$$
\begin{aligned}
\vec{a}\times(\vec{b}\times\vec{c}) &= \vec{b}(\vec{a}\cdot\vec{c})-\vec{c}(\vec{a}\cdot\vec{b}),\\
\vec{a}\cdot(\vec{b}\times\vec{c}) &= \vec{b}\cdot(\vec{c}\times\vec{a})=\vec{c}\cdot(\vec{a}\times\vec{b}),\\
(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d}) &= (\vec{a}\cdot\vec{c})(\vec{b}\cdot\vec{d})-(\vec{a}\cdot\vec{d})(\vec{b}\cdot\vec{c}),\\
\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\psi) &= 0,\\
\boldsymbol{\nabla}\cdot(\boldsymbol{\nabla}\times\vec{a}) &= 0,\\
\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\vec{a}) &= \boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\vec{a})-\nabla^2\vec{a},\\
\boldsymbol{\nabla}\cdot(\psi\vec{a}) &= \vec{a}\cdot\boldsymbol{\nabla}\psi+\psi\boldsymbol{\nabla}\cdot\vec{a},\\
\boldsymbol{\nabla}\times(\psi\vec{a}) &= \boldsymbol{\nabla}\psi\times\vec{a}+\psi\boldsymbol{\nabla}\times\vec{a},\\
\boldsymbol{\nabla}(\vec{a}\cdot\vec{b}) &= (\vec{a}\cdot\boldsymbol{\nabla})\vec{b}+(\vec{b}\cdot\boldsymbol{\nabla})\vec{a}+\vec{a}\times(\boldsymbol{\nabla}\times\vec{b})+\vec{b}\times(\boldsymbol{\nabla}\times\vec{a}),\\
\boldsymbol{\nabla}\cdot(\vec{a}\times\vec{b}) &= \vec{b}\cdot(\boldsymbol{\nabla}\times\vec{a})-\vec{a}\cdot(\boldsymbol{\nabla}\times\vec{b}),\\
\boldsymbol{\nabla}\times(\vec{a}\times\vec{b}) &= \vec{a}(\boldsymbol{\nabla}\times\vec{b})-\vec{b}(\boldsymbol{\nabla}\times\vec{a})+(\vec{b}\cdot\boldsymbol{\nabla})\vec{a}-(\vec{a}\cdot\boldsymbol{\nabla})\vec{b},\\
\boldsymbol{\nabla}\cdot\vec{r} &= 3,\\
\boldsymbol{\nabla}\times\vec{r} &= 0,\\
\boldsymbol{\nabla}\cdot\hat{r} &= 2/r,\\
\boldsymbol{\nabla}\times\hat{r} &= 0,\\
(\vec{a}\cdot\nabla)\hat{r} &= \frac{1}{r}[\vec{a}-\hat{r}(\vec{a}\cdot\hat{r})]=\frac{\vec{a}_\perp}{r}.
\end{aligned}
$$

$$
\begin{aligned}
\int_V d^3r\ \boldsymbol{\nabla}\cdot\vec{A} &= \int_S d\vec{S}\cdot\vec{A},\\
\int_V d^3r\ \boldsymbol{\nabla}\psi &= \int_S \psi d\vec{S},\\
\int_V d^3r\ \boldsymbol{\nabla}\times\vec{A} &= \int_S d\vec{S}\times\vec{A},\\
\int_V d^3r\ (\phi\nabla^2\psi+\boldsymbol{\nabla}\phi\cdot\boldsymbol{\nabla}\psi) &= \int_S \phi(d\vec{S})\cdot\boldsymbol{\nabla}\psi,\\
\int_V d^3r\ (\phi\nabla^2\psi-\psi\nabla^2\phi) &= \int_S (\phi\boldsymbol{\nabla}\psi-\psi\boldsymbol{\nabla}\phi)\cdot d\vec{S},\\
\int_S (\boldsymbol{\nabla}\times\vec{A})\cdot d\vec{S} &= \oint d\vec{\ell}\cdot\vec{A},\\
\int_S (d\vec{S})\times\boldsymbol{\nabla}\psi &= \oint_C d\vec{\ell}\ \psi.
\end{aligned}
$$

$$
\begin{aligned}
\nabla^2 &= \partial_r^2+\frac{2}{r}\partial_r-\frac{\ell(\ell+1)}{r^2},\\
\nabla^2 &= \partial_\rho^2+\frac{1}{\rho}\partial_\rho-\frac{m^2}{r^2},\\
\nabla^2\left(\frac{1}{r}\right) &= -4\pi\delta(\vec{r}).
\end{aligned}
$$

$$
L^\alpha{}_\beta = \begin{bmatrix} \gamma & \gamma v & 0 & 0\\ \gamma v & \gamma & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}.
$$

$$
\begin{aligned}
F_{\alpha\beta} &= \begin{pmatrix} 0 & E_x & E_y & E_z\\ -E_x & 0 & -B_z & B_y\\ -E_y & B_z & 0 & -B_x\\ -E_z & -B_y & B_x & 0\end{pmatrix},\\
F^{\alpha\beta} &= \begin{pmatrix} 0 & -E_x & -E_y & -E_z\\ E_x & 0 & -B_z & B_y\\ E_y & B_z & 0 & -B_x\\ E_z & -B_y & B_x & 0\end{pmatrix}.
\end{aligned}
$$

$$
\begin{aligned}
m\frac{d}{d\tau}u^\alpha &= eF^{\alpha\beta}u_\beta,\\
\frac{d\vec{p}}{dt} &= e\vec{E}+e\vec{v}\times\vec{B},\\
\omega_c &= \frac{eB}{\gamma m}.
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{\nabla}\cdot\vec{E} &= 4\pi J^0,\\
(\boldsymbol{\nabla}\times\vec{B})-\partial_t\vec{E} &= 4\pi\vec{J},\\
\boldsymbol{\nabla}\cdot\vec{B} &= 0,\\
\partial_t\vec{B}+\boldsymbol{\nabla}\times\vec{E} &= 0,\\
\partial_\alpha F^{\alpha\beta} &= 4\pi J^\beta,\\
\partial_\alpha\tilde{F}^{\alpha\beta} &= 0.
\end{aligned}
$$

$$
e^2 = \frac{\hbar c}{137.036},
$$

$$
\begin{aligned}
T^{\alpha\beta} &= \pi^\alpha\partial^\beta\phi-g^{\alpha\beta}\mathcal{L},\\
\pi^\alpha &\equiv \frac{\partial\mathcal{L}}{\partial(\partial_\alpha\phi)}
\end{aligned}
$$

$$
\begin{aligned}
T^{00} &= \frac{1}{8\pi}(E^2+B^2),\\
T^{0i} &= \frac{1}{4\pi}\epsilon_{ijk}E_jB_k,\\
T^{ij}=-T^i{}_j &= \frac{1}{8\pi}(\delta_{ij}(E^2+B^2)-2E_iE_j-2B_iB_j).
\end{aligned}
$$

$$
\begin{aligned}
\vec{E} &= -\boldsymbol{\nabla} A_0 - \partial_t \vec{A} = -\boldsymbol{\nabla}\Phi - \partial_t\vec{A}, \quad \vec{B} = \boldsymbol{\nabla}\times\vec{A}, \\
P_\ell(\cos\theta) &= \sqrt{\frac{4\pi}{2\ell+1}} Y_{\ell m=0}(\theta), \\
Y_{0,0} &= \frac{1}{\sqrt{4\pi}}, \quad Y_{1,0} = \sqrt{\frac{3}{4\pi}}\cos\theta, \\
Y_{1,\pm 1} &= \mp\sqrt{\frac{3}{8\pi}}\sin\theta e^{i\pm\phi}, \quad Y_{2,0} = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1), \\
Y_{2,\pm 1} &= \mp\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta e^{\pm i\phi}, \quad Y_{2,\pm 2} = \sqrt{\frac{15}{32\pi}}\sin^2\theta e^{\pm 2i\phi}, \\
Y_{\ell -m}(\theta,\phi) &= (-1)^m Y^*_{\ell m}(\theta,\phi), \\
\delta_{\ell\ell'}\delta_{mm'} &= \int d\Omega\; Y_{\ell,m}(\theta,\phi) Y_{\ell',m'}(\theta,\phi), \\
P_0(x) &= 1, \quad P_1(x) = x, \\
P_2(x) &= \frac{1}{2}(3x^2-1), \quad P_3(x) = \frac{1}{2}(5x^3 - 3x), \\
P_\ell(x=1) &= 1, \quad \int_{-1}^{1} dx\; P_\ell(x) P_{\ell'}(x) = \frac{2}{2\ell+1}\delta_{\ell\ell'}, \\
(3D)\quad \Phi &= \sum_{\ell m}\left(A_{\ell m} r^\ell + B_{\ell m} r^{-\ell-1}\right) Y_{\ell m}(\theta,\phi) e^{im\phi}, \\
(2D)\quad \Phi &= A_0\ln(\rho) + \sum_m e^{im\phi}\left(A_m r^m + B_m r^{-m}\right), \\
\Phi &= A_0 J_0 = \sum_m e^{im\phi}\left(A_m J_m(k\rho) + B_m N_m(k\rho)\right) e^{\pm kz}, \\
\Phi &= \frac{q}{r} + \frac{\vec{p}\cdot\vec{r}}{r^3} + \sum_m \frac{4\pi}{5r^3} q_{2m}(r) Y_{2m}(\theta,\phi), \\
\vec{E} &= -\frac{1}{r^3}\vec{p} + 3\frac{\vec{p}\cdot\vec{r}}{r^5}\vec{r} + \cdots, \\
\Phi(r,\theta,\phi) &= \sum_{\ell m}\frac{4\pi}{(2\ell+1)r^{\ell+1}} q_{\ell m}(r) Y_{\ell m}(\theta,\phi), \\
q_{22} &= \sqrt{\frac{15}{32\pi}}\int d^3r\; \rho(\vec{r})(x-iy)^2 = \sqrt{\frac{15}{288\pi}}(Q_{11} - 2iQ_{12} - Q_{22}), \\
q_{21} &= -\sqrt{\frac{15}{8\pi}}\int d^3r\; \rho(\vec{r})(x-iy)^2 = -\sqrt{\frac{15}{72\pi}}(Q_{13} - iQ_{23}), \\
q_{20} &= \sqrt{\frac{5}{16\pi}}\int d^3r\; \rho(\vec{r})(3z^2 - r^2) = \sqrt{\frac{5}{16\pi}} Q_{33}, \\
Q_{ij} &\equiv \int d^3r\; (3r_i r_j - r^2\delta_{ij})\rho(\vec{r}), \\
U &= q\Phi_0 - \vec{p}\cdot\vec{E} - \frac{1}{6}Q_{ij}\partial_i E_j,
\end{aligned}
$$

$$
\begin{aligned}
\nabla^2 A^\alpha &= -4\pi J^\alpha, \\
\vec{m} &= \frac{1}{2}\int d^3r\; \vec{r}\times\vec{J} = \frac{I}{2}\int\; \vec{r}\times d\vec{\ell}, \\
\vec{B} &= -\frac{\vec{m}}{r^3} + \frac{3\vec{r}}{r^5}(\vec{m}\cdot\vec{r}), \\
\mu_e &= g_e\frac{e\hbar}{2m_e}, \\
U &= \frac{(\vec{\mu}_N\cdot\vec{\mu}_e)}{r^3} - \frac{3(\vec{\mu}_N\cdot\vec{r})(\vec{\mu}_e\cdot\vec{r})}{r^5} - \frac{8\pi}{3}(\vec{\mu}_N\cdot\vec{\mu}_e)\delta^3(\vec{r}) \\
&\quad -e\frac{(\vec{\mu}_N\cdot\vec{L})}{mr^3}, \\
T_{00} &= \frac{1}{8\pi}\left(|\vec{E}|^2 + |\vec{B}|^2\right) \\
&= \frac{a_i^2 + b_i^2}{8\pi} = \frac{|\vec{a}|^2}{4\pi}\cos^2(\vec{k}\cdot\vec{r} - \omega t), \\
T_{0i} &= \epsilon_{ijk}\frac{E_j B_k}{4\pi} \\
&= \hat{k}_i\frac{|\vec{a}|^2}{4\pi}\cos^2(\vec{k}\cdot\vec{r} - \omega t), \\
T^{ij} = -T^i{}_j &= \frac{1}{8\pi}(\delta_{ij}(E^2 + B^2) - 2E_iE_j - 2B_iB_j) \\
&= \frac{1}{4\pi}\left\{|\vec{a}|^2\delta_{ij} - a_ia_j - b_ib_j\right\}\cos^2(\vec{k}\cdot\vec{r} - \omega t)
\end{aligned}
$$

$$
\omega_s = \omega\sqrt{\frac{1-v}{1+v}}
$$

$$
\begin{aligned}
(TM)\quad E_z &= \psi(x,y)e^{-i\omega t + ik_z z}, \\
\Psi &= 0 \quad \text{at boundary}, \\
\vec{E}_t(x,y) &= \frac{ik_z}{(\omega^2 - k_z^2)} e^{-i\omega t + ik_z z}\boldsymbol{\nabla}_t\psi(x,y), \\
\vec{B}_t(x,y) &= \left(\frac{\omega}{k_z}\right)\hat{z}\times\vec{E}_t, \\
(TE)\quad B_z &= \psi(x,y)e^{-i\omega t + ik_z z}, \\
(\hat{n}\cdot\boldsymbol{\nabla}_t)\psi(x,y)|_S &= 0, \\
\vec{B}_t(x,y) &= \frac{ik_z}{(\omega^2 - k_z^2)} e^{-i\omega t + ik_z z}\boldsymbol{\nabla}_t\psi(x,y), \\
\vec{E}_t(x,y) &= -\left(\frac{\omega}{k_z}\right)\hat{z}\times\vec{B}_t.
\end{aligned}
$$

$$
\begin{aligned}
A^\alpha(x) &= \int d^4x' \, \frac{1}{|\vec{x}-\vec{x}'|} J^\alpha(x')\delta(x_0 - x_0' - |\vec{x}-\vec{x}'|), \\
\vec{E} &= e\left\{ \frac{\hat{n}\times[(\hat{n}-\vec{\beta})\times\dot{\vec{\beta}}]}{(1-\vec{\beta}\cdot\hat{n})^3|\vec{x}|} \right\}, \\
\vec{B} &= \hat{n}\times\vec{E}.
\end{aligned}
$$

$$
\begin{aligned}
P &= \frac{2e^2}{3c}|\dot{\vec{\beta}}|^2 \quad \text{(Non.Rel.)}, \\
\frac{dP}{d\Omega} &= \frac{e^2}{4\pi(1-\vec{\beta}\cdot\hat{n})^6}|(\hat{n}-\vec{\beta})\times\dot{\vec{\beta}})|^2, \\
P &= \frac{2}{3c}e^2\gamma^6\left[\dot{\beta}^2 - |\vec{\beta}\times\dot{\vec{\beta}}|^2\right], \\
\frac{dP}{d\Omega} &= \frac{e^2}{4\pi(1-\beta\cos\theta)^5}|\dot{\vec{\beta}}|^2\sin^2\theta \quad \text{(linear)}, \\
P &= \frac{2e^2\dot{\beta}^2}{3c}\gamma^6 \quad \text{(linear)}, \\
\frac{dP}{d\Omega} &= \frac{e^2}{4\pi(1-\beta n_\beta)^5}|\dot{\vec{\beta}}|^2\left((1-\beta n_\beta)^2 - (1-\beta^2)n_r^2\right) \quad \text{(circular)}, \\
P &= \frac{2}{3c}e^2\dot{\beta}^2\gamma^4 \quad \text{(circular)}.
\end{aligned}
$$

$$
\begin{aligned}
\frac{dP}{d\Omega} &= \frac{1}{8\pi}k^2|\hat{n}\times\vec{p}|^2, \\
P &= \frac{\omega^4}{3}|\vec{p}|^2,
\end{aligned}
$$

$$
\begin{aligned}
\text{(Thomson)} \quad \sigma &= \frac{8\pi e^4}{3m^2}, \\
\frac{\Delta\lambda}{\lambda} &= \frac{\hbar\omega}{m}(1-\cos\theta_s).
\end{aligned}
$$

electron	$-2.00231930436182 \pm 0.00000000000052$
muon	$-2.0023318418 \pm 0.0000000013$
proton	$5.585694702 \pm 0.000000017$
neutron	$-3.82608545 \pm 0.00000090$