Problem Set #16

due Friday, Jan. 26.

PHYSICS 852, SPRING 2001

1. The temperature at the center of the sun is 15 million degrees Kelvin. Consider two protons with a relative kinetic energy characteristic of the temperature,

\[
\frac{\hbar^2 k^2}{2\mu} = \frac{3}{2} kT.
\]

(a) What is the Gamow penetrability factor? Give a numeric value.

(b) If the two particles were a proton and a \(^{12}\text{C}\) nucleus, what would the penetrability factor become?

2. Consider two oscillator levels described by the creation operators, \(a_1^\dagger\) and \(a_2^\dagger\), where the Hamiltonian is

\[
H = \epsilon_1 a_1^\dagger a_1 + \epsilon_2 a_2^\dagger a_2 + \beta (a_1^\dagger a_2^\dagger + a_1 a_2).
\]

Find the values of \(\eta\), \(E_0\), \(E_1\) and \(E_2\) such that the operators

\[
\begin{align*}
b_1^\dagger &\equiv \cosh \eta \; a_1^\dagger + \sinh \eta \; a_2 \\
b_2^\dagger &\equiv \cosh \eta \; a_2^\dagger + \sinh \eta \; a_1
\end{align*}
\]

allow \(H\) to be written as

\[
H = E_0 + E_1 b_1^\dagger b_1 + E_2 b_2^\dagger b_2.
\]