PHY 841

HW 9 Solutions

9.2 Homework Problems

1. For Thomson scattering, show that for un-polarized light the angular distribution of the scattered light $\sim (1 + \cos^2\theta)$, where θ is the scattering angle.

Add contributions from both polarizations

$$
\begin{aligned}
\frac{d\Gamma}{d\Omega} &= \frac{1}{8\pi} k^2 |\hat{n} \times \vec{p}|^2 \left(\frac{8\pi}{E_0^2}\right) \qquad (9. \\
\frac{d\sigma}{d\Omega} &= \frac{e^4}{m^2} (\sin^2\theta \sin^2\phi + \cos^2\theta),
\end{aligned}
$$

the other contribution has $\cos^2\varphi$ instead of $\sin^2\varphi$

$$
\left.\frac{d\sigma}{d\Omega}\right|_{\text{unpolarized}} = \frac{1}{2}\frac{e^4}{m^2}\left\{\sin^2\theta\sin^2\varphi + \cos^2\theta + \sin^2\theta\cos^2\varphi + \cos^2\theta\right\}
$$

$$
= \frac{1}{2}\frac{e^4}{m^2}\left\{\sin^2\theta + 2\cos^2\theta\right\}
$$

$$
= \frac{1}{2}\frac{e^4}{m^2}\left[1 + \cos^2\theta\right] \quad \checkmark
$$

2. Consider the limit that $\Gamma \to 0$ in Eq. (9.11). When $\omega \to \omega_0$ the cross section then diverges. Does the contribution to the integrated cross section,

$$I(\omega_a, \omega_b) \equiv \int_{\omega_a}^{\omega_b} d\omega \, \sigma(\omega),$$

where ω_a and ω_b confine the integral to the region surrounding ω_0, diverge as well?

$$\sim \int d\omega \frac{1}{(\omega^2 - \omega_0^2)^2} \quad \text{as } \Gamma \to 0$$

$$\sim \int \frac{d\omega}{4\omega_0^2} \frac{1}{(\omega - \omega_0)^2}$$

Yes.