Problem Set #17

due Friday, Feb. 2.

PHYSICS 852, SPRING 2001

1. Consider $b$-particles of mass $m$ confined by one-dimensional harmonic oscillator characterized by a frequency $\omega$. The $b$ particles interact with massless and spinless $a$-particles through their respective field operators,

$$H_{\text{int}} = g \int dx \Phi^\dagger(x) \Phi(x) \Psi(x),$$

where $\Phi$ and $\Psi$ are the field operators for the $a$-particles and $b$-particles respectively.

$$\Phi(x) = \frac{1}{\sqrt{L}} \sum_k \frac{1}{\sqrt{2E_k}} \left( e^{ikx} a_k^\dagger + e^{-ikx} a_k \right)$$

$$\Psi^\dagger(x) = \frac{1}{\sqrt{L}} \sum_k e^{ikx} b_k^\dagger$$

(a) What are the dimensions of $g$?
(b) What is the decay rate of a $b$ particle in the first excited state.

2. A spinless particle of mass $m$ and charge $e$ is in the first excited state of a three-dimensional harmonic oscillator characterized by a frequency $\omega$. Assume the harmonic oscillator in the Cartesian state with $n_z = 1$. Using the interaction

$$H_{\text{int}} = \mathbf{j} \cdot \mathbf{A}/c,$$

(a) Calculate the decay rate of the charged particle into the ground state of the oscillator in the dipole approximation.
(b) Calculate $d\Gamma/d\Omega$ as a function of the emission angles of the photon, $\theta$ and $\phi$.
(c) In terms of the unit vectors $\hat{k}$, $\hat{\theta}$ and $\hat{\phi}$, write the two polarization vectors which are allowed for emission of a photon at an angle $\theta$, $\phi$.
(d) For each polarization vector above, calculate $d\Gamma_s/d\Omega$, the probability of decaying via emission of a photon emitted in the $\theta$, $\phi$ direction with polarization $s$.

3. Again consider a spinless particle of mass $m$ and charge $e$ in the first excited state of a three-dimensional harmonic oscillator characterized by a frequency $\omega$. However, this time assume the charged particle is originally in a state with angular momentum projection $m = +1$ along the $z$ axis. Using the interaction

$$H_{\text{int}} = \mathbf{j} \cdot \mathbf{A}/c,$$

and applying the dipole approximation,

(a) Find the decay rate $\Gamma$ of the first excited state.
(b) Find the differential decay rate $d\Gamma/d\Omega$. 