overline{B_y}(x,y) = \frac{1}{\ell}\int_{-\infty}^{\infty} dz\ B_y^a(x,y,z)$$

2D Effective Fields

3D Fields

$$\mathbf{B(z)^* = B_x(x,y) - iB_y(x,y)} = \sum_{n=1}^{\infty} \mathbf{b_n(x+iy)}^{n-1}$$

$$\mathbf{B(z)^*} = \sum_{n=1}^{\infty} \mathbf{b_n}(z)^{n-1}$$

Laurent Series For B Field

n = multipole index

b_n =const(complex)

Cauchy-Riemann Conditions

$$\underline{F} = u(x,y) + iv(x,y)$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \longrightarrow$$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \longrightarrow$$

$\underline{F} = u + iv$ analytic func of $\underline{z} = x + iy$

2D Magnetic Field

$$u = \overline{B_x} \quad v = -\overline{B_y}$$

$$\frac{\partial \overline{B_x}(x,y)}{\partial x} = -\frac{\partial \overline{B_y}(x,y)}{\partial y}$$

$$\frac{\partial \overline{B_x}(x,y)}{\partial y} = \frac{\partial \overline{B_y}(x,y)}{\partial x}$$

$\underline{F} = \overline{B_x} - i\overline{B_y}$ analytic func of $\underline{z} = x + iy$

[3]

B fields

$$\mathbf{B(z)}^* = \sum_{n=1}^{\infty} \mathbf{b_n}(z)^{n-1}$$

$$\mathbf{B(z)}^* = \sum_{n=1}^{\infty} (\mathbf{A_n} - \mathbf{iB_n})(\mathbf{x} + \mathbf{iy})^{\mathbf{n-1}}$$

$$\mathbf{A_n} = \text{"}Skewed\text{"}\,Multipoles$$

$$\mathbf{B_n} = \text{"}Normal\text{"}\,Multipoles$$

$$\mathbf{B(z)}^* = \sum_{n=1}^{\infty} \mathbf{b_n}\left(\frac{z}{r_p}\right)^{n-1}$$

$$\mathbf{r_p} = Aperature\text{"}pipe\text{"}radius$$

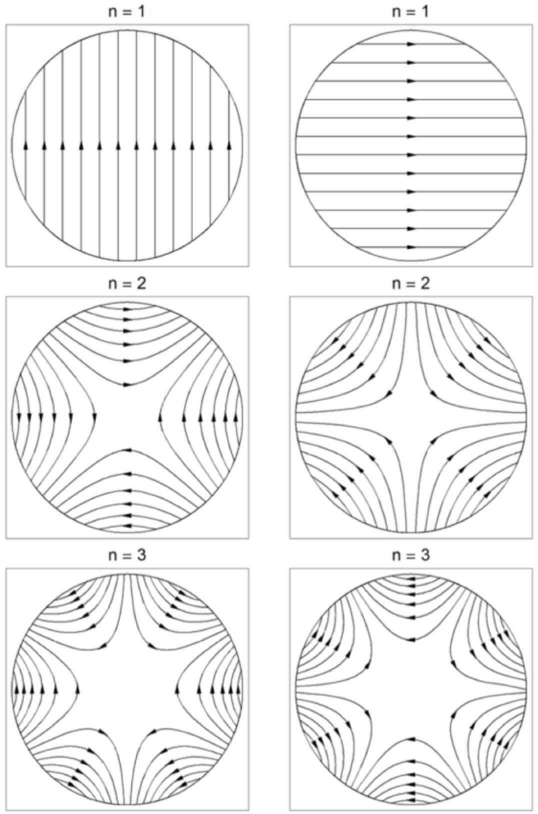
Rescale bn, resulting in all field units (T)

Multipole Equations: The generated fields have different geometric shapes

Cartesian projections: $\overline{B_x} - i\overline{B_y} = (\mathcal{A}_n - i\mathcal{B}_n)(x+iy)^{n-1}$

Index	Name	Normal ($\mathcal{A}_n = 0$)		Skew ($\mathcal{B}_n = 0$)	
n		$\overline{B_x}/\mathcal{B}_n$	$\overline{B_y}/\mathcal{B}_n$	$\overline{B_x}/\mathcal{A}_n$	$\overline{B_y}/\mathcal{A}_n$
1	Dipole	0	1	1	
2	Quadrupole	y	x	x	$-y$
3	Sextupole	$2xy$	$x^2 - y^2$	$x^2 - y^2$	$-2xy$
4	Octupole	$3x^2y - y^3$	$x^3 - 3xy^2$	$x^3 - 3xy^2$	$-3x^2y + y^3$
5	Decapole	$4x^3y - 4xy^3$	$x^4 - 6x^2y^2 + y^4$	$x^4 - 6x^2y^2 + y^4$	$-4x^3y + 4xy^3$

Multipole Fields Normal Right Skewed Right: [1]



Skewed: Normal pole rotated by pi/(2n)

Types of Magnets & Designs

Magnet Types

Some Examples of Iron Dominated Magnets:

- Dipole (Bending)
- Quadrupole (Focus or Defocus)
- Sextuple (correct chromatic aberration due to dispersion in a dipole bends)
- Octupole (Damping of the collective Effects)

Iron Dominated Magnet: Iron Yoke has coils placed in it. Coils Provide flux in the magnet. Magnet fields shaped by the poles of the iron.

[4]

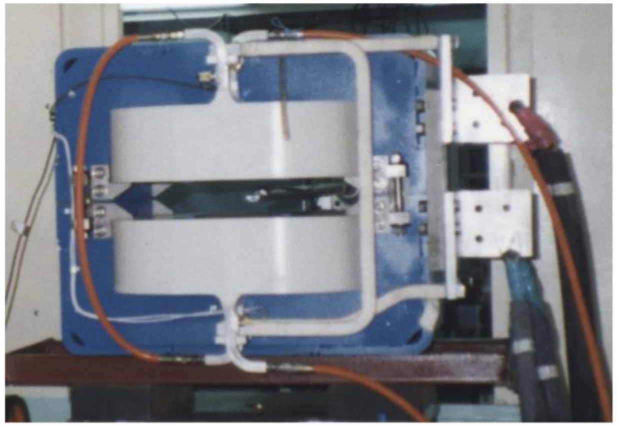


Figure 1 PEPII Low Energy Ring Dipole

Figure 3 SPEAR3 Quadrupole Magnet

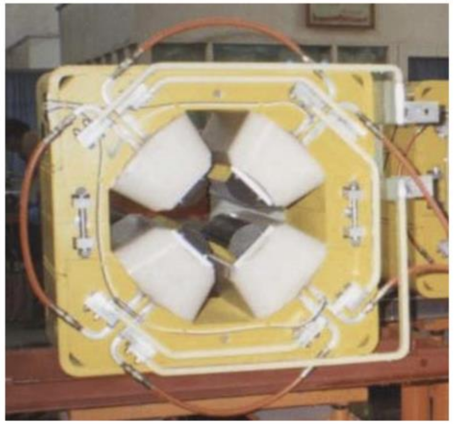


Figure 4 PEPII Quadrupole

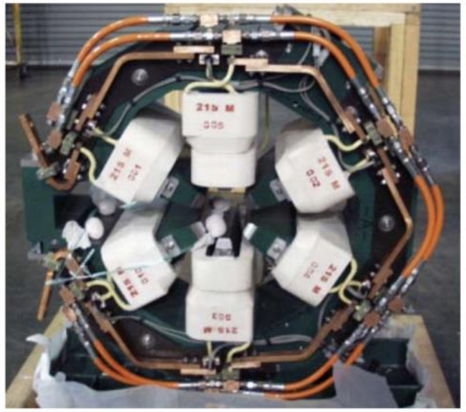
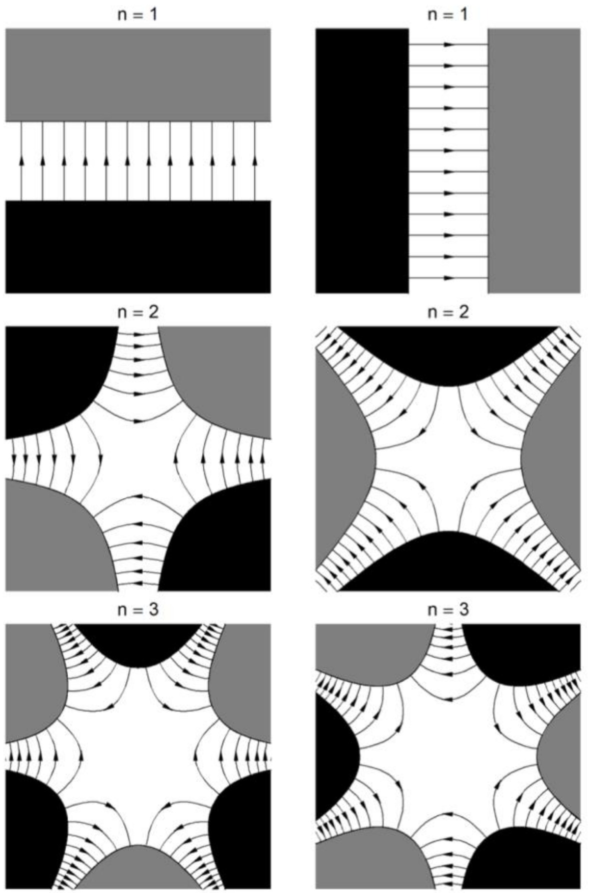


Figure 5 SPEAR3 Sextupole Magnet with Skew Quadrupole Trim Coils

Generating Fields

Ideal/Infinite mu and length

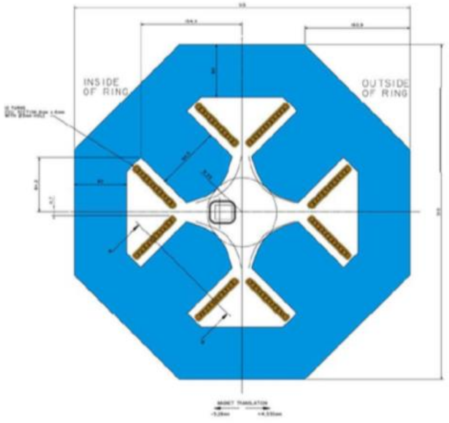


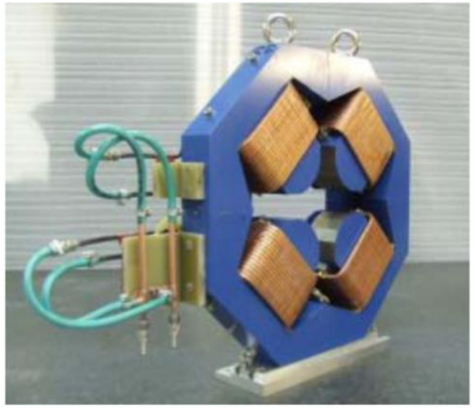
The fields need to have poles North and South. Coils must be inserted with alternate current directions passing through

strength of the magnetic field is determined by the number of amp-turns in coils providing magnet flux into poles

[1]

Iron Dominated Quadrupole magnet (EMMA Fixed-Field Alternating Gradient accelerator at Dares- bury Laboratory) [1]





Quadrupole provides focusing along one axis and defocusing on the other.
The Iron Poles have hyperbolic contours, B field lines are perpendicular to the contours [4]

Boundary Conditions/Magnetic Potential

$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{B} = 0$$

In vacuum the equations for B

$$\mathbf{B} = -\nabla\phi_m$$
$$\nabla^2\phi_m = 0 : \text{Laplace's equation}$$
$$\mathbf{B_x} = Gy$$
$$\mathbf{B_y} = Gx : \mathrm{G} = \text{gradient}$$

B can be expressed as the gradient of a scaler magnetic potential.

Plugging back in leads to Laplace's equation which can be solved, with Boundary conditions.

Geometric designs must satisfy the field gradients and field lines. Truncated Poles and specified geometries etc.

Field errors Non-ideal

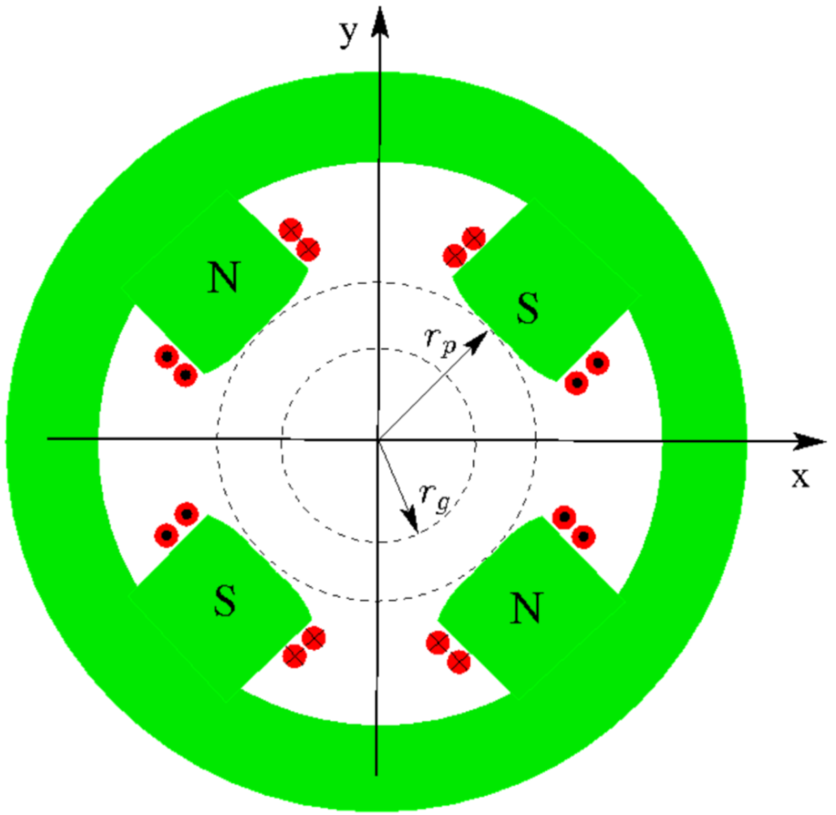
The design field is one that is still in some sense 'ideal'; though the design field for a quadrupole magnet (for example) will contain other multipole components, because the design has to respect practical constraints, i.e., the magnet will have finite longitudinal and trans- verse extent, any currents will flow in wires of non-zero dimension, and any materials present will have finite (and often non-linear) permeability. Usually, one attempts to optimize the design to minimize the strengths of the multipole components apart from the one required: the residual strengths are generally known as *systematic* multipole errors. [1]

The Errors arise from the real magnet designs. Unlike the ideal case, the magnetic fields aren't exactly parabolic, straight, and etc. They deviate from the ideal cases. Only certain terms then are allowed due to symmetries. Harmonics of the Multipoles.

Quadrupole Magnet and Good field Radius

In the expansion Higher order terms lead to nonlinear focusing effects but drop of rapidly with decreasing aperture

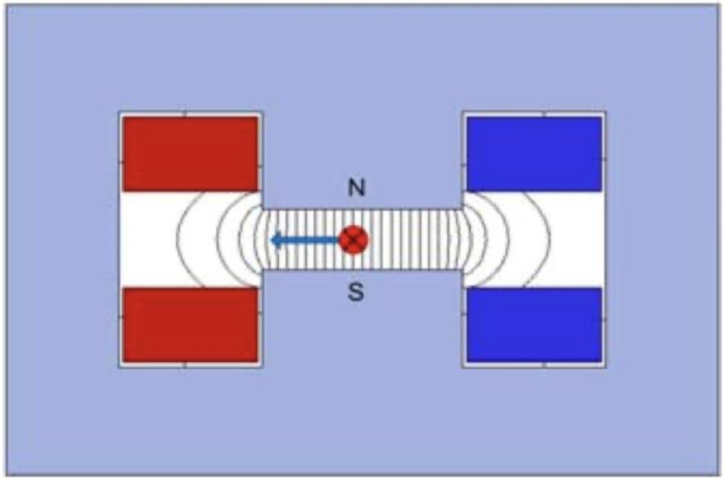
- Generally, a magnet will be designed with a good field radius where the errors are confined to that region.
- In Optimal designs the Good field radius is 70% or more of the the Bore aperture radius.
- ‘Mitigate the effects of the nonlinear terms by making the Magnet bigger (ie. Increase Bore Radius).
- Have more tight focus control over the beam.
- Good field region can be circular, elliptical, rectangular and takes into account the maximum beam size [4].



[3]

Dipoles Magnets

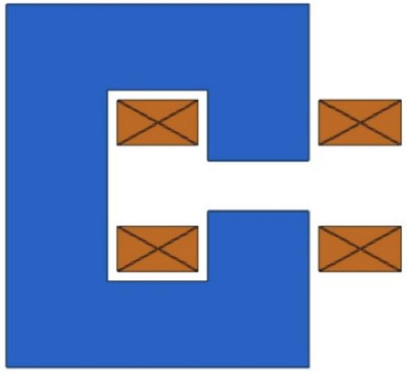
[6]



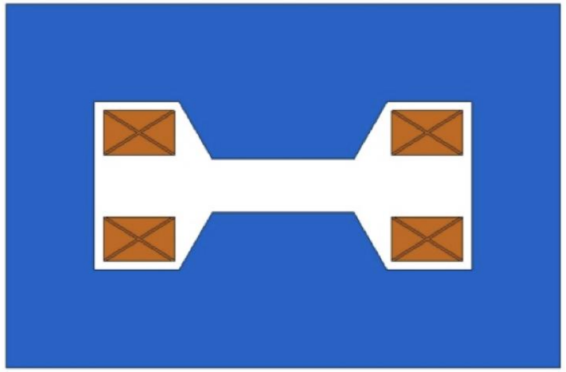
AGS Dipole Magnets [2]

Alternating Gradient Synchrotron

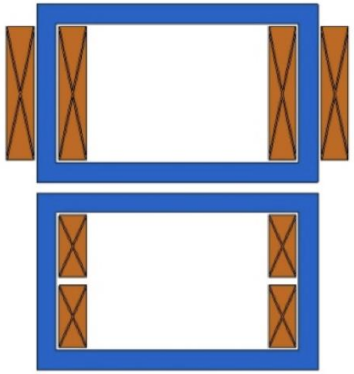
C magnet. Provides good accessibility to beam pipes.
Large Yoke Volume compared to H magnet [4]



H magnet is standard in many accelerators. Hard to access but has good mechanical stability and a symmetric field quality.



Window frame or O type magnets. Produce uniform fields. Top uses racetrack coils while bottom uses saddle coils.



Quadrupole Magnets

a) Standard Type Quadrupole. Provides larger coil windows and less saturation in pole roots

b) Collins Type. Expensive to make Easily Accessible

c)Panofsky, Like window frame magnet, Excellent field quality, but only used as a corrector

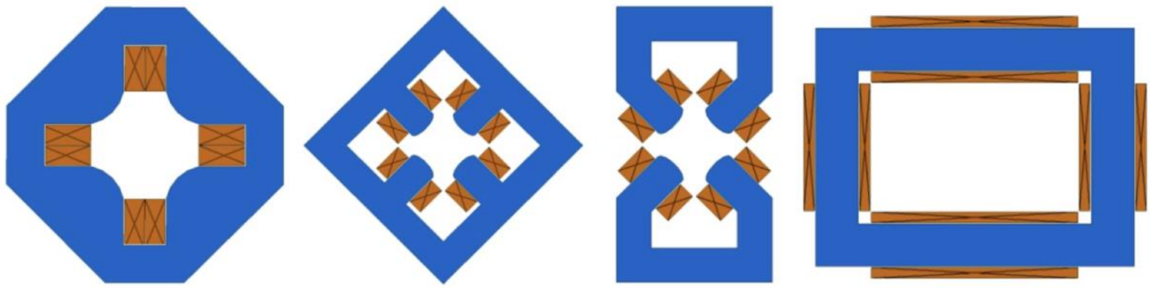
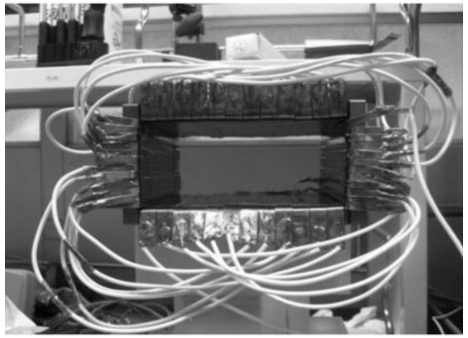


Figure 3 SPEAR3 Quadrupole Magnet

[4]



*Not really and Iron Dominated Magnet since Field is determined by current distribution in the coper conductors and not the iron yoke. [4]

Coil Excitation/Hysteresis

Coil Excitations For Dipole, Quad, Sextupole

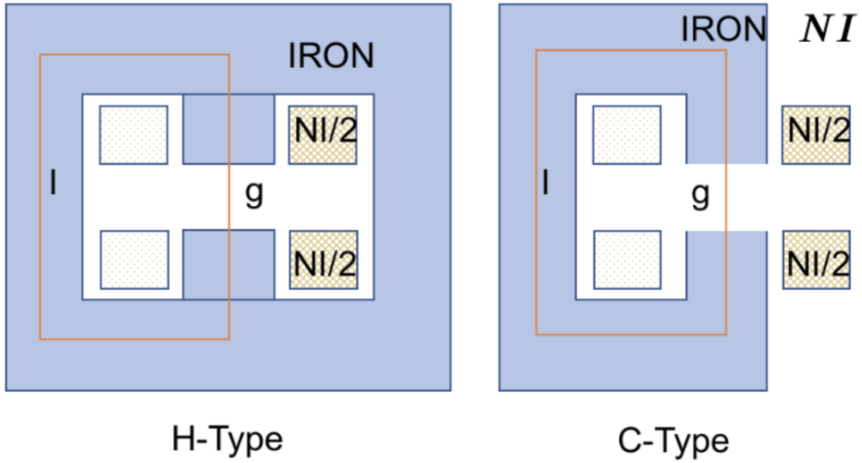
- Apply Amperes Loop law along the Iron magnet to obtain the needed current to the coils to provided the desired Magnetic Flux or field in the magnet.

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{\text{enclosed}}$$

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu I_{\text{enclosed}}$$

n = 2 Quadrupole $NI = \dfrac{B_1 R^2}{2\mu_0}$ [2]

n = 3 Sextupole $NI = \dfrac{B_2 R^3}{3\mu_0}$ [2]

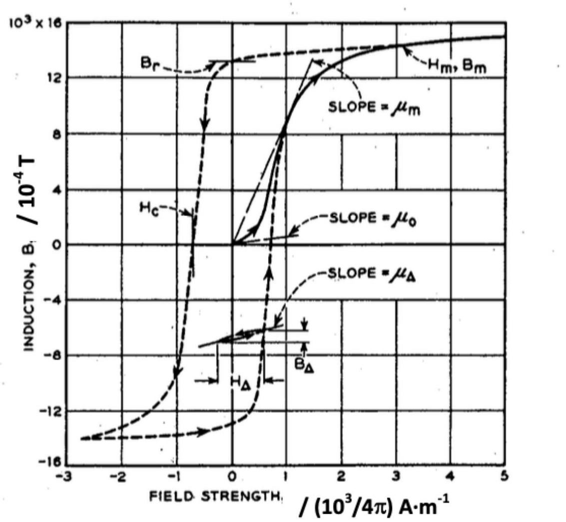


$$NI = B_{air}\,(g + l/\mu_r)/\mu_0 \sim B_{air} g/\mu_0$$

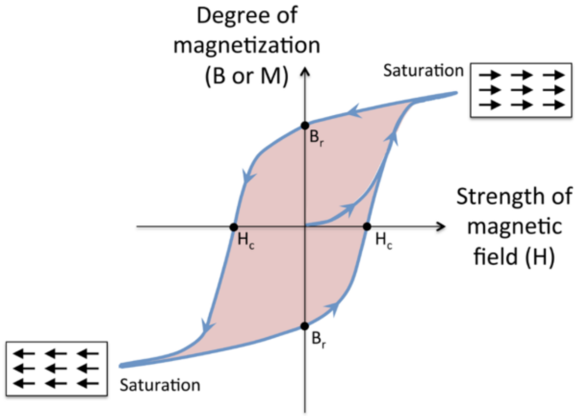
n = 1 Dipole $NI = \dfrac{Bh}{\mu_0}$ Window Magnet

Hysteresis B(H) curves

[1]



[2]



- Flux induction B(H) as a function of field strength is different when Increasing or decreasing strength
- When the current is switched off some magnetic polarizaition of iron Remains (Remanent Field)
- Width of the curve is determined by the coercive force Hc.
- Hc < 1000 A/m Soft Magnetic ex Electro Steel
- Hc > 1000 A/m hard magnetic ex permanent magnets [4]

Electro Steel - It is an iron alloy with silicon as the main additive element (instead of carbon). [5]

When driving a magnet from an unmagnetized state to a magnetized state toward designed point

Conclusion

- Magnets are more complicated than they seem

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