Dynamics of Relativistic Point Particles – Formulas

The equations of motion for a charged relativistic particle in external electromagnetic fields is given by

$$\frac{d(m\gamma v_i)}{dt} = -e\partial_i\phi_0 - e\partial_t A_i + (\vec{v} \times (\nabla \times \vec{A}))_i$$

Given a potential four vector $\vec{A} = (\phi_0, \vec{A})$, the electric field and magnetic field are given by

$$\vec{E} = -(\nabla\phi_0 + \partial_t\vec{A}) \qquad \vec{B} = \nabla \times \vec{A}$$

The electromagnetic field tensor is defined as

$$F^{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha$$

$$F^{\alpha\beta} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

The equations of motion for a charged relativistic particle in external EM fields can then be written as

$$\frac{dp_\alpha}{d\tau} = eF^{\alpha\beta}u_\beta, \qquad p_\alpha = m\gamma v_\alpha, \qquad \tau = \frac{t}{\gamma}, \qquad u_\beta = \gamma v_\beta$$