your name(s)____

Physics 841 Quiz #9 - Wednesday, April 5

The accelerator upgrade at Jefferson Laboratory will provide electron beams with energies of 12 GeV at currents near 100 μA.

1. What is the power of the beam? (in Watts)
2. If the beam passes through a dipole magnet with a 6T magnetic field for a distance of 75 cm, what is the power radiated into photons?
3. If the beam travels through a series of these magnets with oppositely oriented magnetic fields of this strength, and separated by 1.25 m, estimate the net path length required to reduce the beam energy by 99%.

Solutions:

a) Power of beam

$$\begin{aligned} P &= 12 \text{ GeV } \cdot\ 100 \times 10^{-6} \text{ A} \\ &= 1.2 \times 10^{6} \text{ W} \end{aligned}$$

b) Power radiated in one magnet: First consider power radiated by one electron while in magnet:

$$\begin{aligned} P_e &= \frac{2e^2\gamma^4}{3c}\dot{\beta}^2 \\ \gamma &= \frac{12 \times 10^6}{0.511} = 2.35 \times 10^4, \\ m\frac{du_y}{d\tau} &= eBu_x = eB\gamma c, \\ m\gamma\frac{dv_y}{dt/\gamma} &= eB\gamma c, \\ \dot{\beta} &= \frac{eB}{m\gamma} \\ \frac{eB}{m} &= 1.055 \times 10^{12} \ \ \text{s}^{-1}, \\ P_e &= \frac{2e^2\gamma^2}{3c}\left(\frac{eB}{m}\right)^2 \\ \frac{eB}{m} &= 1.055 \times 10^{12} \ \ \text{s}^{-1} \\ \frac{2e^2}{3c} &= 5.130 \times 10^{-37} \text{ J s} \\ P_e &= 3.15 \times 10^{-4} \text{ W.} \end{aligned}$$

Power of N electrons in magnet is

$$\begin{aligned} N &= \frac{I}{e}\frac{L}{c} = 1.562 \times 10^6, && (1) \\ P &= P_e N && (2) \\ &= 492 \text{ W.} && (3) \end{aligned}$$

Solve for energy loss rate,

$$\begin{aligned} E &= mc^2\gamma, && (4) \\ \dot{\gamma} &= -\frac{2e^2\gamma^2}{3mc^3}\left(\frac{eB}{m}\right)^2, && (5) \\ \dot{\gamma} &= -A\gamma^2, \quad A = 6.977 \ \ \mathrm{s}^{-1}. && (6) \end{aligned}$$

For $\gamma_f = 0.01\gamma_i$,

$$\begin{aligned} At &= \int_{t_i}^{t_f} d\gamma/\gamma^2, \\ t &= \frac{99}{A\gamma_i} \\ &= 6.04 \times 10^{-4} \text{ s.} \end{aligned}$$

Now find net length, which is 2.0/0.75 * length,

$$\begin{aligned} L &= \frac{2c}{0.75}t \\ &= 483 \text{ km.} \end{aligned}$$