## PHY 841 HW 8

## Homework Problems 8.8

1. Consider Eq. (8.15) in the case where  $J^{\alpha}$  has no time dependence. Show that one quickly obtains the usual expressions for the potentials in the static cases.

$$A^{\alpha}(x) = \int d^{4}x' \frac{1}{|\vec{x} - \vec{x}'|} J^{\alpha}(x') \delta(x_{0} - x'_{0} - |\vec{x} - \vec{x}'|). \tag{8.15}$$

$$A^{\alpha}(x) = \int d^{4}x' \frac{1}{|\vec{x} - \vec{x}'|} J^{\alpha}(x') \delta(x_{0} - x'_{0} - |\vec{x} - \vec{x}'|). \tag{8.15}$$

Af 
$$\int^{2} has no time dependence, for function integrates to unity  $f$ 

$$A^{2}(x) = \int^{2} d^{3}x \frac{1}{|\vec{x}-\vec{x}'|} \int^{2} (\vec{x}')$$$$

- 2. Using the fact that  $\nabla^2(1/r) = -4\pi\delta^3(\vec{r})$ ,
  - (a) show that any function f(r-t) satisfies the differential equation,

$$\partial^2 \left(rac{f(r-t)}{r}
ight) = 4\pi f(r-t) \delta^3(ec{r}).$$

(b) Now, let  $f(r-t) = \delta(r-t)$ . Show that this satisfies the equation

$$\partial^2 \left( rac{f(r-t)}{r} 
ight) = 0$$

for all t > 0. Also, because r > 0 the function is zero for t < 0.

(c) Show that the form  $f(r-t)=\delta(r-t)$  satisfies the integral of Eq. (8.7).

$$\int_{-\epsilon}^{\epsilon} dt \, \left[ \partial^2 \left( rac{\delta(r-t)}{r} 
ight) 
ight] = 4\pi \int dt \, \delta^4(x).$$

a) 
$$\partial^2 = \partial_r^2 + \frac{2}{r} \partial_r + angular$$
.

$$\frac{\partial^{2}(f)}{\partial z^{2}} = \frac{1}{2} \frac{\partial z^{2}}{\partial z^{2}} + \frac{2}{2} \frac{\partial$$

$$f = f(r-t)$$

$$3(\frac{1}{4}) = \frac{1}{4}(f'' - f'') = 0$$

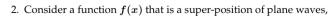
b) From a) 
$$\theta^2 f(r-t) = 0$$
 for all  $t > 0$ 

also = 0 for  $t = 0$ , only non-zero at  $t = 0$ 

$$\int_{-\epsilon}^{\epsilon} dt \left[ \frac{\partial^{2} f(r-\epsilon)}{r} \right] = \frac{\partial^{2} l}{r} = -4\pi \delta^{3}(r)$$

$$= -4\pi \int_{-\epsilon}^{\epsilon} dt \delta^{4}(r)$$

$$\frac{1}{3^2} \frac{f(r-t)}{f} = -4\pi f^4(r^2)$$



$$f(x) = \int dk \ g(k') e^{i\omega(k')t - ik'x + i\phi_0(k')},$$

where g(k') is a narrow function centered about k, e.g.

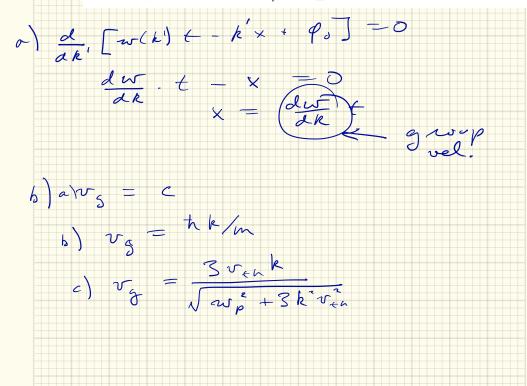
$$g(k') = rac{1}{\sqrt{2\pi a^2}} e^{-(k'-k)^2/2a^2},$$

with a << k.

(a) For a given time t find the position x at which the phase  $[i\omega(k')t-ik'x+i\phi_0(k')]$  is steady as a function of k' at k'=k, i.e.

$$rac{d}{d \emph{k}'} [i \omega(\emph{k}') t - i \emph{k}' x + i \phi_0] = 0.$$

- (b) What are the group velocities for the following cases:
  - a) massless particle in a vacuum,  $\omega = |k|c$
  - b) massive particles in a vacuum,  $\hbar\omega=(\hbar k)^2/2m$
  - c) plasma oscillation,  $\omega^2 = \omega_p^2 + 3k^2v_{
    m th}^2$ .



Show that  $u\cdot a=0$ , where u is the four-velocity and  $a=(d/d\tau)u$  is the acceleration. Then show that  $a_0=\vec u\cdot\vec a/u_0$ .

$$\frac{d}{dt} u^{2} = 0$$

$$= 2 u \cdot \frac{du}{dc} = 2 u \cdot a$$

$$u \cdot a = 0$$

$$u_{0} a_{0} = \dot{u} \cdot \dot{a}$$

$$a_{0} = \ddot{a} \cdot \dot{a} / u_{0}$$

4. Show that the electric field given in Eq. (8.21) is perpendicular to  $\vec{x}$ .

$$E^i \;=\; rac{e}{(u\cdot x)^2}\left\{x^0\left(a^i-rac{u^i(a\cdot x)}{(u\cdot x)}
ight)-x^i\left(a^0-rac{u^0(a\cdot x)}{(u\cdot x)}
ight)
ight\}.$$

$$\vec{E} \cdot \vec{x} = \frac{e}{(\omega \times)^2} \left\{ x_0 \vec{a} \cdot \vec{x} - x_0 \vec{u} \cdot \vec{x} (a \cdot x) - x_0 \vec{u} \cdot \vec{x} (a \cdot x) - x_0 \vec{u} \cdot \vec{x} \right\}$$

$$= \frac{e}{(\omega \times)^3} \left\{ x_0 u \cdot x (\vec{a} \cdot \vec{x}) - x_0 (\vec{u} \cdot \vec{x}) a \cdot x - (\vec{x} \cdot \vec{x}) a \cdot x + (\vec{x} \cdot \vec{x}) a \cdot x + (\vec{x} \cdot \vec{x}) a \cdot x - (\vec{x} \cdot \vec{x}) a \cdot x + (\vec{x} \cdot \vec{x}) a \cdot x - (\vec{x} \cdot \vec{x}) a \cdot x -$$

- (a) Using the fact that  $e^2/(\hbar c)=\alpha$  is dimensionless, show that Eq. (8.25) gives dimensions of energy per time.
- (b) Suppose you had one Coulomb of charge and dropped it off a building, where it accelerated downward with  $g=9.8~\rm m/s^2$ . What power (in W) would be radiated while it fell?

$$P = \frac{2e^2}{3c}|\dot{\vec{\beta}}|^2, (8.25)$$

a) 
$$[t, c]$$
  $[s]$  =  $[e+]$   $[t]$  =  $[e+]$ 

b) 
$$P = \frac{2 + c}{3.137.036} = \frac{(3)^{2}}{(3)^{2}} = \frac{1}{(1.602.10^{-10})^{2}}$$

 $\cline{R}$  , Consider a particle of charge e moving non-relativistically in a synchotron of radius R with the orbit around the z axis, such that

$$x = R\cos\omega t, \ y = R\sin\omega t.$$

- (a) Find  $J_x(\vec{r},t)$  as defined in Sec. 8.7.
- (b) Find  $j_x(\vec{r})$  as defined in Sec. 8.7.
- (c) Find  $p_x$  as defined in Sec. 8.7.
- (d) Using Eq. (8.55), what is the radiated power? Be sure to include contribution from both  $p_x$  and  $p_y$ .
- (e) Compare to the result for a non-relativistic point particle moving in a circle from Eq. (8.37).
- (f) Why should you not apply Eq. (8.55) in the relativistic case?

$$J_{\chi} = -wR \sin wt \cdot e \quad \{(z) \quad \{(\chi - R \cos wt) \quad (g - R \sin wt) \quad (g - R \sin wt) \}$$

$$b) \quad SKIP$$

$$c) \quad S = J_{0} = e \quad S(z) \quad S(\chi - R \cos wt) \quad f(g - R \sin wt) \quad f(g -$$

f) time for hight to traverce system

is not so zty.