Homework, Lectures 7-9

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PHY862 “Accelerator Systems”

Lecture 7.

1. Recently, the researches are underway to use copper travelling-wave acceleration structures at cryogenic temperatures*. The considered option is the structure operating at frequency of 11424 MHz (X-band) at temperature of 77 K.

Estimate for RRR=400:

- Copper conductivity σ at 77 K. Use the data for copper resistivity $\rho = \sigma^{-1}$ versus temperature for different sample purity sown in the slide 16.
- Mean free pass l at 77 K. Use Formula (1), slide 17;
- Classical skin depth δ at 77 K;
- What type of skin effect does one have at 77 K for copper at 11424 MHz?
- Estimate surface resistance using relevant formula from Lecture 8, slides 15 and 18. Compare to the figure at the slide 18. Difference may be explained by copper processing.

\# What surface resistance one may expect for RRR=400 copper at 4K ?

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* Mamdouh Nasr, *et al*., "Experimental demonstration of particle acceleration with normal conducting accelerating structure at cryogenic temperature," PHYSICAL REVIEW ACCELERATORS AND BEAMS 24, 093201 (2021)

\# Complementary problem

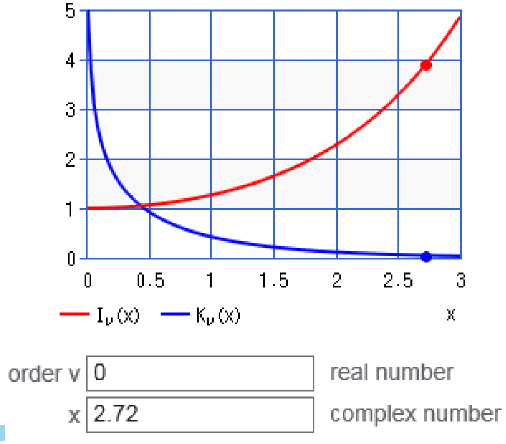
Lecture 7.

2. Why it is impossible to accelerate particles in a straight waveguide?

a. Show that in a waveguide $v_{ph} > c$, and therefore, there is no synchronism of an accelerating EM wave and an accelerated particle. Use dispersion relation for the waveguide, slide 22, Lecture 8: $k_\perp^2 + k_z^2 = \frac{\omega^2}{c^2} \equiv k^2$; $v_{ph}=\omega/k_z$;

b. Show that in a waveguide the product $v_{ph} \cdot v_{gr}=c^2$. Use dispersion relation for the waveguide, see task 2a. Group velocity is $v_{gr} = d\omega/dk_z$;

3. Estimate the ratio of acceleration gain on the cavity axis over the gain at the cavity aperture for $a/\lambda = 0.25$ and $\beta = 0.5$ (Lecture 7, slide 25); note that $k=2\pi/\lambda$; $\gamma = 1/(1-\beta^2)^{1/2}$)



order v 0 real number

x 2.72 complex number

Lectures 7-8.

4.Acceleration TW structure described in the Task 1, operates at 77 K. Operation frequency is 11.424 GHz. Phase shift per cell ψ is $2\pi/3$.

Estimate for this structure:

- a. The cell radius b (slide 46, Lecture 7);
- b. The length of the cell d (slide 47, Lecture 7);
- c. R/Q of the cell (slide 55 and 47, Lecture 7);
- d. G-factor of the cell (slide 51, Lecture 7);
- e. Using results of the Task 1, estimate Q_0 (slide 50, Lecture 7);
- f. Calculate shunt impedance R_{cell} of the cell (slide 54, Lecture 7);
- g. Calculate number of cells per meter N and shunt impedance per meter $R=N\cdot R_{cell}$ (see Lecture 8, slide 16);
- h. Compare to the results from [1], Table I.

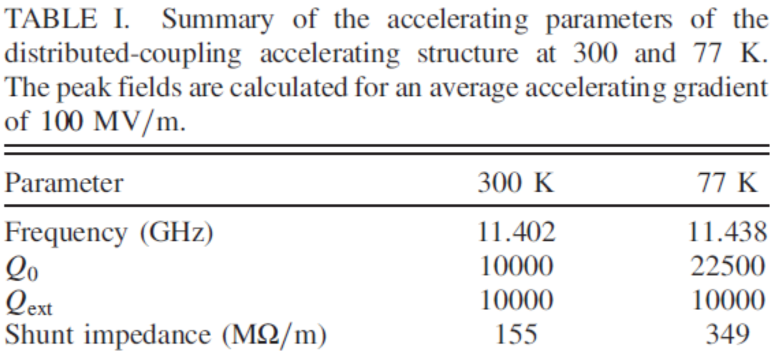


TABLE I. Summary of the accelerating parameters of the distributed-coupling accelerating structure at 300 and 77 K. The peak fields are calculated for an average accelerating gradient of 100 MV/m.

Parameter	300 K	77 K
Frequency (GHz)	11.402	11.438
Q_0	10000	22500
Q_{ext}	10000	10000
Shunt impedance (MΩ/m)	155	349

[1] Mamdouh Nasr, *et al.*, “Experimental demonstration of particle acceleration with normal conducting accelerating structure at cryogenic temperature,” PHYSICAL REVIEW ACCELERATORS AND BEAMS 24, 093201 (2021)

Lectures 7-8.

5. SRF 5-cell cavity designed for PIP II has the following parameters:

- Operating frequency is 650 MHz
- Acceleration voltage V is 20 MV
- R/Q is 620 Ohm
- Q_0 at operation voltage is 3e10
- The beam current I is 2 mA

Estimate for CW operation:

- The cavity loaded Q, $Q_L = V/(I\cdot(R/Q))$, see Lecture 7, slide 71
- The cavity optimal coupling β_{opt}, see Lecture 7, slide 71
- The cavity time constant, $\tau = 2Q_L/\omega$, , see Lecture 7, slide 72
- The cavity bandwidth , $\delta f = f/Q_L$; see Lecture 7, slide 69
- Loss power in the cavity walls, see Lecture 7, slide 54
- The energy stored in the cavity, see Lecture 7, slide 54
- The power transferred to the beam, see Lecture 7, slide 71
- Power required for refrigeration (CoP is 1.e3 W/W), see Lecture 8, slide 46 i.e., in order to remove 1 W from the cavity wall one needs wall plug power of 1 kW);
- Acceleration efficiency, the beam power over the sum of the power delivered by the RF source (which is equal in this case to the beam power, because power dissipated in the cavity is small) and power required for refrigeration.

Lecture 8

6[#]. Calculate the maximum temperature rise δT and maximum surface magnetic field H_b in the cavity, when cavity has thermal breakdown, see slide 82, Lecture 8. For estimations take the following parameters:

- *Operating frequency:* f=3.9 GHz
- *Helium temperature:* $T_0=2^oK$
- *Niobium thickness:* $d=3$ mm
- *Thermal conductivity:* $k(T_0)=30$ W/(m•K)
- *Kapitza resistance:* $h(T_0)=10^4$ W/(m²•K)
- *BCS surface resistance for Nb :*

$$R_s(T) = R_0 \cdot \left[\frac{f\,(GHz)}{1.3}\right]^2 \cdot \left(\frac{T_c}{T}\right) \cdot e^{-\Delta \cdot \frac{T_c}{T}} \quad , T << T_c$$

where: $R_0=10^{-5}$ [Ω]; $\Delta=1.8$; $T_c=9.2^oK$

Use assumption, that temperature rise is small compared to T_0.

Note that thermal equilibrium equations for bulk of niobium and in the niobium-helium transition layer are:

$$\frac{1}{2} \cdot R_s(T_s) \cdot H_s^2(T_s) = k(T_0) \cdot (T_s - T_m)/d \qquad (1)$$

$$k(T_0) \cdot (T_s - T_m)/d = h(T_0) \cdot (T_m - T_0) \qquad (2)$$

where: $T_s,\ T_m$ – temperature of niobium on inside and outside surfaces of the cavity. We assumed here that $k(T_s)= k(T_m) =k(T_0)$

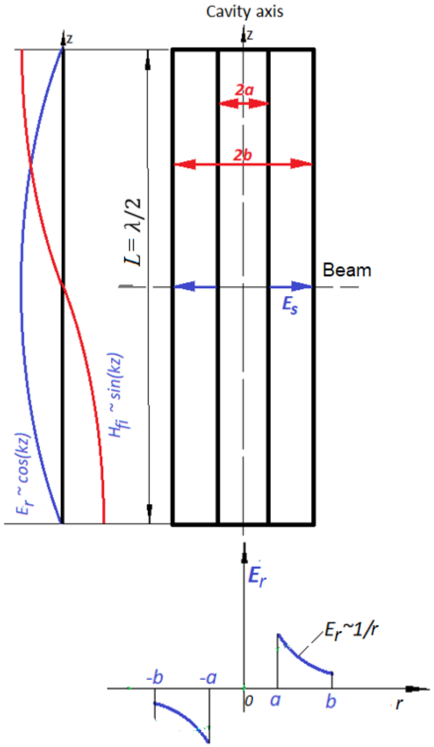
[#] Complementary problem

Lecture 9.

7. The ideal HWR cavity for β = 0.112 operates at frequency f = 162.5 MHz. It provides the energy gain (acceleration voltage) V = 2 MeV. The inner radius is a, the outer radius is b, see slides 28-30, Lecture 9

Estimate:

- The cavity length L;
- Effective cavity length;
- Acceleration gradient
- The optimal ratio b/a to achieve minimal surface electric field (on the inner electrode) at fixed b, taking into account that in a coaxial line $E_r \sim 1/r$;
- b and a to get maximal energy gain;
- The voltage difference U between the inner and outer electrodes at z=0.
- The coaxial impedance Z_c.
- The energy stored in the cavity;
- R/Q;
- The Ohmic loss in the cavity taking R_s=3 nOhm.
- Unloaded quality factor Q_0 and G-factor
- Maximal surface electric and magnetic fields and field enhancement factors K_B and K_E.



$Si(x)$ calculator: https://keisan.casio.com/exec/system/1180573420