1. You observe two events at space-times coordinates x and y. You also observe Sally moving with four-velocity u. Express the time difference between the events that Sally would observe in terms of invariants involving x, y and u.

x_0 in Sally's frame
$= u \cdot x$
y_0 in Sally's frame
$= u \cdot y$
$\Delta t = u \cdot (y - x)$

2. You observe uniform electric and magnetic fields,

$$\vec{E} = E_x \hat{x} + E_y \hat{y},$$
$$\vec{B} = B_x \hat{x}.$$

The strengths of the fields are such that $B_x < E_x$ and $B_x > E_y$. Answer *TRUE* or *FALSE* to the following questions.

(a) There exists a finite velocity by which you can boost to find a frame where $\vec{B}' = 0$
(b) There exists a finite velocity by which you can boost to find a frame where $\vec{E}' = 0$
(c) If you boost along the z axis, the $\vec{B}$ field will stay the same
(d) If you boost along the z axis, the $\vec{E}$ field will stay the same

a) false
b) false
c) false
d) false

3. Beginning with $F^{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha$, and $\tilde{F}^{\alpha\beta} = (1/2)\epsilon^{\alpha\beta\gamma\delta}F_{\gamma\delta}$, express $F^{\alpha\beta}\tilde{F}_{\alpha\beta}$ in terms of $\vec{E}$ and $\vec{B}$ using

$$\begin{aligned}\vec{E} &= -\nabla A_0 - \partial_t \vec{A}, \\ \vec{B} &= \nabla \times \vec{A}.\end{aligned}$$

Show your work (Don't simply write the answer).

$F^{\alpha\beta}\tilde{F}_{\alpha\beta} = \frac{1}{2}\epsilon_{\alpha\beta\gamma\delta}F^{\alpha\beta}F^{\gamma\delta}$

$= \frac{1}{2}\epsilon_{\alpha\beta\gamma\delta}\partial^\alpha A^\beta \partial^\gamma A^\delta \cdot 4$

must have one kind of each index.

by symmetry, assume α or $\beta = 0$, then double answer to account for cases more γ or $\delta = 0$.

$F^{\alpha\beta}\tilde{F}_{\alpha\beta} = (\partial_t A^i)\,\epsilon_{ijk}\partial^i A^k \cdot 4$

$-(\partial^i A^0)\,\epsilon_{ijk}\partial^j A^k \cdot 4$

$= (-E^i)\,\epsilon_{ijk}\partial^j A^k \cdot 4 = -4\,E^i B^i$

$= -4\,\vec{E}\cdot\vec{B}$