

your name(s) _____

Physics 841 Quiz #2 - Monday, Jan. 30

Work in groups of four or fewer. This is open-note, open-book, open-mouth, open-internet, and open-mind.

Turn in one worksheet per group, with all names included.

1. Consider a region with a magnetic field, $A_y = Bx$, which gives a magnetic field in the \hat{z} direction.
 - (a) Consider a boost in the \hat{y} direction by velocity v . Find the new electric and magnetic fields \vec{E}' and \vec{B}' .
 - (b) What is $|\vec{B}'|^2 - |\vec{E}'|^2$?
 - (c) Are there any reference frames in which the magnetic field vanishes?

2. Consider a region with both a magnetic field $\vec{B} = B\hat{z}$ and an electric field $\vec{E} = E\hat{x}$, and $|B| > |E|$.

- (a) Write out the electromagnetic field tensor, $F^{\alpha\beta}$.
- (b) Beginning with the equations

$$m \frac{d}{d\tau} u^\alpha = q F^{\alpha\beta} u_\beta,$$

write the equations of motion for u_x, u_y and u_z , in terms of $d/d\tau$, where τ is the time measured in the frame of the particle. The equations should involve E and B , rather than $F^{\alpha\beta}$. Assume the particle has charge q and mass m .

- (c) Find solutions for $x'(t')$, $y'(t')$ and $z'(t')$, where the primes denote that you are in the frame where there is no electric field. Assume the initial conditions were set up so that $u'_z = 0$ and motion is circular with the center of the circle at the origin, with $x'(t' = 0) = R$, and assume the charge q is positive. Express your answer in terms of R , $B' = \sqrt{B^2 - E^2}$, m , q and τ . Be sure to show how the frequency of the motion depends on R , B' and q .
- (d) Going back to the original frame, where there is also an electric field, find $x(t)$, $y(t)$ and also $t(t')$. (Note it would be difficult to express $x(t)$ and $y(t)$ in closed form.)
- (e) For very large times find $\bar{x}(t)$ and $\bar{y}(t)$ averaged over an oscillation period. I.e. only find the dependence that grows with time. With what velocity does the point $(\bar{x}(t), \bar{y}(t))$ move? How is this answer related to the velocity required to boost away the electric field?