your name(s)____

Physics 841 Quiz #6 - Wednesday, Mar. 6

You may work in groups of 3 (no more than one person from previous group)

Closed note, closed mouth, closed book, open mouth...

Consider a particle of charge e and mass m moving in a circular orbit.

1. Remembering that the magnetic moment is
$$\vec{m} = \frac{1}{2}\int d^3r'\ \vec{r}' \times \vec{J}(\vec{r}'),$$
Derive the constant of proportionality γ such that
$$\vec{m} = \gamma\vec{\ell},$$
where $\vec{\ell}$ is the angular momentum.

2. Remembering that the torque acting on a magnetic moment in a constant magnetic field $\vec{B}$ is
$$\vec{\tau} = \vec{m} \times \vec{B},$$
find equations of motion for ℓ_x, ℓ_y and ℓ_z when $\vec{B}$ is in the $\hat{z}$ direction.

3. At $t = 0$ assume the magnitude of the angular momentum is L and that the direction of the $\vec{\ell}$ is given by
$$\vec{\ell}(t = 0) = L\cos\theta_0\hat{z} + L\sin\theta_0\hat{x}.$$
Find $\vec{\ell}$ as a function of time.

4. For electrons the intrinsic magnetic moment is $\vec{\mu} = (ge\hbar/2mc)(\vec{s}/\hbar)$. If there is no orbital angular momentum, describe the motion of $\vec{s}$ as a function of time in a constant magnetic field, $\vec{B} = B\hat{z}$. If the magnetic field has a strength of 4 T, find the precession frequency $f_p = \omega_p/(2\pi)$ in Hz.

Solutions:

1.

$$\begin{aligned}
\int Jd^2r &= I,\\
m &= (1/2)IR(2\pi R),\\
I &= \frac{ev}{2\pi R}\\
m &= \frac{evR}{2},\\
\ell &= mvR\\
m &= \frac{e}{2m}\ell,\\
\gamma &= \frac{e}{2m}.
\end{aligned}$$

2.

$$
\begin{aligned}
\vec{\tau} &= -\vec{B} \times \vec{m} \\
&= -\gamma \vec{B} \times \vec{\ell} \\
\frac{d\ell}{dt} &= \tau \\
&= -\gamma \vec{B} \times \vec{\ell}, \\
\frac{d\ell_x}{dt} &= \gamma B \ell_y, \\
\frac{d\ell_y}{dt} &= -\gamma B \ell_x, \\
\ell_z &= \text{constant.}
\end{aligned}
$$

3.

$$
\begin{aligned}
\ell_z &= L \cos\theta_0, \\
\ell_x &= L \sin\theta_0 \cos\omega t, \\
\ell_y &= -L \cos\theta_0 \sin\omega t, \\
\omega &= \gamma B.
\end{aligned}
$$

4. Same as #3, $\gamma = e/m$, $\omega = eB/m$. To get units of inverse time using evB as force, one needs

$$
\begin{aligned}
\omega_p &= \frac{eB}{m} \\
f_p &= 1.1196 \times 10^{11} \text{ Hz}
\end{aligned}
$$