PH4841

Chapter S HW Solutions

3.5 Homework Problems

- 1. Consider a sphere of radius ${\it R}$ and charge ${\it Q}$, where the charge is spread uniformly throughout the sphere.
 - (a) Find the strength of the electric field as a function of r.
 - (b) Find the electric potential as a function of r.
 - (c) Find the potential energy required to move the charges to their positions,

$$PE=rac{1}{2}\int d^3r \,
ho(r)V(r).$$

(d) Find the energy contained in the electric fields.

a) Gauss's Law

$$4\pi Q \frac{r^3}{R^3} = E \cdot 4\pi \Gamma^2$$
 $= Q \Gamma , r \cdot R$
 $= Q \Gamma$

$$PE = \frac{1}{2} \frac{3Q^{2}}{4\pi R^{3}} \int_{-\frac{1}{2}}^{4\pi r^{2}} dr \left\{ \frac{3}{2R} - \frac{r^{2}}{2R^{3}} \right\}$$

$$= \frac{3Q}{2R} \left\{ \frac{3}{2} \frac{1}{3} - \frac{1}{2} \cdot \frac{1}{5} \right\}$$

$$= \frac{3}{3} \frac{\alpha}{2} \left\{ \frac{3}{3} - \frac{1}{2} \cdot \frac{1}{5} \right\}$$

$$= \frac{3}{2} \frac{\alpha^{2}}{R}$$

$$= \frac{3}{5} \frac{\alpha^{2}}{R}$$

$$+\frac{Q^{2}}{8\pi}\int_{R}^{\infty}4\pi r^{2}dx\frac{1}{r^{4}}$$

$$= \frac{Q^{2}}{8\pi} \left\{ \frac{1}{5} + 1 \right\} = \frac{3}{5}$$

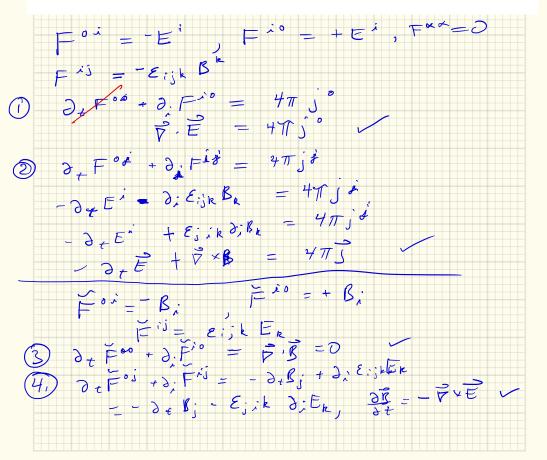
$$=\frac{Q^{2}}{2R}\left\{\frac{1}{5}+1\right\}^{2}=\frac{3}{5}\frac{Q^{2}}{R}$$

2. Beginning with $F^{\alpha\beta}$ written in term of \vec{E} and \vec{B} , restate

$$egin{array}{lll} \partial_{lpha} F^{lphaeta} &=& 4\pi j^{eta} \ \partial_{lpha} ilde{F}^{lphaeta} &=& 0 \end{array}$$

as

$$egin{array}{lll}
abla \cdot ec{E} &=& 4\pi
ho, \
abla imes ec{B} &=& \partial_t ec{E} + 4\piec{j}, \
abla \cdot ec{B} &=& 0, \
abla imes ec{E} &=& -\partial_t ec{B}. \end{array}$$

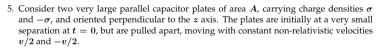


- 3. For a charge-free region, $j^{\alpha} = 0$
 - (a) Use Maxwell's equations to write a wave equation for \vec{E} , and show the speed of propagation is unity (c).
 - (b) For a wave traveling in the \hat{z} direction with the electric field in the $\pm \hat{x}$ direction, write a solution for the propagating plane wave for both $\vec{E}(\vec{r},t)$ and $\vec{B}(\vec{r},t)$.

a)
$$\overrightarrow{P} \times \overrightarrow{E} = -\frac{\partial R}{\partial t}$$
 $\overrightarrow{P} \times \overrightarrow{B} = \frac{\partial E}{\partial t}$
 $\overrightarrow{P} \times \overrightarrow{B} = -\frac{\partial A}{\partial t}$
 $\overrightarrow{E} = -\frac{\partial A}{$

4. First calculate $\hbar c$ in standard mks units. Then, using the fact that the charge on an electron is 1.602×10^{-19} Coulombs, find the constant k in mks units used in Coulomb's law, $PE = kq^2/r$. Use the fact that $PE = e^2/r$, where $e^2 = \hbar c/137.036$.

$$R = \frac{k c}{137.036} (1.602.10^{-19})^2$$



- (a) What is the electric field between the plates?
- (b) Find all four non-zero elements of the stress-energy tensor (T_{xx},T_{yy},T_{zz}) and T_{00} . Check that the stress-energy tensor is traceless.
- (c) In hydrodynamics, the work done by an expanding a gas is PdV. Here, because the expansion is along the z axis the work is $T_{zz}dV$. What is the power required to pull the plates apart at these velocities?
- (d) What is energy density of the field between the plates?
- (e) What is the rate (energy per time) at which the field energy between the plates increases due to the growing volume?

a)
$$E \cdot A = 4\pi 6 \cdot A$$
, $E_{\chi} = 4\pi 6$
b) $T^{\omega} = \frac{E^{2}}{8\pi}$, $T^{\omega} = T^{2} = +\frac{E}{9\pi}$
 $T^{\omega} = \frac{E^{2}}{8\pi}$, $T^{\omega} = T^{2} = +\frac{E}{9\pi}$
c) $T_{\chi\chi} \cdot A = F^{\omega} = F^{\omega} = F^{\omega}$
 $T^{\omega} = \frac{E^{2}}{8\pi} \cdot \sigma$
 $T^{\omega} = \frac{E^{2}}{8\pi} \cdot \sigma$